Knowledge in Physics through Mathematics, Image and Language

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Declaration

I certify that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any university; and that to the best of my knowledge and belief it does not contain any material previously published or written by another person where due reference is not made in the text.

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Abstract

This thesis explores the nature of knowledge in physics and the discourse that organises it. In particular, it focuses on the affordances of mathematics, image and language for construing the highly technical meanings that constitute this knowledge. It shows that each of these resources play a crucial role in physics' ability to generate generalised theory whilst maintaining relevance to the empirical physical world.

First, to understand how mathematics contributes to knowledge-building, the thesis presents a detailed descriptive model from the perspective of Systemic Functional Semiotics that considers mathematics on its own terms. The description builds on O'Halloran's (2005) grammar in order to understand mathematics' intrinsic functionality and theoretical architecture. In doing so, it takes an axial perspective (Martin 2013) that considers the paradigmatic and syntagmatic axes in Systemic Functional theory as the theoretical primitives from which metafunction, strata, rank and all other theoretical categories can be derived. It shows that, when not transposing categories from English but rather deriving them from axial principles, mathematics' theoretical architecture is considerably different to that of any resource previously seen. Looking metafunctionally, mathematics displays a highly elaborated logical component within the ideational metafunction, but shows no evidence for a discrete interpersonal metafunction. Looking at the levels within the grammar, it displays two interacting hierarchies: a rank scale based on constituency and a nesting scale based on iterative layering. Finally, it shows distinct and predictable texts patterns in its interaction with language. From this, the description is able to use genre as a unifying semiotic that strongly predicts the grammatical patterns that occur throughout physics discourse. By developing these models, the thesis offers an understanding of mathematics' unique functionality and the reasons it is consistently used in physics.

Second, the thesis interprets the images of physics from the perspective of the Systemic Functional dimension of field. It shows that much of the power of images comes from the large number of distinct meanings that can be encapsulated in a single snapshot. In one image, large taxonomies, long sequences of activity, extensive arrays of data and various levels of specificity can all be presented. This allows various components of physics' knowledge to be related and coordinated, and aids physics in building a coherent and integrated knowledge structure.

Following the descriptive component of the thesis, the specific functionalities of mathematics, image and language are interpreted through the Legitimation Code Theory dimension of Semantics. This provides an understanding of the organisation of physics' knowledge structure as a whole. It shows how the interaction of mathematics, language and image underpins physics' ability to progressively build ever more elaborated technical meanings, to make empirical predictions from theoretical models and to abstract theoretical generalisations from empirical data. By interpreting the mathematics, image and language used in physics from the complementary perspectives of Systemic Functional Semiotics and Legitimation Code Theory, the thesis offers a detailed model of how physics manages to make sense of and predict the vast physical world.

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Over the last few years I've read numerous acknowledgement sections in various theses and books. Some have been long, some have been short, some have been genuine and some have been generic. But it is only now that I'm realising how difficult it is to adequately encapsulate in a couple of paragraphs the gratitude I feel toward so many people. So I will try to keep it simple.

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For my mum, Sue, and my dad, Carl

Contents

List of Figures

List of Tables

CHAPTER 1

Physics, Knowledge and Semiosis

"Physics is hard." Remarks such as these have been heard by teachers and students of physics innumerable times. Physics of course has its own object of study, its own ways of organising its knowledge and its own ways of expressing its knowledge. In this sense, it is its own unique discipline. But this does not mark physics as different from any other academic subject; every discipline has its intricacies and idiosyncrasies, and every subject has its detractors and its devotees. Nonetheless, physics seems to be regularly positioned as an exceptional case in the academic world. It is often said to be the most fundamental of the sciences, one upon which all others are based (e.g. Feynman et al. 1964, Young and Freedman 2012); this perhaps can be taken to mean that it shares many of the characteristics of the others sciences, but also maintains its own distinctive features. Biglan (1973), for example, classifies physics as a pure science, along with geology, chemistry and botany, but he positions it as the 'hardest' of the pure sciences. Kolb (1981) characterises it as a reflective (non-applied) discipline, like geography, bacteriology and biochemistry, but he portrays it as the most 'abstract' of the reflective disciplines. And those following Bernstein (1999) identify it with other natural sciences as a discipline that develops generalised theories and integrates empirical phenomena, but they regularly use physics as the exemplar of such a discipline (Maton and Muller 2007, O'Halloran 2007a, Martin 2011b).

There is thus a sense that physics is both a natural science, and as such shares many of the features of the natural sciences; but at the same time physics is in some sense the most 'sciencey' of the natural sciences. Exactly how this recurrent characterisation of physics arises, however, is not clear. We might even ask whether it is truly the case that physics maintains a special position within the sciences? And if so, what gives rise to this special position? Questions such as these go to the heart of the disciplinary organisation of physics, and so are not born of idle curiosity. They hold strong significance for the development of educational programs that acknowledge and target disciplinary knowledge. If disciplines vary in the way they organise their knowledge, vary in the discourse they use to construe this knowledge and vary in the means of judging and comparing competing knowledges, the pedagogic approach for teaching these disciplines must take this into account.

1.1 Knowledge and education

In response to the disciplinary nature of knowledge, the last few decades have seen the development of an influential educational linguistics program, known as 'Sydney School' genre pedagogy.[1] This approach arises from the linguistic theory generally referred to as Systemic Functional Linguistics (hereafter SFL) and specifically targets knowledge differences across the disciplinary spectrum. The program develops explicit pedagogy across all areas of schooling and aims to ensure access for all students regardless of their background. In order to do this, it addresses the specialised ways each subject organises its knowledge, as well as the literacy practices that are associated with it; this is instead of offering a generic pedagogy that generalises across disciplinary differences (for an introduction to this pedagogy, see Rose and Martin 2012).

Sydney school genre pedagogy developed out of research into the types of texts students need to read and write across subject areas in primary (elementary) and high school. These projects were known as the *Writing Project*, *Language and Social Power*, and *Write it Right* projects (for an overview of these projects, see Veel 2008 and Rose and Martin 2012; for a collection of foundational papers in SFL educational linguistics, see Martin and Doran 2015e). This research showed that each subject regularly utilised only a small set of text types to organise their disciplinary knowledge. Science, for example, involved factual texts known as reports and explanations that were geared toward 'content' knowledge. These genres built taxonomies that organise phenomena in terms of classification and composition, and established sequences of processes these phenomena were involved with (Martin and Rose 2008, Met. East DSP 1995a). Visual arts, on the other hand, more commonly required texts that gave a student's response to or evaluation, interpretation or critique of an artwork (Met. East DSP 1994, 1995b). This necessitated students develop the ability to judge a pre-existing artwork or the process that led to its creation. With the difference in text type came concomitant differences in the language used. For example, where visual art's evaluative responses required students to marshal a broad range of evaluative language to appreciate the

[1] As Martin (2006a) and Rose and Martin (2012), who were two of the key developers of the Sydney School, acknowledge, this name is misleading, since by the time the name was first used (Green and Lee 1994) the development of the program was by no means confined to Sydney, having spread across Australia as far as practice, innovation and development were concerned.

artwork, judge the artist or express their emotional responses, scientific texts were relatively non-attitudinal[2] (see Martin and White 2005 for a detailed exploration of the evaluative resources in English). Over time, research in this tradition has expanded its breadth to include a large range of subject areas, and in doing so has progressively elaborated a picture of the varying literacy demands placed on students across the curriculum.

With significant variation in each subject's literacy requirements comes differences in each discipline's knowledge itself. What is accepted as valid knowledge and the means for judging competing knowledges in one discipline is typically very different to that of another discipline. In developing a discipline-sensitive pedagogy these variations need to be carefully considered. However, as Maton (2014) argues, despite knowledge-building being at first sight the *raison d'être* of education, educational research tends to have a blind-spot when it comes to actually seeing knowledge; like language, knowledge structure is often taken for granted. This 'knowledge-blindness' means that the principles underpinning the various educational and literacy practices of disciplines have frequently not been made explicit for teachers and students.

Rather than considering knowledge as an object of study, Maton argues that education tends to reduce knowledge to knowing. In physics education (and science education in general) this tends to be grounded in cognitive models of student understanding that foreground the varying ways students conceptualise and frame the knowledge of physics (see diSessa 2006 for an overview of research of this kind). These models have been important in drawing attention to the fact that students do not come into physics (or indeed any discipline) with a clean slate, but rather maintain intuitive conceptions of much of the phenomena physics aims to teach more technically. However by focussing primarily on the student as the framer of knowledge, this tradition of research often obscures how knowledge is structured in the discipline in general (see Maton 2014, Georgiou et al. 2014). These models thus fail to formulate the underlying principles that underpin why a discipline is how it is, how it can progress and what forms of knowledge need to be taught to students for them to be successful.

If we wish to develop a discipline-sensitive pedagogy these structuring principles of knowledge must be understood. We need a way of answering, for example, why it is that the type of writing in English literature is not appropriate in science, or why the methods of investigation of science are not utilised in English. Moreover, if we wish this pedagogy to

[2] See however Hood on appraisal in scientific writing (e.g. 2010).

inform the reading students do to learn the knowledge and the writing they produce to show they have this knowledge, we need a method for understanding this knowledge in terms of the language and other resources used in each discipline. That is, we need to understand knowledge semiotically.

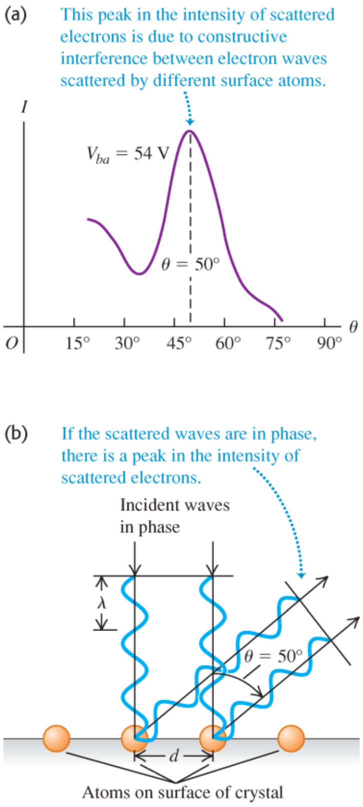
This thesis takes a step toward interpreting knowledge in physics from a semiotic perspective. It considers how physics is organised to develop a coherent and multifaceted knowledge structure, and how this knowledge is construed and distributed across language and other semiotic resources such as mathematics and image. Not only does this give insights into how physics works for the sake of physics education, but given physics' nominally special position within the academic world, it allows an understanding of one of the 'poles' in the cline from the science and humanities. It thus broadens our understanding of academic knowledge in general.

1.2 Mathematics, images and language in physics

To investigate how physics manages its apparently special knowledge structure, this thesis examines its discourse in classrooms, in textbooks and in student work. What is immediately apparent when considering this discourse is the large emphasis on mathematics and images throughout almost all contexts. Figure 1.1 shows a typical page from a university physics textbook that involves three images and numerous equations permeating the page.

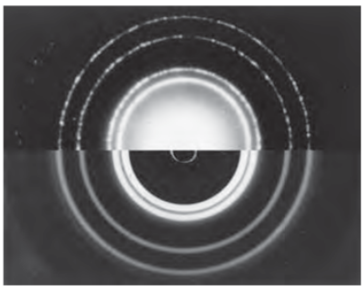
1288 CHAPTER 39 Particles Behaving as Waves

39.3 (a) Intensity of the scattered electron beam in Fig. 39.2 as a function of the scattering angle θ. (b) Electron waves scattered from two adjacent atoms interfere constructively when $d \sin\theta = m\lambda$. In the case shown here, $\theta = 50°$ and $m = 1$.



39.4 X-ray and electron diffraction. The upper half of the photo shows the diffraction pattern for 71-pm x rays passing through aluminum foil. The lower half, with a different scale, shows the diffraction pattern for 600-eV electrons from aluminum. The similarity shows that electrons undergo the same kind of diffraction as x rays.

Top: x-ray diffraction



Bottom: electron diffraction

During the experiment an accident occurred that permitted air to enter the vacuum chamber, and an oxide film formed on the metal surface. To remove this film, Davisson and Germer baked the sample in a high-temperature oven, almost hot enough to melt it. Unknown to them, this had the effect of creating large regions within the nickel with crystal planes that were continuous over the width of the electron beam. From the perspective of the electrons, the sample looked like a *single* crystal of nickel.

When the observations were repeated with this sample, the results were quite different. Now strong maxima in the intensity of the reflected electron beam occurred at specific angles (Fig. 39.3a), in contrast to the smooth variation of intensity with angle that Davisson and Germer had observed before the accident. The angular positions of the maxima depended on the accelerating voltage V_{ba} used to produce the electron beam. Davisson and Germer were familiar with de Broglie's hypothesis, and they noticed the similarity of this behavior to x-ray diffraction. This was not the effect they had been looking for, but they immediately recognized that the electron beam was being *diffracted.* They had discovered a very direct experimental confirmation of the wave hypothesis.

Davisson and Germer could determine the speeds of the electrons from the accelerating voltage, so they could compute the de Broglie wavelength from Eq. (39.1). If an electron is accelerated from rest at point a to point b through a potential increase $V_{ba} = V_b - V_a$ as shown in Fig. 39.2, the work done on the electron eV_{ba} equals its kinetic energy K. Using $K = (\frac{1}{2})mv^2 = p^2/2m$ for a nonrelativistic particle, we have

$$eV_{ba} = \frac{p^2}{2m} \qquad p = \sqrt{2meV_{ba}}$$

We substitute this into Eq. (39.1), the expression for the de Broglie wavelength of the electron:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV_{ba}}} \qquad \text{(de Broglie wavelength of an electron)} \quad (39.3)$$

The greater the accelerating voltage V_{ba}, the shorter the wavelength of the electron.

To predict the angles at which strong reflection occurs, note that the electrons were scattered primarily by the planes of atoms near the surface of the crystal. Atoms in a surface plane are arranged in rows, with a distance d that can be measured by x-ray diffraction techniques. These rows act like a reflecting diffraction grating; the angles at which strong reflection occurs are the same as for a grating with center-to-center distance d between its slits (Fig. 39.3b). From Eq. (36.13) the angles of maximum reflection are given by

$$d \sin\theta = m\lambda \qquad (m = 1, 2, 3, \ldots) \qquad (39.4)$$

where θ is the angle shown in Fig. 39.2. (Note that the geometry in Fig. 39.3b is different from that for Fig. 36.22, so Eq. (39.4) is different from Eq. (36.16).) Davisson and Germer found that the angles predicted by this equation, using the de Broglie wavelength given by Eq. (39.3), agreed with the observed values (Fig. 39.3a). Thus the accidental discovery of **electron diffraction** was the first direct evidence confirming de Broglie's hypothesis.

In 1928, just a year after the Davisson–Germer discovery, the English physicist G. P. Thomson carried out electron-diffraction experiments using a thin, polycrystalline, metallic foil as a target. Debye and Sherrer had used a similar technique several years earlier to study x-ray diffraction from polycrystalline specimens. In these experiments the beam passes *through* the target rather than being reflected from it. Because of the random orientations of the individual microscopic crystals in the foil, the diffraction pattern consists of intensity maxima forming rings around the direction of the incident beam. Thomson's results again confirmed the de Broglie relationship. Figure 39.4 shows both x-ray and electron diffraction

Figure 1.1 Mathematics, images and language in a university textbook (Young and Freedman 2012: 1288)

This page is typical of physics texts throughout the data set used for this study (described in Section 2.5 of Chapter 2 and Appendix B). Indeed Parodi (2012), in his quantitative study of textbooks across multiple academic disciplines, suggests that, like other sciences, physics regularly utilises images such as graphs and diagrams to present information, and, of the basic sciences, it is by far the most reliant on mathematics. Based on these findings, Parodi suggests that physics is the most predominately graphic-mathematical of the disciplines he studied. This amplifies the characterisation of physics by Biglan as the 'hardest' of the pure sciences and by Kolb as the most abstract of the non-applied disciplines, and reinforces its exceptional form. Parodi's study is backed up by Lemke's (1998) survey of articles in the prestigious physics research journal *Physical Review Letters*. Within this corpus, Lemke found that on average, around four images and equations occurred per page (2.7 equations, 1.2 images); this is significantly higher than the rate of images and equations in the corresponding journal for the biological, earth and space sciences, *Science,* or for medicine, *Bulletin of the New York Academy of Medicine* (Lemke 1998: 89).[3] Images and equations are thus clearly a regular feature of the discourse of physics.

In order to understand the discourse of physics, then, it is necessary to comprehend the full range of resources involved – language, mathematics and images, as well as gesture, demonstration apparatus, various symbolic formalisms and numerous others. This thesis moves in this direction by considering mathematics and images in relation to language as crucial components of the discourse and knowledge of physics. It thus offers a more exhaustive analysis of physics texts than would be possible if our gaze was restricted to language. In addition, a detailed study of each resource makes it possible to understand why each is used. The pervasiveness of each resource throughout physics across a broad range of levels in schooling and in research suggests that each plays a crucial role in developing the knowledge of physics. By taking each resource seriously and considering their roles in detail, we can begin to understand their functionality for organising this knowledge. More specifically, we can investigate whether the particularly predominant use of mathematics and images plays any role in the distinctive knowledge structure that physics maintains.

[3] In the case of *Science*, the non-linguistic resources used were primarily images, with only a handful of articles using equations. As such, *Science* contained slightly more images per page than *Physical Review Letters*. This is echoed by Parodi's study, that found chemistry and biotechnology involved more images than physics (but fewer equations). However, importantly, both studies found that images are nonetheless still standard elements of physics discourse, alongside equations.

Before we can investigate these resources, however, we need a common method for understanding them. Beginning with scientific language, we note that it has a long history of research in linguistics. For example in the tradition of Systemic Functional Linguistics and the agnate approach of Social Semiotics, there have been several decades of research into its peculiarities (e.g. Huddleston et al. 1968, Lemke 1990, Halliday and Martin 1993, Martin and Veel 1998, Halliday 2004). These studies and the educational programs developed from them have been based upon elaborate and wide ranging descriptions of language developed by Halliday, Martin and colleagues (consolidated in Halliday and Greaves 2008 (phonology), Halliday and Matthiessen 2014 (lexicogrammar), Martin 1992a (discourse semantics), Martin and Rose 2008 (register and genre)). These descriptions were not necessarily developed with science or education explicitly in mind (though some did arise in relation to these considerations), but were built with an eye to the range of variation across different contexts and the extrinsic functionality that language serves. They have proven immensely useful to many researchers who wish to study the language of science in addition to its other applications. These descriptions are being continually developed and improved as further avenues of research come to light.

In recent decades, descriptions drawing on SFL have been developed for modalities of communication alongside language. Kress and van Leeuwen's (1990) and O'Toole's (1994) grammars of images, and O'Halloran's (1996, 2005) description of mathematics have widened our gaze and allowed us to understand these resources as meaning-making systems in their own right (see Chapter 2 for a detailed discussion). These and related descriptions have fostered the growing field of multimodality and have encouraged scholars to take seriously the roles of extra-linguistic semiotic resources. Some of these descriptions, however, require further development in order to achieve the comprehensiveness and robustness of the descriptions of English mentioned above. For example, O'Halloran's description of mathematics has yet to be fully systematised and the range of variation in mathematical symbolism has not yet been fully mapped. In addition for images, there still remain significant areas yet to be fully developed, including a more thorough exploration of abstract graphs and the highly complex diagrams used in the sciences to explain physical phenomena. Before we can understand the functionality of these resources and their role in building the knowledge in physics, we need an understanding of what they can and cannot do in various contexts. This requires thorough descriptions to map the choices available and the typical text patterns in each context.

One challenge we need to face is that the various models of semiotic resources produced to date (whether focused on language, image, mathematics, gesture, film, sound or space), rarely begin from the same starting point. Each description tends to make its own assumptions and develops according to its own criteria. Moreover, with the exception of linguistic studies, descriptions tend to have little explicit discussion of the principles guiding them. If we are looking to compare the functionality of various resources and determine why it is they are used, this becomes an issue. With different starting points and different methods of development, it is often difficult to determine whether similarities or differences apparent in resources are due to the intrinsic nature of these resources or simply due to the informing theory and descriptive methodology. Just as the classification of species in biology or the explanation of phenomena in physics needs to be based on systematic principles, so does the description and comparison of resources in semiotics. By way of facing this challenge, this thesis proposes principles for description based on systemic functional theory that provide a basis for systematic and thorough descriptions of semiotic resources. These principles offer a method through which descriptions can bring out each resource's intrinsic and unique functionality, rather than assuming categories developed from other resources. They will also underpin a detailed description of mathematical symbolism that explores the range of variation and the detailed potential available across contexts. And they will be used to generate larger theoretical architecture that allows a comparison of the broader functions of resources.

This thesis thus incorporates two components. The first is descriptive: it develops a comprehensive formalised description of mathematical symbolism both at the micro level of its grammar and at the broader level of text patterns arising from its interaction with language. In doing so, it illustrates a set of descriptive principles that can be applied widely as a shared basis for semiotic description. In addition, it considers both images and mathematics in terms of the meanings they can make and their unique functionality in the discourse of physics. The second component of the thesis uses these descriptions to investigate the knowledge structure of physics. It shows how physics manages to build abstract theory while maintaining its grasp on the empirical world, and highlights the crucial role that language, mathematics and images each play in this. The thesis thus investigates the knowledge of physics by examining the semiotic resources that organise it.

1.3 Organisation of the thesis

This thesis is organised into six chapters. Following the foundations chapter (Chapter 2), Chapters 3-5 progressively build the descriptive component of the thesis by steadily widening its gaze. Chapter 3 begins with a monomodal study of mathematics; Chapter 4 considers mathematics and language together; and Chapter 5 brings images into the picture to contrast their role with that of mathematics and language. As the descriptive lens broadens, the thesis increasingly focuses on the overall knowledge of physics and the role each resource plays in its structure. The final chapter brings each component of the thesis together to underscore the interplay between the descriptive and knowledge-building foci, as well as their implications for the broader field of semiotics and our understanding of knowledge.

Chapter 2: Theoretical and Descriptive Foundations

Chapter 2 establishes the foundations for both the descriptive and knowledge-structure components of the thesis. In doing so, it introduces the two main theoretical frameworks that inform the study: Systemic Functional Linguistics (SFL) (or more broadly, Systemic Functional Semiotics) and Legitimation Code Theory (LCT). First, it presents the theoretical architecture of SFL and its conception of scientific language in terms of the concept of *field*. Second, it introduces a tradition within the sociology of education that has taken knowledge seriously as an object of study in its own right, and will consider how it has positioned physics. This tradition emanates from the work of Bernstein (e.g. 1999), and has been developed by the increasingly influential Legitimation Code Theory (LCT). Third, it will introduce the Systemic Functional descriptions of image and mathematics that form the platform upon which the descriptions in this thesis build. And finally, it offers a detailed account of some of the issues that have arisen in trying to systematise these descriptions and presents the descriptive principles and methodology that underpin the thesis. The main point of this section will be that many of large-scale theoretical categories often assumed in semiotic description can be derived from a single dimension of Systemic Functional theory, known as axis. This orientation accordingly grounds a methodology for testing whether these categories are indeed appropriate for any particular semiotic resource.

Chapter 3: A Grammar of Mathematical Symbolism

Based on the principles spelled out in Chapter 2, Chapter 3 builds a systematised and comprehensive grammar of mathematical symbolism. This grammar shows that mathematics is based on a series of recursive systems where each choice can be repeated indefinitely. Its corresponding structural realisation is thus also indefinitely iterative. This produces a significantly different system to that commonly seen in language, or indeed in any semiotic resource so far described. In particular, it will show that there are two different hierarchies of units involved (known as ranks and nestings) that are based on different types of structure. Further, it will illustrate that the broad functional divisions in the grammar (known as metafunctions) are different from those in language (which are generally posited as common across all semiosis). By building a grammar based on the principles described in Chapter 2, the description is able to show the unique functionality of mathematics in comparison to language and other semiotic resources.

Chapter 4: Genres of Mathematics and Language

Chapter 4 considers the broader text patterns associated with mathematical symbolism and language, known as genre. It will show that, in general terms, physics involves two types of mathematical genre. These genres have their own systems and their own structures. However like the categories in mathematics' grammar these genres can be repeated indefinitely to produce large and complex texts. Each genre will be shown to strongly coordinate with different components of the grammar, meaning that the text patterns are largely predictable from the texts' overall purpose. The model of genre is based on the same descriptive principles as those of the grammar (given in Chapter 3), which will also be used to show the usefulness of a generalised model of genre that unifies both primarily mathematical and primarily linguistic genres. Finally, the chapter uses the models of grammar and genre to trace the development of mathematics from primary (elementary) school, through high (secondary) school and into university physics. The development will be interpreted in terms of the Legitimation Code Theory dimension of Semantics (introduced in Chapter 2) and used to explore mathematics role in building the knowledge-structure of physics.

Chapter 5: Images and the Knowledge Structure of Physics

Chapter 5 turns its focus to images to examine the distinct meanings they make for physics in comparison to mathematics and language. It does this by considering each resource from the perspective of SFL's concept of field. This allows the discussion of images and mathematics to relate to the model of scientific language in SFL that has typically been framed in terms of field (discussed in Chapter 2). In doing so, it shows the unique affordances of each resource for construing knowledge and offers an explanation for why each is used in physics. When framed in terms of Legitimation Code Theory's dimension of Semantics, the chapter will show the crucial role each resource plays in allowing physics to develop generalised theory and keep this theory in touch with the empirical world.

Chapter 6: Multisemiosis and the Knowledge Structure of Physics

Chapter 6 consolidates the thesis by bringing together the threads that have arisen in the previous chapters. First, it compares and contrasts the affordances of mathematics, language and image, before discussing a model of physics' knowledge as a whole. Second, it reflects on the ramifications of the model of mathematics developed in this thesis for claims about the pervasiveness of metafunctions across all semiosis. In addition, it considers the possibility of the SFL concepts of genre and register as a method for unifying the diverse semiotic resources that occur in discourse. And finally, it looks ahead to argue for the development of a general semiotic typology that can make explicit the parameters of variation and contexts of use of various semiotic resources that permeate human culture.

The underlying motivation for this thesis is educational. However the models developed are not tied exclusively to educational contexts. Rather, they have been developed with an eye to broader appliability, and, most importantly, with theoretical integrity and descriptive rigour in mind. By maintaining these, we can step further toward a generalised theory of semiosis that is appliable across both education and the wider world.

CHAPTER 2

Theoretical and Descriptive Foundations

Physics knowledge is multifaceted. It involves extensive and complex relations between innumerable elements that describe, explain and predict the physical world. Similarly, the discourse that organises this knowledge is equally as multifaceted. It involves language, mathematics, image, nuclear symbolism, gesture, demonstration apparatus and many other semiotic resources that each bring their own functionality and their own particular construal of knowledge. To understand how physics organises its knowledge, it is important to understand the roles these various semiotic resources play. For physics, the most ubiquitous of these resources are language, mathematics and image (Parodi 2012). They are consistently used throughout all fields of physics, they occur across much of schooling and research, and they have done so for many centuries (see O'Halloran 2005: Chapter 2; and papers by Copernicus, Galileo, Kepler, Newton, Planck, Einstein and others in Hawking 2002, 2011). With such consistent use it is fair to suggest that each resource plays a distinct and crucial role in building physics' knowledge.

Understanding physics' knowledge, therefore, involves seeing the functionalities of each semiotic resource and relating this functionality to the knowledge of physics in general. In this thesis, the functionality of mathematics, language and image for organising physics knowledge will be viewed through the complementary perspectives of Systemic Functional Linguistics (SFL) and Legitimation Code Theory (LCT). One the one hand, Systemic Functional Linguistics offers a theory and descriptive apparatus for viewing the intricate knowledge-building potential of each semiotic resource, as well as their actual use in text. On the other hand, Legitimation Code Theory presents a theorisation of the structure of physics knowledge itself by making manifest the organising principles that underpin this knowledge and its discourse. The two approaches offer complementary perspectives of the intricate functionality of physics discourse and the semiotic resources that constitute it, as well as the broader structuring principles of physics knowledge that coordinate it.

Systemic Functional Linguistics has had a long-standing concern for scientific discourse. Its encounters with scientific language date back to Huddleston et al.'s (1968) grammar of sentences and clauses in scientific English and have provided one stimulus for Halliday's seminal work on grammatical metaphor (as consolidated in Halliday 2004). In recent decades, SFL's focus on science has been intertwined with its deep concern with education and literacy, resulting in a series of books and papers probing the nature of scientific discourse across various educational contexts (e.g. Halliday and Martin 1993, Martin and Veel 1998, Rose et al. 1992, Christie and Martin 1997 – often in interaction with the agnate social semiotic perspective, e.g. Lemke 1990). In connection with its educational focus, SFL has also often interacted with sociology developing out of the work of Basil Bernstein (e.g. Christie 1999). The most illuminating connection for understanding the nature of physics is the recent work on the structure of knowledge (e.g. Christie and Martin 2007, Christie and Maton 2011) During this more recent phase of interaction, varying forms of knowledge across academic disciplines have been explored through close collaboration between SFL and LCT (Maton 2014). During this phase, LCT and SFL have developed in creative tension with each other, with each approach pushing the others' explanatory framework and expanding their horizons (for the history of the interaction between SFL and this tradition of sociology known as code theory see Maton and Doran in press 2017, Maton et al. 2015, Martin 2011b).

At the same time as SFL's focus on science developed and its interaction with code theory entered a new phase, it became actively involved in the development of multimodal discourse analysis. The concern with multimodality nudged linguistics to look outside language and consider a broad range of meaning-making resources. The field began in earnest with the seminal studies of images by Kress and van Leeuwen (1990) and O'Toole (1994), followed quickly by descriptions of various other resources such as physical action (Martinec 1998, 2000, 2001), sound (van Leeuwen 1999) and, importantly for this thesis, mathematical symbolism (O'Halloran 1996). This tradition has highlighted the importance of non-linguistic semiotic resources for organising meaning both in everyday life and in specialised academic discourse, including science. Physics in particular has been shown to be heavily reliant on mathematics and images for its discourse, potentially more so than most academic disciplines (Parodi 2012). If we wish to understand how physics organises its knowledge, therefore, it is important that we develop a rich appreciation of the role mathematics and images play alongside language in its discourse. In pursuing this goal, the developments in multimodality over the last three decades have prepared a solid foundation upon which this thesis can stand.

This chapter establishes the foundations that underpin this thesis. First, it reviews the relevant theoretical architecture of SFL and considers scientific discourse from the perspective of SFL's concept of field. Second, it explores how knowledge is structured across academic disciplines, and introduces the tradition of code theory that has culminated in Legitimation Code Theory. Third, it introduces developments in multimodality that inform this study and raises some key issues concerning the description and comparison of non-linguistic semiotic resources. Finally, it outlines the data set used for this thesis.

2.1 Systemic Functional Linguistics

Systemic Functional Linguistics is a multifaceted theory of language with a large interconnected architecture. It considers language in its social context and describes it in terms of both the functions it plays and the possible options available in any situation. It is by far the most elaborated component of the broader field of Systemic Functional Semiotics and is the basis for most of the theoretical principles and descriptive mechanics in Systemic Functional theory. As will be argued in Section 2.4, most Systemic Functional descriptions of semiotic resources other than language have taken their lead from the work on English by Halliday and his colleagues, beginning in the 1960s. It is thus important to understand the architecture of language from the perspective of Systemic Functional Linguistics in order to understand how accounts of non-linguistic semiotic resources have developed. This section will focus on four main dimensions of language in SFL: stratification, metafunction, axis and rank, before considering scientific language from the perspective of field in Section 2.2. Each of these dimensions is crucial to the model of mathematics developed in Chapters 3 and 4 and the role it plays alongside language and image in Chapter 5.

2.1.1 Stratification

SFL views language as a stratified system arranged on a cline of abstraction (e.g. Halliday 1985). Under this model, language contains a content plane which is divided into the strata of discourse semantics and lexicogrammar. Lexicogrammar is concerned with meanings made within a clause, whereas discourse semantics is concerned with meanings made through entire texts (beyond the clause). In addition, language contains an expression plane that realises the meanings made by discourse semantics and lexicogrammar, of either phonology (for spoken language) or graphology (for written language) (following Martin's 1992a model). Every instance of language necessarily makes a choice from all strata, and every stratum contributes its own meanings. The result of this is that choices made in discourse semantics are realised by choices in lexicogrammar, and choices in lexicogrammar are realised in phonology/graphology. The stratal organisation of language is typically represented through cotangential circles, as shown in Figure 2.1.

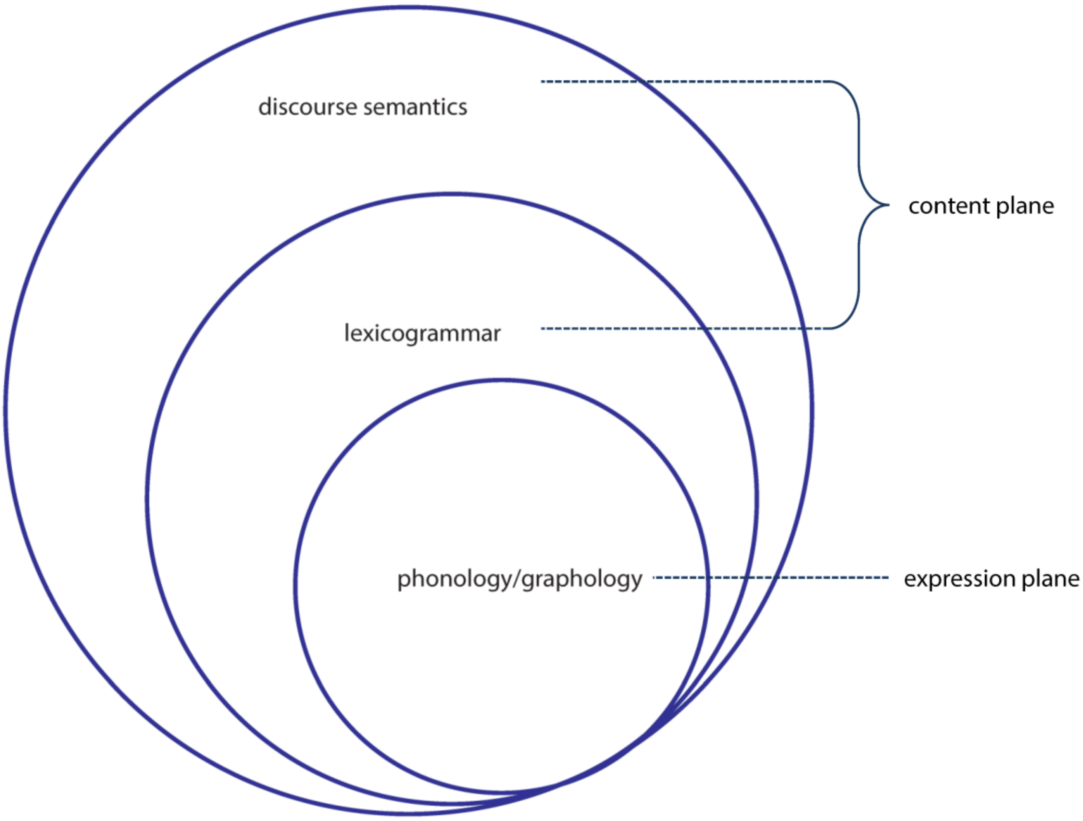


Figure 2.1 Strata of language (following Martin 1992a)

The way we use language changes depending on the context. In order to account for this, SFL proposes further strata above language. In Martin's model (1999), two further strata, register and genre, coordinate choices in language. Genre is the highest stratum and describes a text's global social purpose (Rose and Martin 2012). Genres tend to unfold in distinct stages and coordinate the meanings at the stratum below, termed register. Register encompasses three variables: field, tenor and mode (Martin 1992a). Field is concerned with what is happening in a social activity, tenor is concerned with the relationship between participants and mode is concerned with the role language plays in the situation (Martin and Rose 2008). Figure 2.2 shows register and genre in relation to the strata of language.

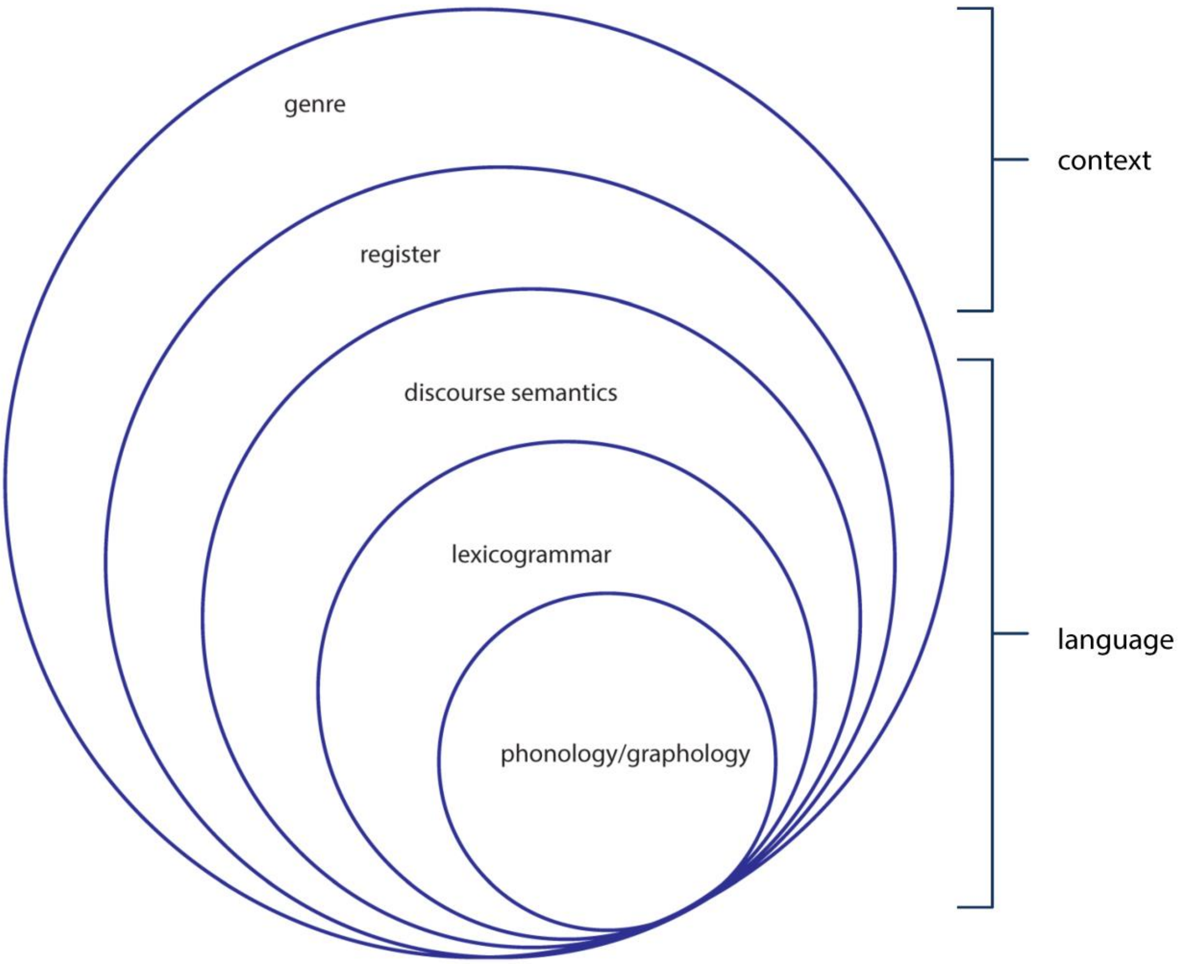


Figure 2.2 Strata of language and context (following Martin 1999)

Crucially for this thesis, genre and register are not strata of language. Rather, they are semiotic systems in their own right that are expressed through language. In Hjelmslev's (1943) terms, genre and register are connotative semiotics, while language is a denotative semiotic (Martin 1992a). As connotative semiotics, genre and register have language as their expression plane, whereas language, as a denotative semiotic, has its own expression plane – phonology or graphology. This connotative/denotative distinction is important for Chapters 4 and 5 below where mathematics, language and image are viewed from the perspective of a shared system of genre (Chapter 4, involving mathematical symbolism and language only) and field within register (Chapter 5, which deals with mathematical symbolism, language and image). By positing a semiotic system above language (genre and register), the regular uses of mathematics, language and image in physics can be explained.

2.1.2 Metafunction

SFL proposes that language makes three broad types of meaning: ideational, interpersonal and textual meanings (Halliday 1969, 1973, 1985). Ideational meanings construe the outside world, interpersonal meanings organise our social relationships and textual meanings organise language in terms of information flow and salience. Ideational meanings are further distinguished into two subtypes: experiential meanings that construe our experience of the outside world and logical meanings that specify general iterative relations between different elements of this experience. These different types of meanings are referred to as metafunctions and are reflected in different components of the grammatical systems of languages.

The metafunctional distribution of meanings in language coordinates with the tripartite distinction of register into field, tenor and mode (Halliday 1970a, 1978b). Field, as the variable concerned with the ‘content’ of what is being said, tends to coordinate ideational meanings. Tenor, as the variable concerned with the social relations encoded through language, tends to coordinate interpersonal meanings. Mode, as concerned with the role of language in any situation, tends to coordinate textual meanings. This relation between register and the metafunctions of language relates extrinsic functionality to intrinsic functionality of language: it suggests that language is as it is because of the functions it has evolved to do (Martin 1991). This metafunction-register hook-up is often configured as in Figure 2.3.

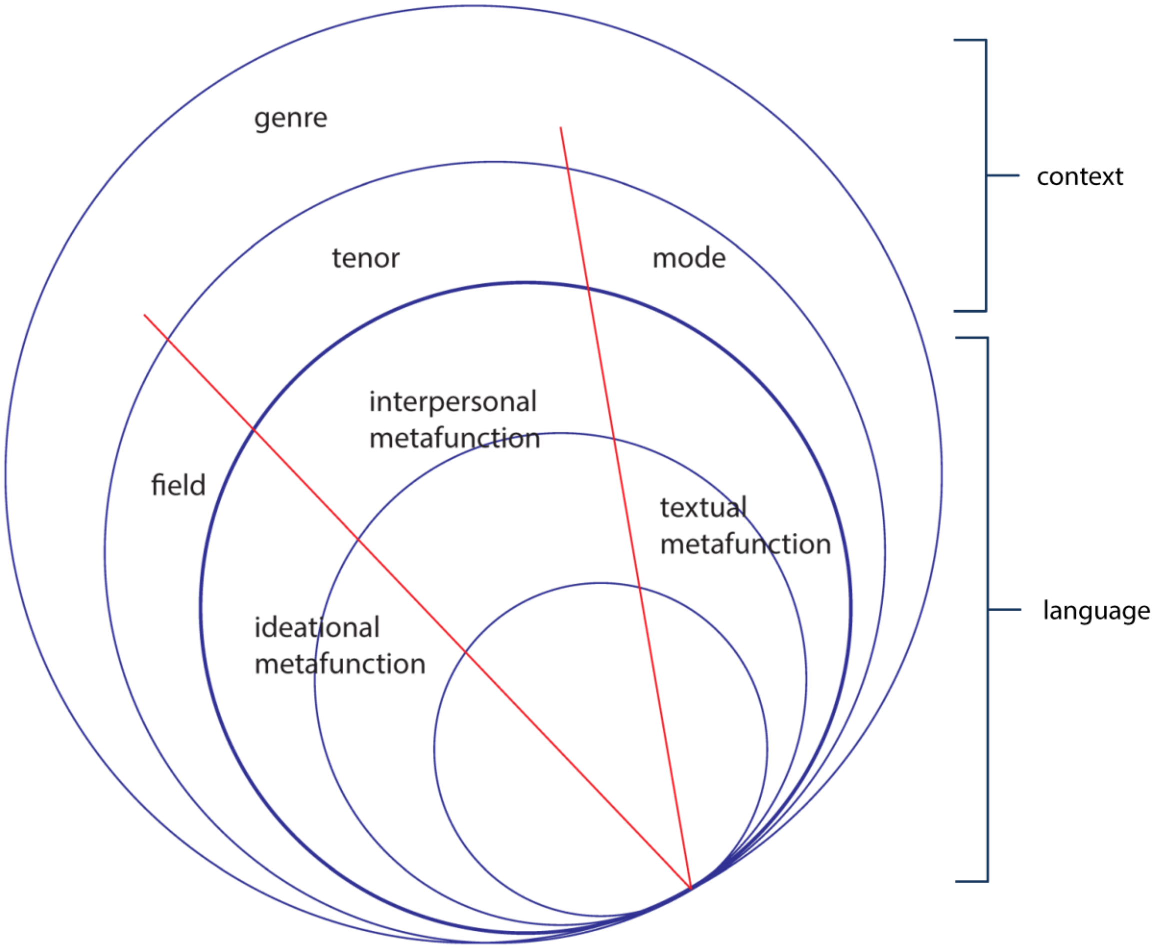


Figure 2.3 Metafunctional relations between language, register and genre.

Halliday's notion of metafunction arises from his work on the grammatical organisation of language where each component is made up of relatively discrete bundles of systems. As Halliday (1978b: 187) puts it, '*within* one component there is a high degree of interdependence and mutual constraint, whereas *between* components there is very little: each one is relatively independent of the others.' In English, this is shown through the relatively independent systems of MOOD (interpersonal), THEME (textual) and TRANSITIVITY (ideational) (Halliday 1967a, b, 1968, 1969, Martin 1983). Across these systems in general terms, a choice in one system can be combined with any choice in the others. In Section 2.4.4 this systemic independence will be used to show how metafunctions can be derived from another theoretical category termed axis.

In addition to their paradigmatic independence, Halliday (1979) suggests that metafunctions also tend to be realised by particular types of structure. Ideational meanings tend to be realised by discrete particulate structures wherein each constituent function is clearly distinguished from the others and plays a complementary role in the structure. For example the TRANSITIVITY system involves Participants such as Actor, Goal etc, where the boundaries between each are relatively clear-cut and each tends to occur only once (Halliday and Matthiessen 2014). Interpersonal meanings, on the other hand, are realised by prosodic structures that cut across units. For example negation in English clauses is realised across the Finite and any subsequent indefinite deixis in other elements, e.g. *I wo<u>n't</u> eat <u>any</u> more* (c.f. the positive *I will eat more*) (Martin 2008). Finally, textual meanings are realised by periodic structures whereby text is ordered into peaks and troughs of informational salience. At clause rank, this is represented in English through the thematic prominence at the beginning of the clause (labelled Theme) and the intonationally marked newness prominence typically placed at the end of the clause (labelled New) (Halliday and Matthiessen 2014). Figure 2.4 represents these three types of structure for the same clause.

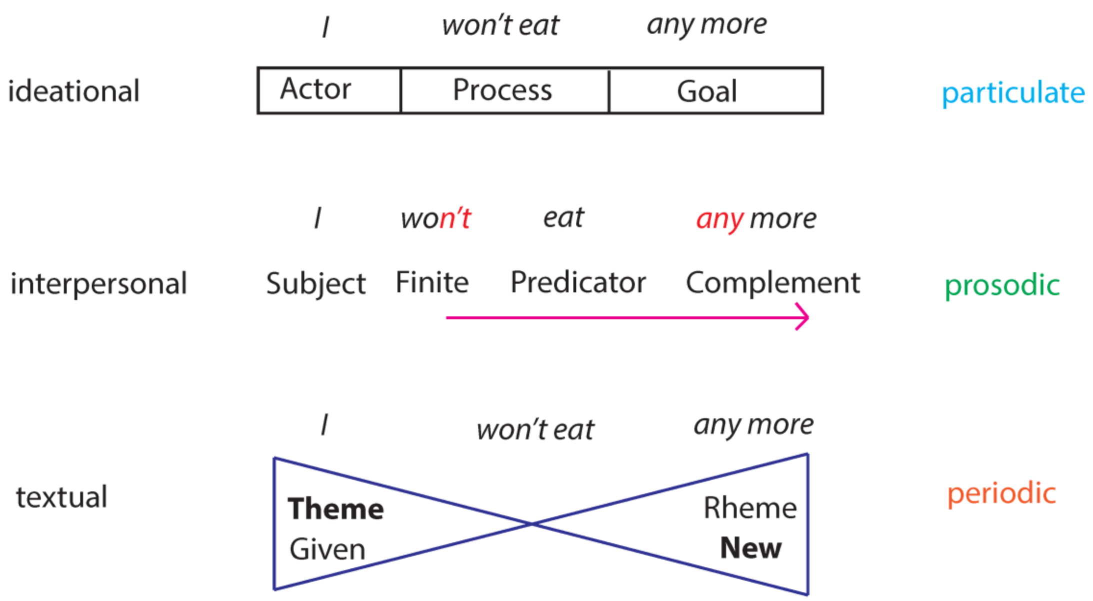


Figure 2.4. Types of structure for each metafunction

For this thesis, the most pertinent structures are particulate structures. These are associated with the ideational and field-based meanings developed in physics, and form the core of the grammatical organisation of mathematics (see Chapter 3). For the discussion it is important to further distinguish two subtypes of particulate structure, which Halliday (1965) calls multivariate and univariate structures. These types of structure are associated with the two components of the ideational metafunction – multivariate with the experiential component, univariate with the logical component (Halliday 1979).

Multivariate structures involve multiple variables that normally occur only once (Halliday 1965). For example, the transitivity structure of the English clause *We saw him* is a multivariate structure involving three functions in sequence Senser (*We*) ^ Process (*saw*) ^ Phenomenon (*him*) (Halliday and Matthiessen 2014).[4] In contrast, univariate structures involve a single variable which can be repeated (often indefinitely). For example, the clause complex <u>*The Cronulla Sharks bought Michael Ennis this year*</u> *and* <u>*next year they have James Maloney*</u> involves two clauses (underlined) linked by *and* (realising paratactic extension, Halliday and Matthiessen 2014). In principle, any number of elements could be related in this way, creating very complex sequences. Univariate structures can either be paratactic, where each element is of the same status (such as the clause complex above), or hypotactic where one element is dependent on another. An example of a hypotactic structure is the clause complex *after buying Ennis, Cronulla bought Maloney*. In this complex, the clause *buying Ennis* is dependent on *Cronulla bought Maloney* in the sense that it cannot occur on its own.

It is common in systemic functional descriptions for multiple structures to be mapped onto the same unit. For example, the English clause in Figure 2.4 above has structures arising from TRANSITIVITY (Actor^Process^Goal), MOOD (Subject^Finite^Predicator^Complement), THEME (Theme^Rheme) and INFORMATION (Given^New). An example involving both multivariate and univariate structures arises in the English nominal group. In this case a hypotatic univariate structure arising from the logical metafunction is mapped onto a multivariate structure from the experiential metafunction. Considered hypotactically, the nominal group *the second Saints album*, for example, can be seen simply as a series of words modifying the head *album*. Following the conventions in Halliday and Matthiessen 2014, *album* is labelled α and is modified by *Saints* which is labelled β, which is further modified by *second* (γ) and finally by *the* (δ), giving a structure δ (*the*) ^ γ (*second*) ^ β (*Saints*) ^ α (*album*). Complementing this, each word can be viewed multivariately as performing a distinct function in a set sequence and with specific possibilities for variation. Under this analysis, *the* is a Deictic, *second* is a Numerative, *Saints* is a Classifier and *album* is a Thing

[4] Sequence is indicated by a caret ^ . Non-sequenced functions are indicated by a dot •

(Halliday and Matthiessen 2014). This analysis attends to the strict ordering of the words; in English these functions do not occur in a sequence such as *second the album Saints*. Both the multivariate and univariate analyses bring out different features of the variation that is possible for each element and so provides a rich interpretation of the English nominal group. For this example the full multifunctional analysis is shown in Table 2.1.

	the	*second*	*Saints*	*album*
univariate	δ	γ	β	α
multivariate	Deictic	Numerative	Classifier	Thing

Table 2.1 Univariate and multivariate analysis of an English nominal group

In addition to multivariate and univariate structures, Martin (1992a) offers a third type of particulate structure to account for discourse relations. These structures account, for example, for relations deriving from taxonomies where multiple elements are mutually dependent on each other. Martin's example considers the relation between *robot* and *model* in the short text:

> *I'm not pleased with this robot, but that model looks fine.* (1992a: 24)

In this example, *model* is a superordinate of *robot* (that is, *robot* is a type of *model* in this case). This sets up a cohesive tie in the text, whereby *model* and *robot* enter into the same taxonomy and thus hold some relation in the text. This relation is of mutual interdependence; *model* is as dependent on *robot* for its role in helping the text cohere as *robot* is on *model*. Following Lemke 1985, Martin terms these relations covariate structures.

The multivariate, univariate and covariate distinction is important for the grammatical description of mathematics in the following chapter. Each type of structure is associated with a particular bundle of systems in the grammar, and will be used in developing a metafunctional model for mathematics. Figure 2.5 presents the different types of structure relevant for this thesis (following Martin 1992a, covariate and univariate structures are grouped together as interdependency structures).

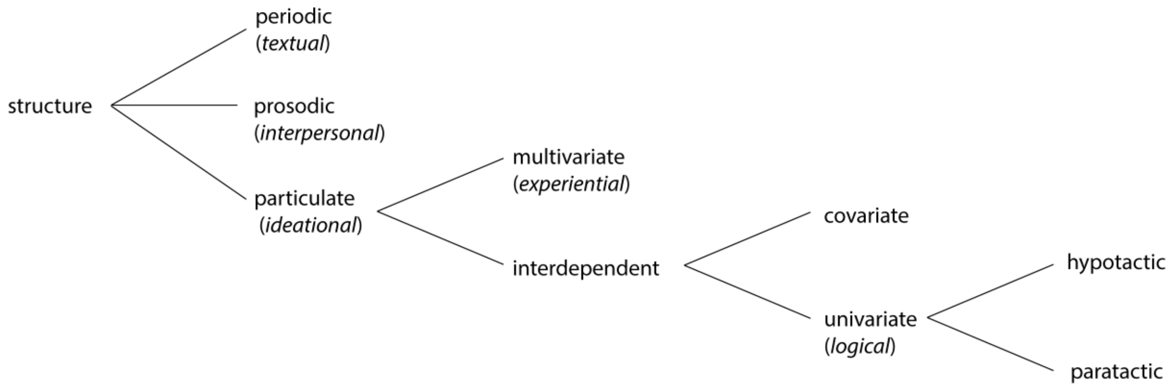


Figure 2.5 Types of structure and their coordinating metafunction

2.1.3 Rank and nesting

Within strata, SFL proposes a second set of levels known as ranks. Ranks form a constituency hierarchy whereby every rank is made up of one or more units of the rank below (Halliday 1963). In English phonology for example, there are four ranks: the tone group is the highest and contains one or more feet, which in turn contain one or more syllables, which in turn contain one or more phonemes (Halliday 1963, 1994). At the higher stratum of lexicogrammar, English also has four ranks. Clauses contain groups (or phrases), which contain words, which contain morphemes (Halliday and Matthiessen 2014). A rank scale's constituency organisation is what distinguishes it from strata. As mentioned above, strata are related in terms of abstraction; this means that a unit on a higher stratum is not compositionally related to a unit in another stratum (e.g. a morpheme in the grammar does not consist of phonemes in phonology, Hockett 1961). In contrast, by definition, the units in a rank scale will necessarily be related through a part-whole relation (Halliday 1961).[5] In Systemic Functional modelling, every rank is obligatory; this means, for example, that a clause must be interpreted as consisting of one or more groups and phrases, which consist in turn of one or more words, which consist in turn of one or more morphemes – even where the entire clause is a single morpheme long (e.g. *Help!*). The clause *The second Saints album contains thirteen tracks* can thus be divided into three functions: Carrier (*The second Saints album*), Process (*contains*) and Attribute (*thirteen tracks*). Each of these functions are

[5] It is worthwhile noting in addition that not all languages (nor semiotic resources) will necessarily have the same rank scale, or indeed a rank scale at all. Halliday (1992a), for example argues against the need for a rank of phoneme in Beijing Mandarin. Similarly, for mathematics O'Halloran (2005) argues for four ranks, while for images, Kress and van Leeuwen (1990) do not use a rank scale. In this sense, any particular rank scale is not a part of the SFL theory of language (or semiosis), but rather a descriptive tool.

realised by a single unit at the rank of group/phrase. The Carrier is realised by a nominal group (*The second Saints album*), the Process is realised by a verbal group (*contains*) and the Attribute is realised by another nominal group (*thirteen tracks*). Each of these groups is in turn realised by their own functions, and then by words until the final rank of morpheme. The justification for analysing each instance of language at all ranks is to account for the possible variation that may occur at all ranks. Although the minimal clause *Help!* consists of only one group with one word with one morpheme, it could be expanded to include multiple groups (e.g. *you please help me!*), with each group potentially containing multiple words (*Will you both pretty please help us all!*) with many words potentially containing multiple morphemes (*Won't you both be helping us all!*). By proposing a rank scale, the possible variation at each tier can be described.

In addition to the obligatory ranks, a process of 'rankshift' may occur, where a unit of the same or higher rank may realise a function at a given rank. This may involve additional compositional depth for the analysis of a particular unit. For example, it is relatively common for a clause to occur within a nominal group: in the clause *we went through the experimental evidence that led Rutherford to come up with his model*, the nominal group *the experimental evidence that led Rutherford to come up with his model*, includes a rankshifted clause, *that left Rutherford to come up with his model*, functioning as a Qualifier. Clauses (or any unit) that are rankshifted are more commonly referred in current SFL literature as embedded (Halliday and Matthiessen 2014).

The rank scale is organised through a constituency hierarchy involving multivariate structures (Huddleston 1965). However in addition to the rank scale, univariate structures can also produce additional depth in a process commonly known as layering or nesting (Halliday 1965). This arises from the fact that (in English) every rank can iterate: clauses can become clause complexes as we've seen above, groups and phrases can form group/phrase complexes (e.g. *<u>the hydrogen atom</u> and <u>the helium ion</u>*), words can form word complexes (*<u>hydrogen</u> and <u>helium</u>*) and morphemes can form morpheme complexes (*<u>pre</u>- and <u>post</u>-discovery*). Through this iterative structure, complexes of units can form single elements within larger complexes, producing further depth. Halliday (1965) illustrates this through the nominal group complex *soup, a main dish, sweet or cheese and biscuits, and coffee*. This complex sets up a univariate structure with four immediate elements: (1) *soup*, (2) *a main dish*, (3) *sweet or cheese and biscuits* and (4) *coffee*. The third element, however, itself contains two elements: (1) *sweet* and (2) *cheese and biscuits*, with *cheese and biscuits* in turn containing two elements: (1)

cheese and (2) *biscuits*. This sets up three layers arising from its univariate structure, shown in Table 2.2.

soup, a main dish, sweet or cheese and biscuits, and coffee

<table>
<tr><td>layer 1</td><td>soup</td><td>a main dish</td><td colspan="3">sweet or cheese and biscuits</td><td>coffee</td></tr>
<tr><td>layer 2</td><td colspan="2"></td><td>sweet</td><td colspan="2">cheese and biscuits</td><td rowspan="2"></td></tr>
<tr><td>layer 3</td><td colspan="3"></td><td>cheese</td><td>biscuits</td></tr>
</table>

Table 2.2 Nesting in *soup, a main dish, sweet or cheese and biscuits and coffee*

The nesting in this case all occurs at the same multivariate rank; the entire complex would function as a whole within a clause (e.g. *The menu today is soup, a main dish, sweet or cheese and biscuits, and coffee*).[6] As discussed in the previous section, univariate structures (and the nesting that arises from them) are associated with the logical metafunction. This metafunction, including its univariate structure, nesting and the recursive systems (see below), is particularly important for the discussion of mathematics in Chapter 3, and so we will return to it periodically as it becomes relevant.

2.1.4 Axis: System and structure

Underpinning each of the dimensions of rank, strata and metafunction is the complementarity of system and structure, which together form the dimension of axis. Systems (constituting the paradigmatic axis) arrange descriptive features in opposition to each other. Options in systems are realised by particular configurations in structure (the syntagmatic axis). For example, in the English system of MOOD, the paradigmatic axis opposes indicative clauses to imperative clauses. To distinguish these two types, indicative clauses are realised on the syntagmatic axis by having a Subject and a Finite, while an imperative typically has neither.

[6] Though not all iterative structures will necessarily be complexes of whole units. English agency, for example, iterates a combination of a Participant (and Agent) and an extra causative verbal group in the Process, rather than a full clause or a single group, such as those underlined in *The President allowed his advisors to pressure the Speaker to give up her opposition.*

To formalise these relations, SFL uses system networks. A simplified MOOD network is outlined in Figure 2.6.

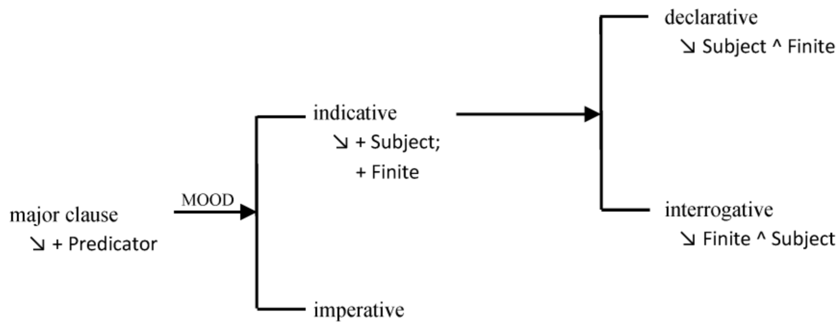


Figure 2.6 Simplified system network of MOOD
(adapted from Halliday and Matthiessen 2014: 162)

Read from left to right, the entry condition for the system is a major clause. The downward slanting arrow ↘ indicates a realisation statement (relating system to structure). In this case, a major clause is realised by the insertion of a Predicator (indicated by +). Predicator is a function label and is written with an initial capital, MOOD is a system label and is written in small caps and major clause is a feature (class) label and is written entirely in lower case. The square bracket indicates that if major clause is chosen, then either indicative or imperative must be chosen. These options are known as features and are written entirely in lower case. If indicative is chosen, then a Subject and a Finite is inserted, and the choice of declarative or interrogative arises. If declarative, then the Subject is sequenced before the Finite (sequence is indicated by ^); if interrogative, then the Finite comes before the Subject. Movement from the left to the right of the network is a movement in delicacy. Systems further to the right are more delicate options of those to the left.

Figure 2.6 above illustrates a series of ‘or’ brackets (e.g. for the system with the options indicative vs imperative). In addition to these, system networks can also show simultaneous systems through curly ‘and’ brackets. This allows multiple systems to cross-classify an entry condition (another feature or rank) and is the mechanism through which Systemic Functional Linguistics models multiple strands of independent variation. Figure 2.7 illustrates a simplified account of the three simultaneous systems of MOOD, TRANSITIVITY and THEME.

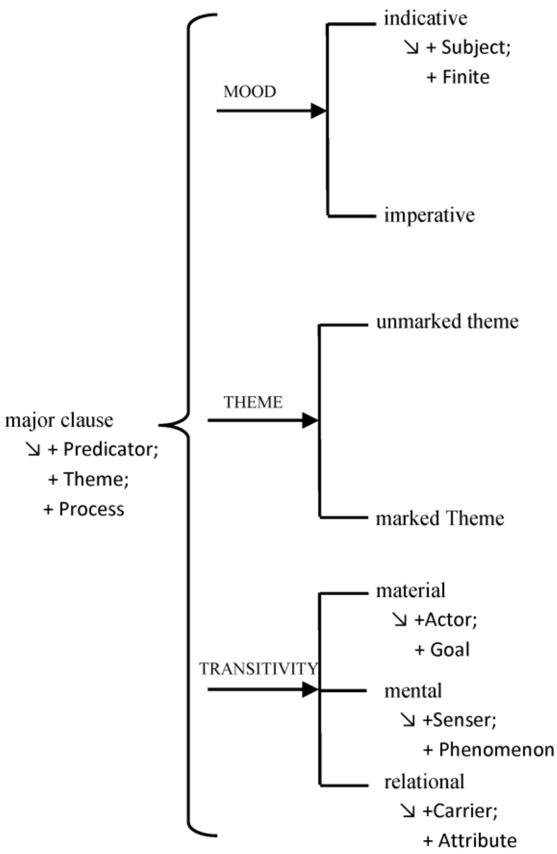


Figure 2.7 Simultaneous systems of MOOD, TRANSITIVITY and THEME

This network says that for all major clauses, a choice must be made from each of the systems of MOOD, THEME and TRANSITIVITY.

Systems may also be recursive, whereby a choice in one feature can be repeated indefinitely. Figure 2.8 exemplifies this for a simplified network of English clause complexing.

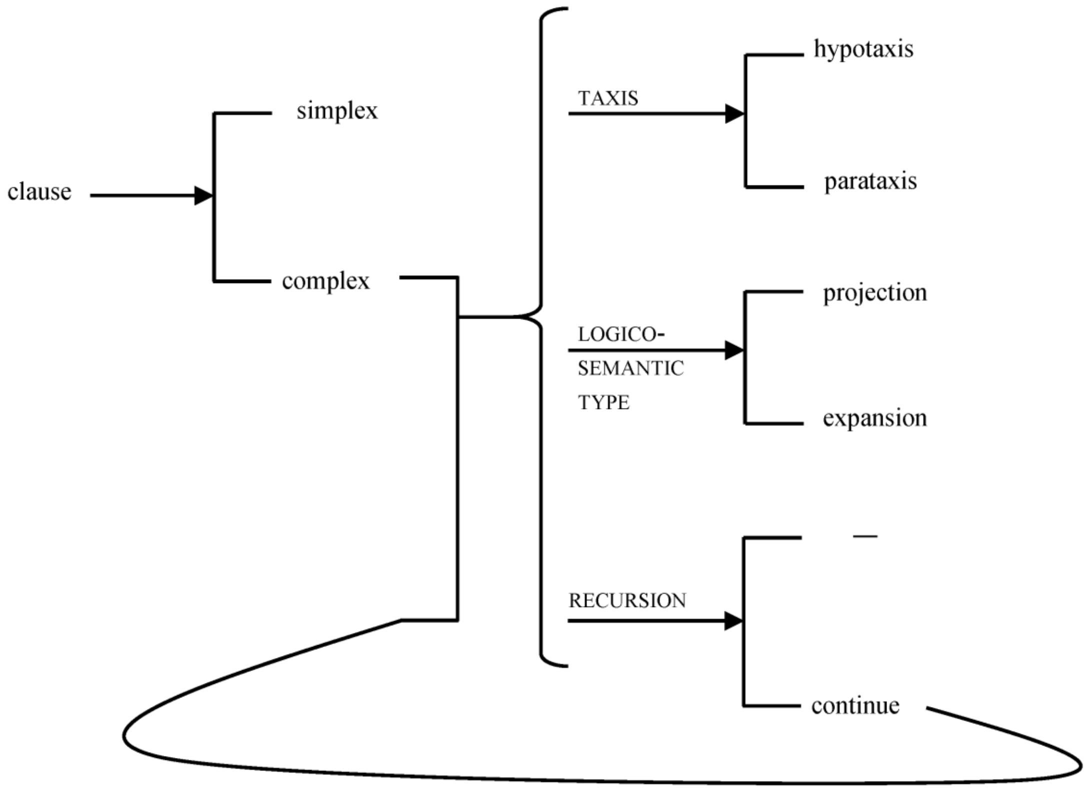


Figure 2.8 Recursive system in a simplified network of English clause complexing. (adapted from Halliday and Matthiessen 2014: 438)

This system indicates that if complex is chosen, a choice must be made from each of the systems of TAXIS, LOGICO-SEMANTIC TYPE and RECURSION. Within the RECURSION system the wiring that emanates from the feature *continue* indicates that if this feature is chosen another choice from each of these systems is needed. This offers an indefinitely recursive loop with any number of choices available until the feature "**—**" (glossed as *stop*) is chosen. As this system is indefinitely recursive, it is realised by an indefinitely iterative structure (not shown); i.e. the recursive system produces a univariate structure. As mentioned above, recursive systems such as this and the univariate structures that realise them are associated with the logical metafunction. We will see in Chapters 3 and 4 that recursive systems permeate the description of mathematics, and so form an important component of this thesis. The full set of conventions for system networks is outlined in Appendix A.

The systems and structures constituting the dimension of axis play a pivotal role in the architecture of Systemic Functional descriptions. Martin (2013) argues that the paradigmatic and syntagmatic axes are the theoretical primitives of Systemic Functional theory from which metafunction, rank and strata can be derived. This is a powerful claim and will be explored in more detail in Section 2.4.4 in relation to descriptive issues in the broader field of Systemic Functional Semiotics.

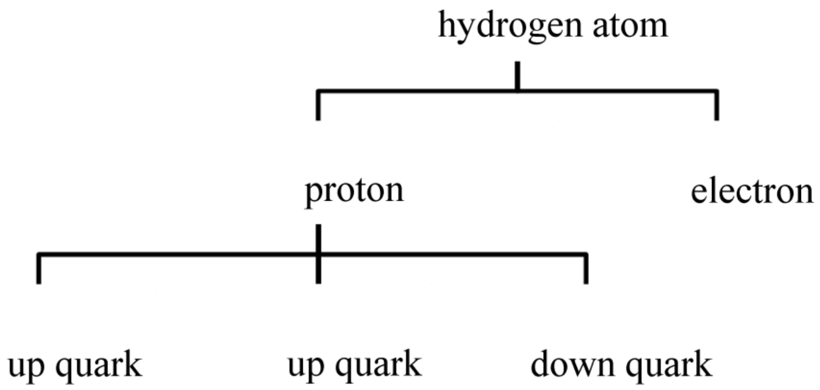
Now, however, we will turn to the Systemic Functional view of scientific language. The concepts of strata, rank, axis and metafunction will be drawn on throughout this discussion and in subsequent chapters and so will be elaborated on where relevant.

2.2 Science as viewed from the Systemic Functional dimension of field

The discourse of science and the knowledge it construes is far removed from that of everyday life. Science uses distinct technical terminology, particular linguistic patterns and specific text types that are rarely seen outside academic or vocational discourse. Halliday (1989), for example, outlines a number of features of scientific English that distinguish it from everyday language. These include interlocking definitions of terminology (where multiple technical terms are dependent on each other for their meaning), delicate technical taxonomies (where technicality is elaborated through classification or composition), a high density of lexical items per ranking clause (where a large degree of information is consolidated in a relatively short space), and high use of grammatical metaphor (whereby, for example, processes and qualities are repacked as things). In addition, Lemke (1982, 1990) highlights that scientific discourse puts to use a large set of intricate semantic relations in organising its technical knowledge (known as thematic patterns). Lemke's (and other's) studies exploring thematic patterns (across language and other semiotic resources, e.g. Tang et al. 2011, Fredlund, 2015, Fredlund et al. 2012, 2015) unveil both the complexity of meaning underpinning scientific knowledge and the difficulty in reconciling apprentices' understanding of this knowledge with that of the broader field.

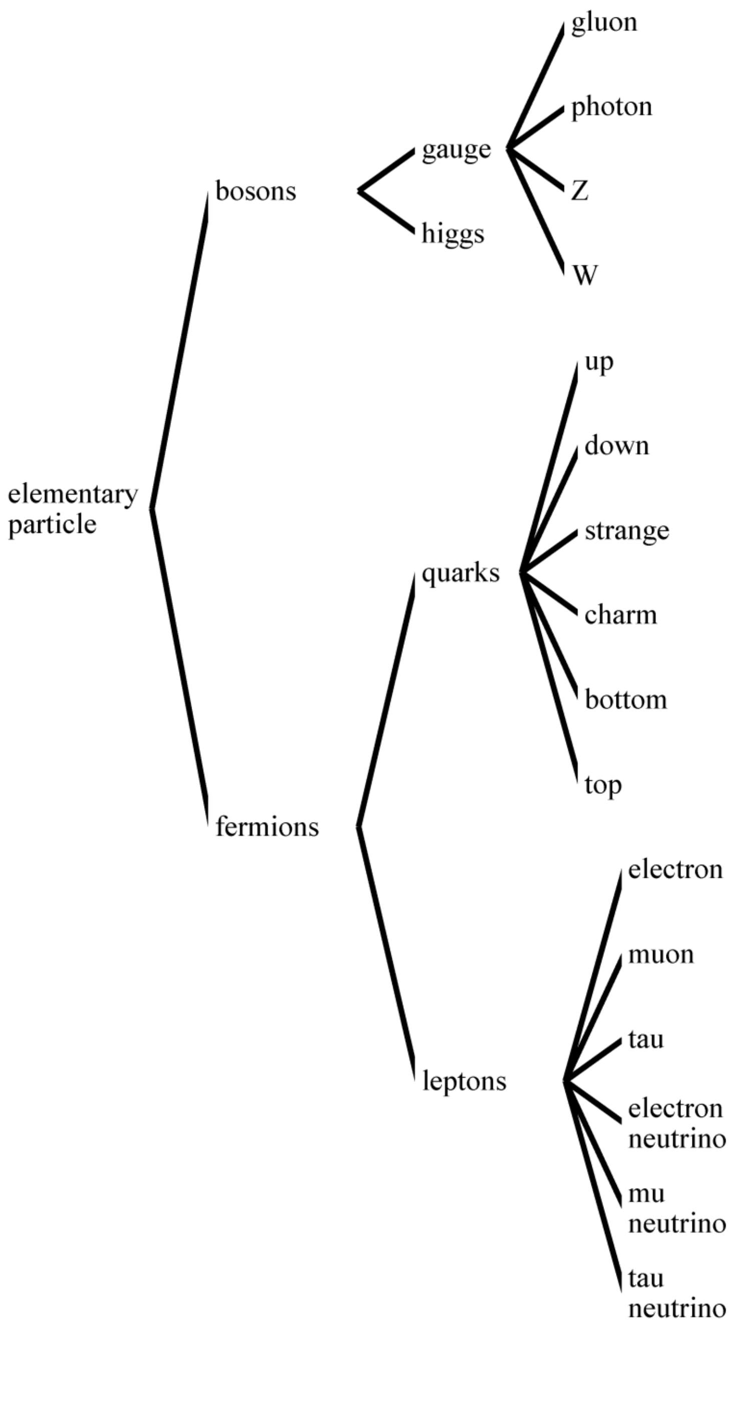
Physics knowledge, as conveyed through language, mathematics and images, is central to this thesis. As one way into this knowledge, physics will be viewed from the SFL's register variable *field*. Field is a component of the stratum of register (see Section 2.1.1 above) and is concerned with the nature of the social activity realised through language. In understanding physics, a view from field can be roughly interpreted as offering a semiotic perspective on its content. To interpret this, Martin considers field as 'a set of activity sequences oriented to some global institutional purposes, alongside the taxonomies involved in these sequences (organised by both classification and composition)' (2006b: 1). Studies of science using field have shown that the its language involves deep taxonomies and intricate activity sequences that encode very precise and field-specific meanings (Wignell et al. 1989, Martin 1993a, Rose 1998, Rose et al. 1992, Hao 2015).

Taxonomies of science are either compositional, arranging terms into part-whole relations, or classificational, arranging terms into type-subtype relations. An example of a relatively small compositional taxonomy in physics is the structure of a hydrogen atom. The atom is composed of an electron and a proton, with the proton composed of two up quarks and one down quark, represented in 2.9.



2.9 Compositional taxonomy of a hydrogen atom

In addition to compositional taxonomies, physics is constituted by a range of classification taxonomies. For example, the above compositional taxonomy already indicates two types of *quark* – *up quarks* and *down quarks*. These quarks are positioned in a much larger classification taxonomy of elementary particles, shown in 2.10.



2.10 Classification taxonomy of elementary particles

Taxonomies are concerned with the relations between individual entities in field and are complemented by activity sequences that account for a field's unfolding events. Activity sequences can be divided into relations of expectancy and implication. Expectancy sequences indicate activities in which events typically follow one another in a particular sequence. In language, the expectancy links between events in these activities are normally realised as temporal relations. In science, expectancy sequences are often associated with procedural texts that detail the steps of an experiment (Martin and Rose 2008, Rose et al. 1992). Text 2.1(a), for example, shows a university physics lecture recounting Rutherford's experiment that led to the discovery of the nucleus. In doing so, it builds an expectancy sequence of the events involved in the experiment (segments of the text not relevant to the activity sequence of the experiment have been left out).

> So 1911 Rutherford comes along and does a scattering experiment…. He took some atoms, ah in fact what he did was he had a very thin gold foil which he knew if he hit it with alpha particles, whatever alpha particles are, hit it with particle radiation, … the film was thin enough that the particles would go through, but some would be reflected but basically they would go through. He constructs the experiment so that he has a narrow beam of these particles [alpha particles]. And then he basically detects what comes up here… they did the experiment and found a number of things is that many many of them went through, and then the occasional one bounced back at them… these people analysed it in a sophisticated way…

Text 2.1 (a) Procedural recount of the Rutherford experiment

The expectancy sequence constituting Rutherford's activity is outlined below:[7]

> So 1911 Rutherford comes along
>
> ^
>
> (Rutherford) does a scattering experiment
>
> =
>
> what (Rutherford) did was (Rutherford) had a very thing gold foil which (Rutherford) knew if (Rutherford) hit it with alpha particles, whatever alpha particles are, hit it with particle radiation, … the film was thin enough that the particles would go through, but some would be reflected but basically they would go through.
>
> ^
>
> (Rutherford) constructs the experiment so that he has a narrow beam of these particles
>
> ^
>
> (Rutherford) basically detects what comes up here
>
> =

[7] A caret ^ between lines indicates that the line below temporally follows the line above, = indicates that they occur at the same time. In this example, the two instances of = involve a general activity encapsulating a number of more specific ones (*does a scattering experiment*; *did the experiment*).

(Rutherford) did the experiment

^

and (Rutherford) found a number of things is that many many of them went through and the occasional one bounced back at him

^

these people (Rutherford + others) analysed it in a sophisticated way

Text 2.1(b) Expectancy sequence from the Rutherford experiment procedural recount

This sequence shows activities involving Rutherford unfolding temporally. Each event in this expectancy sequence follows the previous one, but this sequence does not arise from logical necessity in the field. Although this particular sequence recounts a past sequence and so is necessarily fixed, at the time of its original unfolding (or in any recreation of the sequence in a lab), other events could have occurred (see Martin 1992a: 537 ff. who follows Barthes 1977: 101-104 in this regard). In science, the possibility for expectancy sequences to be diverted along another path opens the way for variations in experimental procedures (whether through error or design). It is this possibility for differing sequencing that distinguishes expectancy sequences from the other type of activity sequence, known as implication sequences. Implication sequences detail the unfolding of events in a field whose sequence arises through logical necessity (Martin 1992a: 323). These sequences tend to underpin scientific explanations where one event implies the next. Text 2.2(a) shows an example of an implication sequence being jointly developed by a high school teacher and a class. The sequence outlines what classical physics predicts should happen to an electron in an atom.[8]

Teacher: This electron by definition is accelerating. Why is it? Who can tell me, Tony?

Student: It changes direction

Teacher: Right, it is continually changing direction, moving in a circular motion and circular motion is a type of acceleration. What did Maxwell say that accelerating charges do? They emit?

[8] Note that this is an implication sequence within the field of *classical* physics. The fact that it does not accurately predict what happens in the atom, (i.e. that the implication sequence here does not match what is observed), is what makes the classical model wrong. It doesn't change the fact that this is an implication sequence specified by the field.

Student: Emit EMR

Teacher: They emit EMR. So this electron should be emitting radiation. And if it was emitting radiation, it is emitting energy. And if it is emitting energy it must be losing energy, by law of conservation of energy, and if it is losing energy, sooner or later it has to slow down. And if it slows down, John, what's it going to do?

Student: Ah, crash into the nucleus?

Teacher: It'll crash.

Text 2.2 (a) Classical physics' model of an electron in an atom

Again we can reconstitute the text to show the implication sequence being realised. This time, the implication sequence considers the electron. Each line is ordered in terms of the causal sequence among events established in the text, where the following line is a logical result of the previous.

(The electron) changes direction

^

This electron by definition is accelerating

^

This electron should be emitting radiation

^

(The electron) is emitting energy

^

(The electron) must be losing energy, by law of conservation of energy

^

sooner or later (the electron) has to slow down

^

(The electron will) crash into the nucleus

Text 2.2 (b) Implication sequence of classical physics' model of an electron in an atom

Under the classical theory being explained, the relation between these events is not one of expectancy where one would probabilistically follow the other, but rather of implication, where one necessarily follows the other. Implication sequences such as this are the primary means of explaining natural phenomena in science (Martin 1993a), and, naturally, are often built through explanation genres (Unsworth 1997, 2001, Rose 1998, Veel 1997, Martin and Rose 2008). In contrast, as shown above, scientific expectancy sequences occur in procedures and procedural recounts associated with experiment, while taxonomies of both classification and composition are built through various types of report (Rose et al. 1992, Veel 1997, Martin and Rose 2008).

In addition to these field dimensions, Zhao (2012) highlights an important relation between technical meanings in physics that has not yet been adequately captured in the model of field to this point. This relation is between technical terms such as *force* and *acceleration*, or *kinetic energy* and *speed*. Terms such as these are regularly coupled in physics discourse, and they hold a close connection that students must learn as they move through schooling. For example, the greater the speed of an object (assuming everything else remains the same), the greater the kinetic energy. Relations such as this are taught in physics schooling and are necessary to understand the intricate network of meanings associated with the field, but they do not display composition or classification relations (*force* is not a type nor a part of *acceleration*), nor do they necessarily involve events that can be related in sequences. To account for these terms, Zhao proposes a new type of taxonomic relation that she terms causation. In the following chapters, we will see that terms in these relations are often related mathematically. Due to this, Chapter 5 will argue that rather than including these under taxonomic relations, their mathematical nature requires a reinterpretation of the field dimension of implication.

The research into science from an SFL perspective has shown that scientific language organises a large set of deep taxonomies of composition and classification, and a series of intricate activities involving entities organised in these taxonomies and related in successive events to one another either through implication or expectancy. Technical meanings in science therefore involve large swathes of meaning deriving from these intersecting dimensions, with any instances of technicality in the field engaging in indefinitely large numbers of relations with other elements of the field. In the next section, this interconnectivity will be explored in relation to the ongoing dialogue between SFL and a branch of sociology known as code theory that is concerned with the structuring of knowledge. As part of this, it will introduce the modern incarnation of code theory which forms the other theoretical foundation of this thesis, Legitimation Code Theory.

2.3 Code theory and the structuring of knowledge

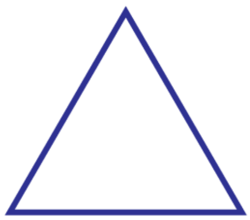
Every academic discipline has its own ways of organising its knowledge. Disciplines have their own objects of study, their own methods of discovery, their own bases for argumentation and their own principles for judgement. Through its studies of registerial variation, Systemic Functional Linguistics has revealed many of the differing ways of meaning of various academic disciplines across the sciences, humanities and vocational education (much of which is collected in various volumes such as Halliday and Martin 1993, Christie and Martin 1997, Martin and Veel 1998, Christie 1999, Martin and Wodak 2003, Halliday 2004, O'Halloran 2005, Coffin 2006, Rose et al. 1992, Christie and Martin 2007, Wignell 2007, Feez et al. 2008, Christie and Maton 2011, Martin 2012 and a range of material arising from the Write it Right Project, see refs in Martin 1993b)[9]. Along the way, SFL has had regular dialogue with a branch of sociology deriving from Basil Bernstein, known as code theory. This dialogue has been long standing and has taken many forms, but in recent decades it has focused on the nature of knowledge (for the history of dialogue between SFL and code theory see Martin 2011b, Maton et al. 2015, Maton and Doran in press 2017). Where SFL has focused on the way knowledge is construed through language and other semiotic resources, code theory has offered a theorisation of knowledge itself. This theorisation has allowed SFL to understand the principles that underpin the different ways of meaning across academic disciplines, while SFL's descriptive apparatus has offered code sociology a way in to understanding the discursive resources at stake in any particular form of knowledge.

In theorising knowledge, Bernstein (1999) makes a distinction between horizontal and vertical discourse. Horizontal discourse is associated with everyday, common sense knowledge and tends to be 'oral, local, context dependent and specific, tacit, multi-layered and contradictory across but not within contexts' (1999: 159). In contrast, vertical discourse is associated with academic knowledge and takes the form of 'a coherent, explicit, and systematically principled structure, hierarchically organised, as in the sciences, or it takes the

[9] Many of the Write it Right resources focusing on science, history, English, art, geography and mathematics can be found at www.educationalsemiotics.wordpress.com

form of a series of specialised languages with specialised modes of interrogation and specialised criteria for the production and circulation of texts, as in the social sciences and humanities' (1999: 159).

His description of the difference between the sciences on the one hand and the social sciences and humanities on the other leads Bernstein to make a further distinction within vertical discourse. This distinction is between hierarchical knowledge structures, most typically associated with the sciences, and horizontal knowledge structures, associated with the social sciences and the humanities. Hierarchical knowledge structures (the natural sciences) attempt to 'create very general propositions and theories, which integrate knowledge at lower levels, and in this way show underlying uniformities across an expanding range of apparently different phenomena' (1999: 162). To symbolise hierarchical knowledge structures, Bernstein uses an image of a triangle, where the generalised and integrated theories at the peak encompass a wide range of phenomena at the base.



2.11 Hierarchical knowledge structures (Bernstein 1999: 162)

Horizontal knowledge structures (the humanities and social sciences), on the other hand, 'consist of a series of specialised languages with specialised modes of interrogation and criteria for the construction and circulation of texts' (1999: 162). To represent these, Bernstein uses a series of Ls:

$$L^1\ L^2\ L^3\ L^4\ L^5\ L^6\ L^7 \ldots\ L^n$$

2.12 Horizontal knowledge structure (Bernstein 1999: 162)

As mentioned above, the natural sciences are said to typify hierarchical knowledge structures. In problematising this, Muller (2007) argues that hierarchical knowledge structures (and thus the sciences) develop through integration - toward ever more integrative and general propositions that subsume multiple statements. This capacity he terms strong verticality. In addition, hierarchical knowledge structures involve strong grammaticality whereby they are able to stably generate unambiguous empirical correlates. In these terms, hierarchical knowledge structures are said to progress both integratively through expanding explanatory sophistication and empirically through worldly corroboration (2007: 71).

Of all the hierarchical knowledge structures, physics is the one most regularly positioned as the ‘archetypical’ example (e.g. Bernstein 1999, Maton and Muller 2007, Martin 2007a, O’Halloran 2007a, Martin 2011b). This classification of physics makes potentially fruitful suggestions as to how it organises its knowledge and forms of discourse it takes. How (and indeed whether) physics achieves this knowledge structure, however, has yet to be thoroughly explored.

Complementing the field-based perspective given in the previous section, this thesis will also investigate physics in relation this code theory perspective of knowledge structure. In order to do this, however, we need tools that allow for fine-grained analyses of its discourse. Bernstein’s formulation provides an eye-opening first step, but it does not detail how we ‘see’ this in data. As Maton (2014: 109) argues, categorising a discipline such as physics in terms of knowledge structure is a good tool to think with, but it does not offer analytical principles to explore how this arises in the discourse itself. To understand the organising principles of physics knowledge, the next section will introduce the incarnation of code theory known as Legitimation Code Theory.

2.3.1 Legitimation Code Theory

Legitimation Code Theory (LCT) offers a series of tools for analysing the varying forms taken by knowledge and other social practices across the disciplinary spectrum and in broader society. It is being increasingly used in cooperation with SFL to analyse knowledge practices, allowing each theory to speak back to the other and catalysing theoretical innovation (see Maton et al. 2015, Maton and Doran in press 2017). LCT is made up of five dimensions (Maton 2014), of which two are particularly important for understanding physics’ knowledge structure: Specialisation and Semantics.

Specialisation concerns what the basis for knowledge is in a discipline. It distinguishes between epistemic relations (ER) between knowledge and its object of study, and social relations (SR) between knowledge and its authors or subject (Maton 2014: 29). Disciplines with stronger epistemic relations (ER+) are said to emphasise specialised knowledge of specific objects of study, while those with weaker epistemic relations (ER-) downplay these skills. In contrast, those with stronger social relations (SR+) emphasise the attributes of actors in the field as the basis of achievement (such as having a cultivated gaze from prolonged immersion in art), while those with weaker social relations (SR-) downplay the attributes of actors. Epistemic and social relations are independently variable and form gradable continua. This means that a given discipline can have any strength of ER or SR at the same time. As a means of categorising in general terms the possible combinations of stronger and weaker epistemic and social relations, Maton (2014) presents four distinct codes:

- ‘*knowledge codes* (ER+, SR–), where possession of specialised knowledge of specific objects of study is emphasised as the basis of achievement, and the attributes of actors are downplayed;
- *knower codes* (ER–, SR+), where specialised knowledge and objects are less significant and instead the attributes of actors are emphasised as measures of achievement, whether these are viewed as born (e.g. ‘natural talent’), cultivated (e.g. artistic gaze of ‘taste’) or socially based (e.g. the notion of gendered gaze in feminist standpoint theory);
- *élite codes* (ER+, SR+), where legitimacy is based on both possessing specialist knowledge and being the right kind of knower (here, ‘élite’ refers not to social exclusivity but rather to possessing *both* legitimate knowledge *and* legitimate dispositions); and
- *relativist codes* (ER–, SR–), where legitimacy is determined by neither specialist knowledge nor knower attributes – a kind of ‚‘anything goes’.’ (Maton 2014: 30-31)

In terms of this categorisation, physics is classified as a knowledge code that emphasises epistemic relations (Maton 2014). As part of this, it maintains a cohesive integrated theoretical organisation and develops through accurately describing and explaining the physical world.[10] It is physics' relatively strong epistemic relations, therefore, that underpin its hierarchical knowledge structure (Maton 2014: 92). By fostering strong relations with its object of study, physics can develop ever more integrative theory that encompasses an expanding range of empirical phenomena. This, however, brings us back to our previous question: how do we 'see' this in physics' discourse? For this, we will use a second dimension of LCT known as Semantics.

Semantics is concerned with how meanings relate to their context and to each other. It involves two variables, semantic gravity (SG) and semantic density (SD). Semantic gravity conceptualises the degree to which meanings depend on their context (Maton 2014: 110). If semantic gravity is stronger (SG+), meanings are more dependent on their context; if it is weaker (SG–), meanings are less dependent on their context. For example in physics, a specific numerical measurement of an instance of a physical phenomenon (say a force), displays significantly stronger semantic gravity than a generalised theoretical principle holding across multiple contexts (e.g. a generalised equation $F = ma$). Semantic density, on the other hand, refers to the degree of condensation of meaning in an item (be it in a word, a symbol, a concept or a theory etc.). Stronger semantic density (SD+) indicates more meaning condensed; weaker semantic density (SD–) indicates less meaning condensed. For example in physics the technical term *star* holds relatively strong semantic density as it contains a large degree of specialised meanings for the field: a star is a spherical mass of plasma held together by gravity, it involves multiple types that are classified by their effective temperature, absolute magnitude, luminosity and various other features, and the light from many of these stars arises from the release of energy during the thermonuclear fusion of hydrogen into helium. In contrast, in everyday language, *star* is rarely used in relation to these meanings, and rather refers to a shiny point of light in the night sky. For this reason, in everyday discourse, the term *star* has relatively weak semantic density. Maton and Doran (in press 2016a) argue that the key marker of semantic density is the degree of relationality a meaning has. This involves the degree to which a meaning is multiply interconnected with other meanings in a field. In the case of the term *star* in physics, it resonates out to a vast interconnected network of meanings in the field and so involves a large degree of relations for those trained in the field. The everyday meaning of *star* on the other hand does not resonate out to such a large degree and so involves fewer relations and thus has weaker

[10] In terms of the more delicate 4K model of LCT (Maton 2014: 175), this means that it likely also offers both relatively strong ontic relations to its physical object of study, and relatively strong discursive relations between its various theoretical components.

semantic density.[11] Semantic gravity and semantic density have been usefully applied to physics education to show the large constellations of meaning regularly at play in classrooms (Lindstrøm 2010), and the regular shifts from generalised theoretical meanings to specific empirical examples that students must negotiate when learning physics (Georgiou et al. 2014, Conana 2015, Georgiou 2015).

Like the epistemic and social relations of Specialisation, semantic density and semantic gravity are independently variable, allowing any strength of one to be combined with the other. This is crucial for understanding the knowledge of physics. For physics to develop a hierarchical knowledge structure that integrates knowledge across a range of empirical phenomena, it needs a mechanism for generating and displaying relatively strong semantic density. It must be able to cohesively relate a series of concepts associated with the physical world, so as to avoid segmental and ad hoc explanations. At the same time, in order to keep its theory in touch with empirical data, it must involve a large range of semantic gravity. In the first place, physics needs a tool to make empirical predictions from its theoretical descriptions; it needs a way to generate stronger semantic gravity (empirical predictions) from weaker semantic gravity (its theory) (a movement known as *gravitation*, Maton 2014: 129). But it also needs to be able to move the other way; it needs a method for allowing empirical data to speak back to and develop the theory. In terms of semantic gravity, it also needs a mechanism for moving from stronger semantic gravity to weaker semantic gravity (what Maton's terms *levitation*).

Thus in order to develop a hierarchical knowledge structure, physics must be able to generate new theoretical meanings and link these meanings with its empirical object of study. In terms of Semantics, it needs to display strengthening semantic density (condensation) and the ability to move between stronger and weaker semantic gravity. In Chapters 4 and 5, we will see that each of mathematics, language and images are crucial to these movements. No semiotic resource by itself allows physics to develop its hierarchical knowledge structure, but together they do.

[11] The everyday *star* may, however, hold astrological or spiritual significance. Such meanings display *axiological* condensation (Maton 2014: 153) more associated with values and dispositions than those of empirical description and explanation (known as *epistemological* condensation). As physics knowledge is ostensibly less concerned with organising moral values and personal dispositions and more with empirical prediction and explanation, this thesis will not consider axiological meanings, only epistemological meanings. Thus any reference to semantic density or semantic gravity in this thesis refers to epistemological semantic density and epistemological semantic gravity.

This brings us to the final major component underpinning this thesis. Physics knowledge is expressed not just through language, but through mathematics, images and many other resources. In order to see the knowledge being construed, we must have an understanding of the meanings made by each of these resources. In the last few decades, major advances have been made in understanding how non-linguistic semiotic resources such as mathematics and images organise their meanings. These developments form the third platform upon which much of this thesis stands, and so it is to these that we will now turn.

2.4 Physics as a multisemiotic discipline

Physics knowledge is construed through spoken and written language, mathematics, image, gesture, demonstration apparatus and a number of other semiotic resources. If we understand only the language used in physics, we are only seeing a partial picture. Indeed Parodi's (2012) study of non-linguistic resources used in six academic disciplines (chemistry, biotechnology, physics, history, linguistics, literature) suggests that physics is a discipline which more than most relies on non-linguistic semiotic resources. This study found that in a university textbook corpus, physics had by far the highest use of mathematical formulae and also a relatively high use of graphs and diagrams compared to the other disciplines under study. If mathematics, images and the other resources are consistently used in various contexts in conjunction with language, it is fair to say that they likely do something that complements language. Indeed Airey and Linder (2009) argue that the particular constellation of semiotic resources in the physics discourse is one of the crucial factors organising its specific disciplinary ways of knowing.

To understand the roles various semiotic resources play alongside language throughout academic discourse and in the broader culture, the past few decades have seen the establishment and rapid growth of the field of multimodality. Multimodality was sparked by the seminal accounts of images by Kress and van Leeuwen (1990) and O'Toole (1994) and aims to understand the multitude of meaning-making resources available in human culture (for an introduction to multimodality, see Machin 2007 and Bateman 2014a, for overviews of the field see Jewitt 2010, Norris 2015). Multimodal research quickly moved into the area of mathematical and scientific discourse through the studies of mathematics and images by O'Halloran (1996, 2005) and Lemke (1998, 2003), both of which strongly influence this thesis. The development of multimodality paralleled a similar development within physics education research. This tradition has highlighted that non-linguistic semiotic resources play an integral role in organizing the knowledge of physics and that educational interventions must therefore take these into account when building pedagogy (see e.g. van Heuvelen 1991, Åberg-Bengtsson and Ottosson 2006, Fredlund et al. 2012, 2015).

As mathematics and images are the most salient and pervasive non-linguistic resources used in physics, this thesis will focus in particular on these resources. To this end, the following sections will introduce the models of image and mathematics that this thesis builds on. First, it will discuss the role of images in organising scientific meanings, focusing in particular on Kress and van Leeuwen’s (1990/1996/2006) image grammar, and the different types of meanings images make in science (from Lemke 1998) (Section 2.4.1). Second, it will present O’Halloran’s grammar of mathematics, which forms the direct antecedent and primary influence on the mathematical description developed in Chapter 3 (Section 2.4.2). Third, it will discuss the different ways of reading multimodal discourse based on Bateman’s (2008) distinction between text flow and page flow (Section 2.4.3). Fourth, it will discuss some general descriptive issues in multimodality in relation to abstracting theoretical categories such as metafunction, rank and strata and further develop the axial principles on which the descriptions in this thesis are based (see Section 2.4.4). Finally, it will introduce the multisemiotic corpus underpinning the thesis (2.5). Each of the models introduced in this section will be further elaborated in Chapters 3-5 as they become relevant.

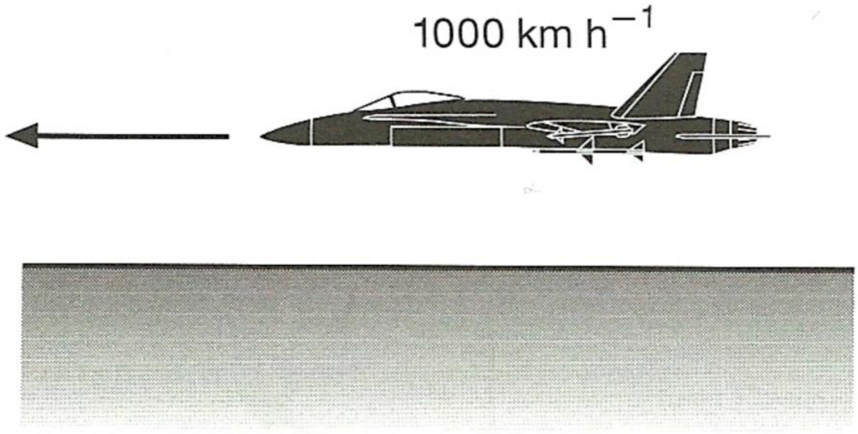
2.4.1 Images in physics

Images permeate all areas of physics. They are used through schooling from primary school to university, and form a regular component of research publications. Physics education research has convincingly shown that images play a crucial role in organising the knowledge of physics, but that despite this they do not necessarily offer uncomplicated access to this knowledge for students (e.g. Åberg-Bengtsson and Ottosson 2006, Fredlund et al. 2012, 2015, Fredlund et al. 2014, Meltzer 2005, Rosengrant et al. 2007, 2009, van Heuvelen and Zou 2001). Significantly for their utility, Lemke suggests that the meanings made by images are of a different order to those construed in language. Under this model, images more readily construe ‘topological’ meanings of ‘degree, quantity, gradation, continuous change, continuous co-variation, non-integer ratios, varying proportionality, complex topological relations of relative nearness or connectedness, or non-linear relationships and dynamical emergence' in contrast to language's 'typological' meanings of categorical difference (at least ideationally) (1998: 87). The topological meanings in images arise from the spatial arrangement of elements on a page (or screen), which allows indefinitely small gradations to be presented and relative size/distance to be construed as meaning.

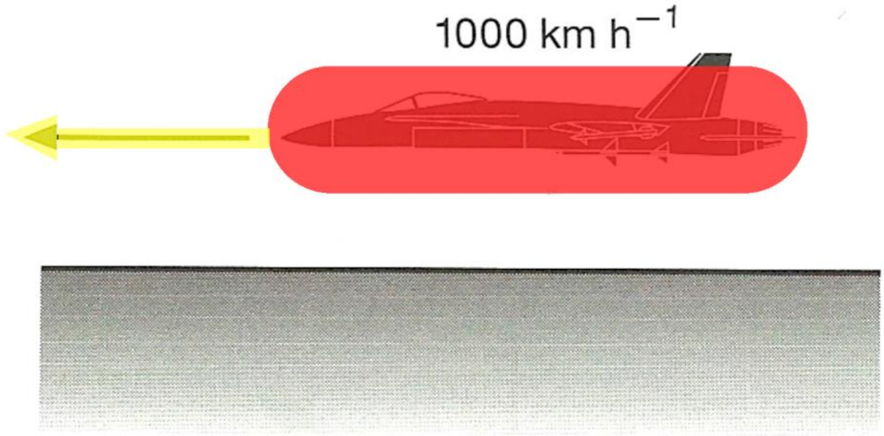
This topological meaning also allows images to construe relations between multiple elements across multiple dimensions in a single snapshot (these relations will be termed *arrays* in Chapter 5). In particular, it gives rise to graphs that, as O'Halloran (2005: 137) highlights, are central for conveying patterns of variation by arranging data points and lines along multiple axes. These graphs often intersect with various types of diagram to present a large range of meanings in a relatively small stretch of discourse. To understand the role images play in physics, Chapter 5 will interpret them in terms of the Systemic Functional dimension of field. This interpretation in turn requires a model that can interface with SFL's conception of field. The model of images that most easily does this is Kress and van Leeuwen's (2006) grammar of images.

Unlike SFL models of language, Kress and van Leeuwen do not present a stratal model that distinguishes clearly between grammar, discourse semantics or any other strata, nor do they suggest a rank scale (c.f. O'Toole 1994 who presents a rank scale, see Zhao 2010 for discussion). They do, however, adopt a metafunctional approach to images, dividing their systems into ideational, interpersonal and textual systems. The ideational component of Kress and van Leeuwen's grammar will be the main avenue through which Chapter 5 will view field, and so it will be introduced here.

The broadest ideational distinction in Kress and van Leeuwen's grammar is between narrative and conceptual images. Narrative images show unfolding actions or events, or some sort of dynamic change. Conceptual images, on the other hand, relate participants through their class, composition or symbolic meaning. The criterial element of a narrative image is the presence of a Vector (2006: 59). Vectors indicate some sort of movement or direction and often emanate from an Actor (which usually indicates the direction of motion of the Actor). 2.13 (a), from a senior high school textbook, illustrates a narrative image such as this with the plane functioning as the Actor (highlighted in red in 2.14 (b)) and the arrow functioning as the Vector (in yellow in 2.14 (b)).

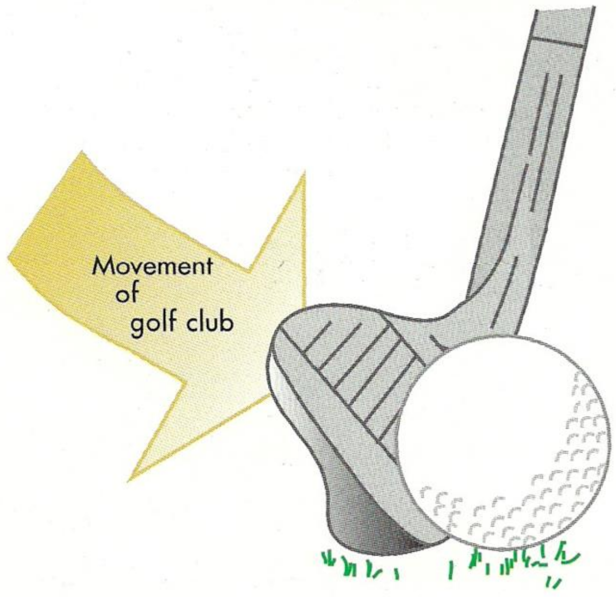


2.13 (a) Narrative image (Wiecek et al. 2005: 179)

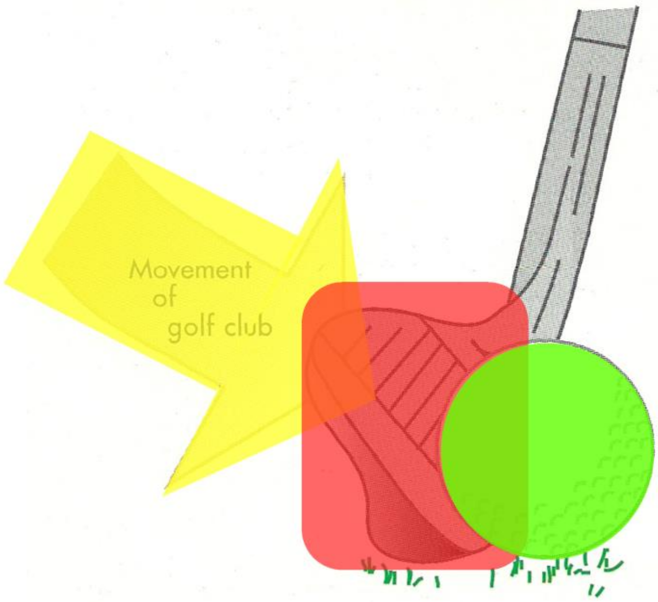


2.13 (b) Analysed narrative image (Actor in red, Vector in yellow) (Wiecek et al. 2005: 179)

Images with only an Actor and a Vector are known as non-transactional images. In addition to these elements, images may include another participant known as a Goal - to which the Vector is directed. Narrative images that include both an Actor and a Goal are known as transactional images. 2.14 (a), from a junior high school textbook, illustrates a transactional image with the golf ball functioning as the Goal (highlighted in green in 2.14 (b)), the golf club functioning as the Actor (red) and the arrow once more functioning as the Vector (yellow).



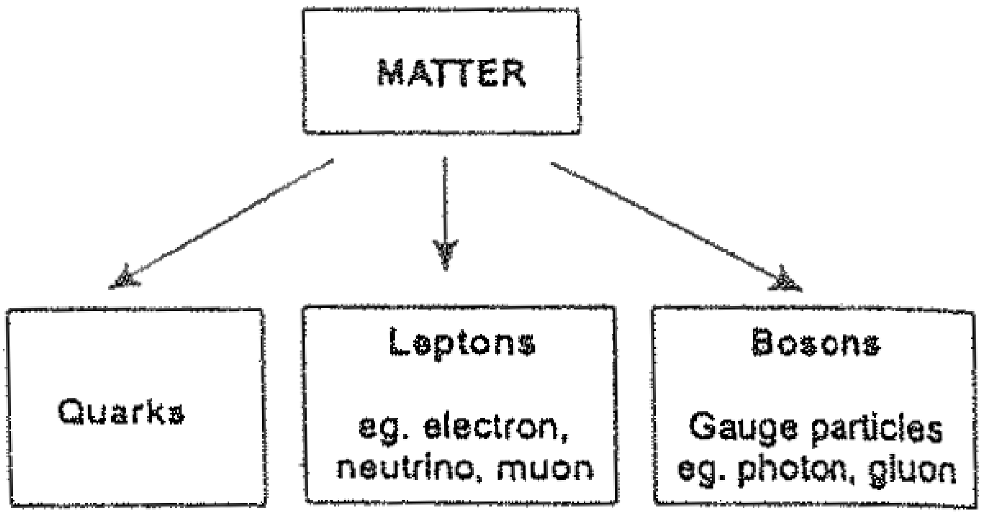
2.14 (a) Transactional narrative image. (Mau 1999: 13)



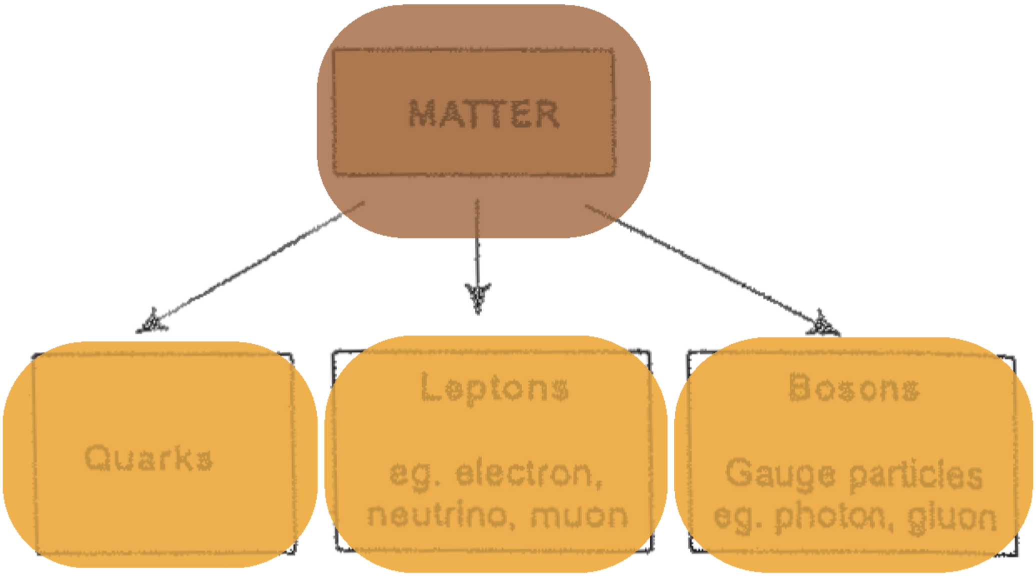
2.14 (b) Analysed transactional narrative image.
(Actor in red, Vector in yellow, Goal in green)
(Mau 1999: 13)

Narrative images such as these are important in physics for realising activity sequences (further discussed in Chapter 5). Conceptual images on the other hand tend to organise taxonomic relations. The two types of conceptual image most important for this study are known as classificatory and analytical images (the third type, symbolic images, are less prevalent in construing the technical knowledge of physics and so will not be reviewed here).

Classificatory images arrange participants in terms of type-subtype relations. They include two types of participants, Subordinates (sub-types) and Superordinates (types). These images do not appear as often in physics as narrative and analytical images, but still perform an important function presenting the classification taxonomies of physics. 2.15 (a) illustrates a classificatory image of types of matter from a high school textbook. The Superordinate is on top (highlighted in brown in 2.15 (b)), and the Subordinates are at the bottom (highlighted in brown).

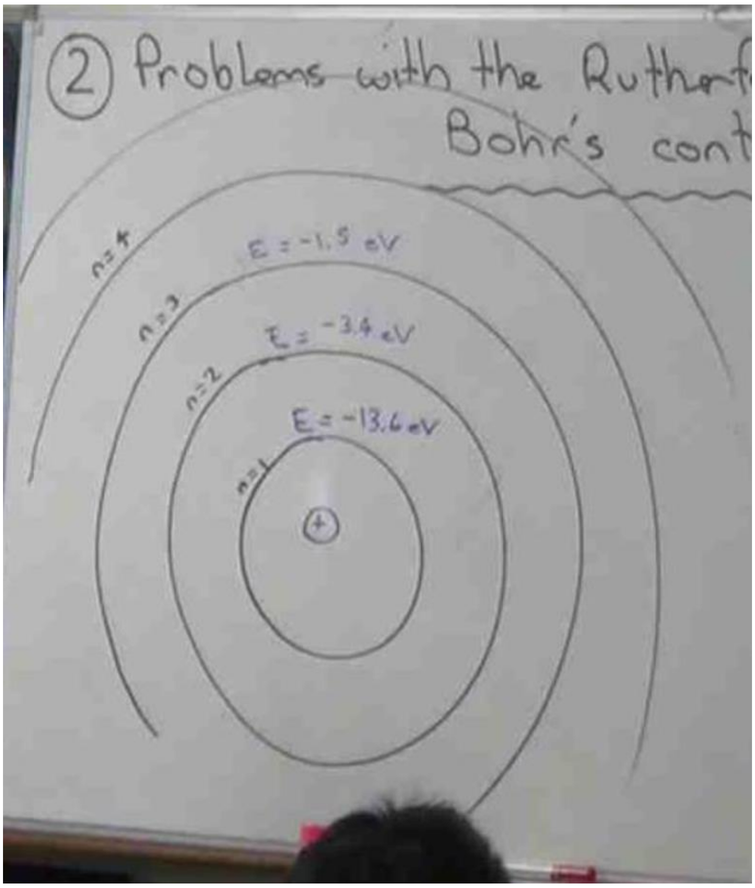


2.15 (a) Classificatory image. (Warren 2000: 155)

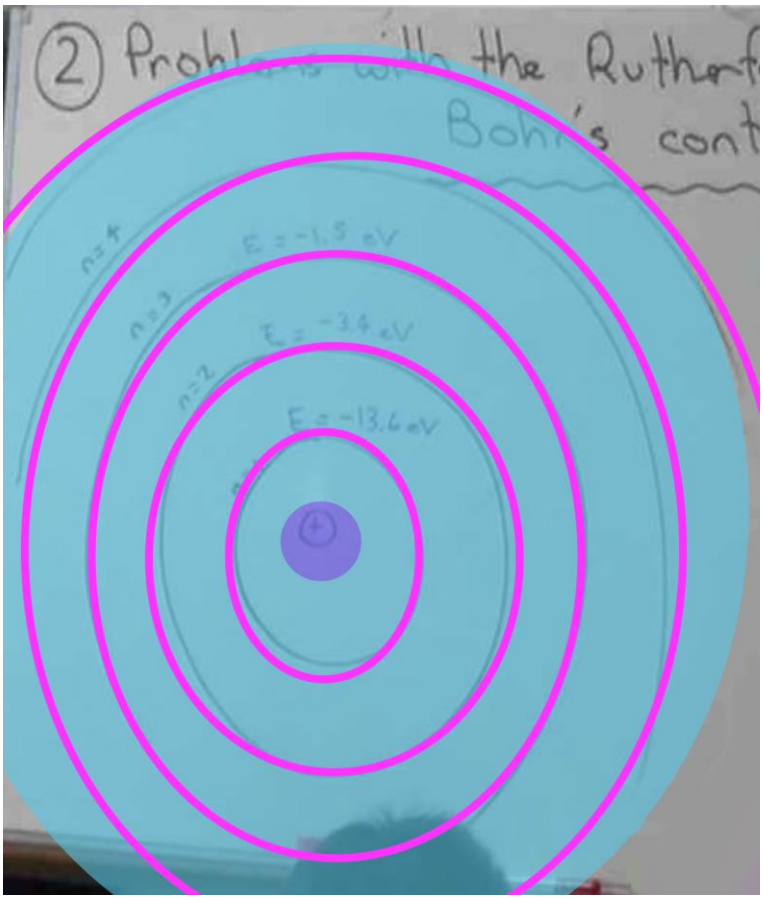


2.15 (b) Analysed classificatory image
(Superordinate in brown, Subordinates in orange)
(Warren 2000: 155)

The second types of conceptual image, analytical images, also display taxonomic relations, but of composition rather than classification. They present part-whole relations between entities and in doing so show the internal make up of physical things. The whole is known as the Carrier, with each constituent part called a Possessive Attribute. 2.16 (a) shows an analytical image of the Bohr model of the hydrogen atom drawn on a whiteboard in a high school classroom. The entire image is the Carrier (in light blue in 2.16 (b)), to which each line (orbital) and the inner circle (nucleus) are the Possessive Attributes (in purple).



2.16 (a) Analytical image.

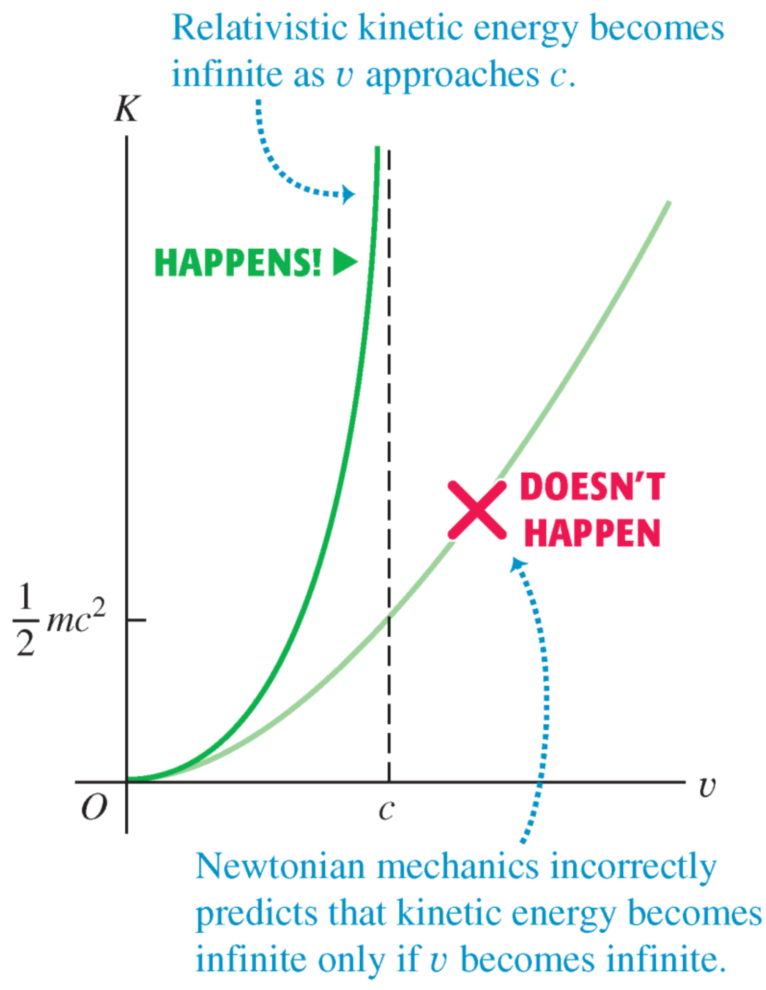


2.16 (b) Analysed analytical image.
(Carrier in light blue, Possessive Attributes in purple)

In Chapter 5, we will see that much of the power of images comes from the fact that they can display multiple structures at once. In a single image, part-whole and classification relations can be combined with narrative images and graphs.

In Kress and van Leeuwen's model, graphs are classified as a particular type of analytical image, where the points on the graph indicate Possessive Attributes of the particular dimensions of the graph. For example, in 2.17 below, each point on the two green lines indicates a particular measurement (the Possessive Attribute) of both K (the vertical axis, glossed as *kinetic energy*) and v (the horizontal axis, glossed as *velocity*) (the Carriers). However in addition, Kress and van Leeuwen describe line graphs such as this as also being 'quasi-vectorial' and having a 'quasi-narrative structure' (2006: 102). This is because the lines appear to show dynamic change from the left to the right of the graph. This creates indeterminacy between analytical and narrative readings of graphs.

O'Halloran (1996, 2005, see also Guo 2004) illustrates that one of the functions of graphs is to abstract away from raw data to show generalised patterns. For example in 2.17, the curves on the graph (shown in green) are not presented as corresponding to any particular numerical measurements. Rather, they show a generalised pattern relating the two variables, K (kinetic energy on the vertical axis) and v (velocity on the horizontal axis). This abstraction plays an important role in the knowledge of physics, allowing generalised theories to be developed from raw empirical data. In Chapter 5, we will return to the nature of graphs and interpret this possibility for generalisation from the perspective of field, which will also allow us to understand the significant relationship between the meanings made in mathematics and those made by graphs.



2.17 Graph in a university textbook. (Young and Freedman 2012: 1247)

Extending Systemic Functional theory and description to images significantly broadens the horizon of SFL text analysis. It allows language and image to be discussed in comparable terms, and thus offers an insight into each resource's affordances and uses. For this thesis, Kress and van Leeuwen's grammar of images gives a model that can be readily interpreted in terms of the register variable field, and thus understood in terms of the knowledge images construe in physics. Chapter 5 will bring together such a field-based view of images with the field-based model of scientific language (see Section 2.2 above) and a field-based interpretation of mathematics.

As 2.17 above shows, images are regularly coupled with mathematics in physics. In order to understand how mathematics organises its meanings, we need a thorough description built on comparable terms to that of language and image. Chapters 3 and 4 are devoted to developing such a description, which extends and enhances a model developed by O'Halloran (2005). O'Halloran's model will be introduced now.

2.4.2 Mathematics in physics

As Parodi's (2012) study of academic disciplines shows, mathematics permeates physics. As for images, mathematics holds its own functionality and organises the knowledge of physics in its own ways. Lemke (2003) argues that one of the broader functions of mathematics is to mediate between the typological meanings of language and the topological meanings of images (in particular graphs). Individual mathematical symbols encode typological meanings associated with categorical and qualitative distinctions in language. For example F (glossed as *force*) is qualitatively and categorically distinct from another symbol E (glossed as *energy*). However when symbols are placed within mathematical formulae, the interdependencies between symbols give rise to patterns of covariation. These patterns offer variation by degree (topological meaning) that can be plotted onto graphs. In this way, mathematics sits between images and language, facing each way and allowing their meanings to be reconciled. Chapter 3 will detail the mathematical interdependencies that construe covariation, while Chapter 5 will discuss how these are interpreted in graphs.

This function arises from the grammatical organisation of mathematics. As O'Halloran (1996, 2005) shows, mathematics' grammar varies significantly from that of language and of image. O'Halloran's model is the most elaborated Systemic Functional description of mathematics to date and is the main foundation on which the grammar in Chapter 3 builds. It offers a range of insights into the possible variation and functionality of mathematics that underpin the model developed in this thesis. For reasons of space, we cannot recapitulate her entire grammar, however this section will introduce a number of the most salient observations that directly influence the model developed in the following chapters.

O'Halloran presents a language-based approach to mathematical symbolism that interprets the organisation of mathematics through the three metafunctions: textual, interpersonal and ideational (including both the experiential and logical components) (2005: 96). The justification for a language-based approach comes from the fact that mathematical symbolism evolved out of language (2005: 96; for a detailed account of the evolution of mathematical symbolism, see Chapter 2 of the same book). In addition to metafunctionality, O'Halloran's language-based model also proposes three strata (discourse semantics, grammar and graphology) and four ranks in the grammar that organise the range of possible variation in each metafunction.

The lowest grammatical rank is the *element* (O'Halloran 2005 terms this rank *component,* before renaming it element in O'Halloran 2007b), which organises the variation in individual symbols. Different types of elements, such as pronumerals (e.g. *F, x, m* etc.) and numerals (e.g. 2, 3 10.1 etc.), coordinate with broader grammatical and text patterns and will be systematised in Chapter 3. Elements form constituents of the next higher rank in O'Halloran's model, *expression*. Expressions involve one or more elements related by operations such as + (addition), – (subtraction), ÷ (division) and × (multiplication). O'Halloran terms these relations operative processes and shows that they can be indefinitely iterative. Any number of elements can recur within an expression with every element linked by with the others through an operative process. That is, at its simplest, an expression may contain only a single element (e.g. *m*), or alternatively it may contain two (e.g. $m \times a$) or more, (e.g. $\frac{a}{b} + \frac{c}{d} + \frac{e}{f}$). This potential for indefinite expansion and iteration of elements is a motif that runs through both the grammar and text patterns of mathematical symbolism and has significant repercussions for the overall organisation the description developed in Chapters 3 and 4.

Above expressions, the next rank in O'Halloran's grammar is the *clause*, which links expressions through items such as the equals sign =, for example $F = ma$. Continuing the language-based approach, O'Halloran terms = a relational identifying process (2005: 106).

This draws a likeness with relational identifying clauses in English that regularly involves the verb *to be*, for example in *Impulse is the time rate of change of momentum* (Halliday and Matthiessen 2014). As part of this analysis, O'Halloran utilises the same structural functions for relational identifying clauses in mathematics as those in Halliday's description of English. This produces analyses such as:

$(xyz)^n$	=	$x^n y^n z^n$
Token/Identified/Medium	Process	Value/Identifier/Range

Table 2.3 O'Halloran's analysis of an equation (2005: 106)

This analysis gives a multivariate interpretation (see Section 2.1.2 above) where each expression on the left and right side of the equation hold distinct functions (e.g. the left side in Table 2.3 functions as the Token, while the right side functions as the Value). Although this analysis brings out much of the likeness between equations and relational identifying processes, it poses a number of problems, including how to identify which expression is, for example, the Token and which is the Value, and what to do when there are more than two expressions. This will be considered in detail in Chapter 3, leading to an alternative interpretation as a univariate structure.

The final rank in O'Halloran's grammar, termed *statement*, accounts for mathematical formulae where there are more than two expressions (and thus more than one relational process), such as $\frac{a}{\sin A} = \frac{b}{sinB} = \frac{c}{\sin C}$. Like elements within expressions, the number of expressions in a statement is in principle indefinitely iterative. Despite this, the actual number and order of expressions in any given statement is tightly coordinated by mathematical symbolism's information structure and the text patterns that arise from these. This will be discussed in Chapter 3 and form the basis for distinguishing different genres in Chapter 4.

O'Halloran's full rank scale for the grammar of mathematical symbolism is summarised in Table 2.4.

Rank	Example
statement	$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{104.1 \times 10^6} = 2.9$
clause	$\lambda = \frac{v}{f}$
expression	$\frac{v}{f}$
element	v

Table 2.4 Rank scale of mathematics in O'Halloran (2005)

Each rank offers a range of variation within each metafunction. O'Halloran argues, however, that it is the experiential component of the ideational metafunction where the most significant expansions of meaning occur. As mentioned above, the rank of expression involves an indefinite number of elements (symbols) being related by operative processes such as +, –, ×, ÷. This indefinite iteration allows for expressions with a large number of symbols to occur such as the two expressions on the left and right side of (2:1):

(2:1) $$4\pi\varepsilon_0 \frac{n^2\hbar^2}{mZe^2} = \frac{n^2}{Z} a_0$$

O'Halloran highlights that there is no simple linear unfolding of the operative processes (+,–, ×, ÷ etc.). Rather, expressions regularly show a large degree of rankshift (see Section 2.1.2 above). An example of this in (2:1) above involves $n^2\hbar^2$ on the left hand side. This sequence of elements involves a multiplication relation between n^2 and $\hbar^2$ of multiplication (though with the × elided as per convention).[12] In addition, this expression in its entirety is related to another group mZe^2 through division (shown by the vinculum —). That is, each group of symbols is effectively functioning as a single symbol; expressions are rankshifted within another expression. The degree of rankshift is again theoretically indefinite and regularly

[12] We will not consider the power indicated by the superscript [2], here. This will be looked at in detail in the next chapter.

becomes quite deep. (2:2) shows this by indicating the rankshift in (2:2) through single red square brackets. Expressions within smaller square brackets are rankshifted within those with larger brackets. In this case there are three levels: the highest (unbracketed as this level is not rankshifted) and two rankshifted levels (bracketed).

(2:2) $$4\pi\varepsilon_0\left[\frac{[n^2\hbar^2]}{[mZe^2]}\right]$$

This possibility for indefinite iteration and rankshift allows mathematics to set up indefinitely complex relations between indefinitely many symbols. By doing this, many elements in the field of physics can be related, allowing its various components to be integrated. This indefinitely prolonged iteration and the integration that arises from it is a constant theme throughout each of the following chapters and impinges significantly on the overall model of mathematics developed.

Complementing the iterative nature of operative processes, O'Halloran also highlights variation associated with both the textual and interpersonal metafunction. Textually, she suggests that the left side of the equation typically functions as Theme, and thus as the point of departure for the mathematical message, while the right side functions as the Rheme (following the description of English in Halliday 1994). Interpersonally, O'Halloran shows that compared to language, the meanings possible in mathematics are significantly contracted. Mathematics may shift speech function from its typical form of a statement to a command with the aid of language (e.g *Let* $x = 5$), however it cannot do this on its own. Further, there appears some variation in polarity between, for example, = glossed as *equals* and ≠ glossed as *not equals*, but this does not appear to regularly cut across all relations. And finally, modality is given in the form of digitised measures of probability, (e.g. $p > 0.5$) These variations, however, exhaust the scope of interpersonal meaning internal to the grammar of mathematics. Thus in comparison to the elaborated interpersonal meanings available in English (e.g. Halliday and Matthiessen 2014, Martin 1992a, Martin and White 2005) the interpersonal metafunction in mathematics is quite narrow. Possible interpersonal and textual dimensions of mathematics will be further explored in Chapter 3.

O'Halloran's description presents a significant step forward in our understanding of mathematics and the meanings it construes. As O'Halloran concedes, however, there is still much work to be done to develop as comprehensive a model as that offered for English. Taking up this challenge, this thesis, in particular Chapters 3 and 4, takes a further step. In particular, it presents systems for much of the variation highlighted by O'Halloran and the structural realisations that arise from these systems, as well as reinterpreting certain components of the grammar in light of these systems. The overview of O'Halloran's grammar given here has been necessarily brief; however given the strong influence it has had on the model of mathematics developed in this thesis (in particular in Chapter 3), it will be revisited where relevant in the following chapters.

2.4.3 Text flow and page flow

Much of the discussion in this thesis concerns the meaning-making patterns internal to individual semiotic modes (mathematics, image, language); what O'Halloran (2005) terms *intrasemiosis* (as opposed to *intersemiosis* - the shift between representations in multiple semiotic resources).[13] The intrasemiotic models developed here aim to understand the intrinsic functionality of each resource for organising the knowledge of physics as a step toward a more comprehensive picture of physics discourse. Chapter 3, for example, builds a grammatical model of mathematical symbolism that considers its system in isolation from other semiotic resources, whereas Chapter 5 views each of mathematics, language and image individually from the perspective of field to see the specific types of meaning they contribute to the overall knowledge of physics.

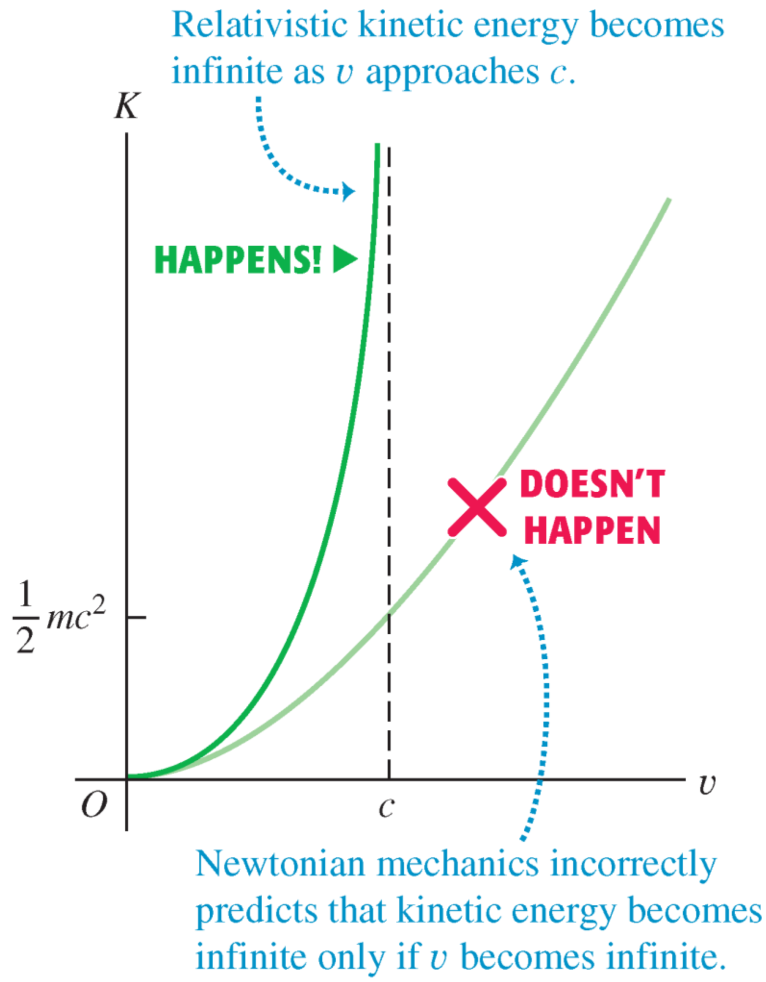
The main exception to the primarily intrasemiotic descriptions developed in subsequent chapters is the model of genre built in Chapter 4. This model views mathematical symbolism and language as working in interaction to build certain recurrent patterns of text. This model is based upon the fact that both mathematical symbolism and written language tends to unfold through what Bateman (2008, 2011) terms *text flow*. Text flow refers to the linear unfolding of written text, whereby there is a relatively definite order of unfolding; in English, the text is read from left to right in a single line, with each line read in succession down the page (an

[13] The distinction between intrasemiosis and intersemiosis is similar to that made by Duval (2006) in his distinction between *treatment* – processes that happen within the same semiotic resource – and *conversion* – the representation of the same object (roughly the same ideational meaning) across multiple different semiotic resources in a text.

example is this page).[14] Although, as O'Halloran (2005) points out, the grammatical organisation of mathematical symbolism is such that within expressions multiple dimensions are used to indicate meaning (e.g. left to right in multiplication and addition, $2 \times y$, $x + 1$, top to bottom in division, $\frac{x}{2}$ or the use of subscripts or superscripts in various positions, x^2, y_1), the overall unfolding of multiple mathematical statements in text strongly tends to follow the same text-flow pattern as that of English.

This is in contrast to the *page-flow* of many images and highly multimodal text (Bateman 2008: 176). Page-flow utilises a page's inherent two-dimensional spatiality to make meaning, and in doing so, tends to confound any linear reading. In the graph given in 2.17, for example, the horizontal (*v*) and vertical (*K*) axes both contribute meaning such that any point on either green line gains meaning from both. In addition, the mathematical and linguistic labels distributed across the entire image all function in their own particular ways, with the meanings and ordering determined not by linear sequence but by a range of other relations (for discussion of image-text relations, see Bateman 2014a). As we will see in Chapter 5, the page flow used in graphs and diagrams offers the potential for a large number of different field-specific meanings to be presented in a single snapshot. However as Bateman (2008) shows, these images and the pages they are situated in can become highly complex, which means that primarily sequential models of genre (such as those developed in SFL for English, Martin and Rose 2008) need significant revision if transferred over to page-flow resources. In contrast, the shared text-flow of mathematical symbolism and English lends itself relatively nicely to a genre model that involves sequential staging (shown in Chapter 4).

[14] This is not to say, however, that the eye must definitely follow this pattern. Bateman (2008) is at pains to point out that this indeed does not always happen, even in text-flow written language. All it suggests is that the inherent two-dimensional spatiality of the page is not utilised to its full extent as it is in, for example, tables where two intersecting dimensions (rows and columns) show distinct meanings.



2.17 Graph in a university textbook. (Young and Freedman 2012: 1247)

Notwithstanding this, the attempt in this thesis to build a model of genre for mathematics and language, and to further develop O'Halloran's grammar raises a number of descriptive issues, not just for the description of mathematics but for Systemic Functional Semiotics in general. As such, the following section will overview these issues and suggest some solutions that will function as principles of description for this thesis.

2.4.4 Semiotic description

With the advent of multimodal discourse analysis, there has been an explosion of research into and description of non-linguistic semiotic resources. This has expanded the horizons of text analysis and allowed a much broader view of semiosis to arise. Despite this, the intricate comparison and classification of semiotic resources (what could be called semiotic typology) is still in its infancy. A typology that considers both the functionalities and formal features of various semiotic resources is vital for understanding why different semiotic resources are used or not used in certain circumstances (and thus for informing an appliable semiotics). In order to develop an adequate typology, rigorous and comprehensive descriptions of multiple semiotic resources need to be developed that can be compared across a number of dimensions. To this point, however, this endeavor has been constrained by the fact that in the Systemic Functional tradition, there has been little consensus in practice as to how such descriptions should be developed.

A survey of the points of departure for the most prominent descriptions in the last few decades illustrates this lack of consensus. Kress and van Leeuwen's (1990/1996/2006) description of images takes as its starting point the linguistic metafunctions: ideational, interpersonal and textual. They argue that the metafunctions 'apply to all semiotic modes, and are not specific to speech or writing.' (1990/2006: 42, though van Leeuwen later withdraws this claim for sound and music, 1999:189-91). In this assumption of metafunctionality, they are followed by Martinec's (1998, 2000, 2001) description of bodily action and Painter et al.'s (2013) approach to images in children's picture books. In his description of visual art, O'Toole (1994) also assumes metafunctionality (though with relabeling of the functional components); and on top of this, O'Toole additionally assumes a hierarchical rank scale (with four ranks). As discussed above, O'Halloran's work on mathematical symbolism and images pushes one step further, assuming both metafunctionality and rank, as well as a tri-stratal allocation of each resource involving discourse semantics, grammar and an expression stratum (termed the display plane: graphics for images, and graphology and typography for mathematic symbolism). In contrast, van Leeuwen's (1999) description of sound is not organised with respect to metafunction, rank or strata (indeed he explicitly addresses and rejects the possibility of metafunctions, pp. 189-191).

In response to this lack of consensus across descriptions, discussions of the principles upon which description should take place and how we are to judge and compare competing descriptions have begun to emerge, in particular in relation to images and visual documents (e.g. Zhao 2010, Martin 2011a and the work of Bateman, e.g. 2008, 2011, 2014a, b). This discussion, however, is only at a very early stage of development and is yet to engage in a detailed manner with semiotic resources outside of images and visual documents. The relative nascence of discussion surrounding descriptive matters can be contrasted with the study of language within linguistics, where the principles of description and theorising have been the site of considerable debate throughout the field's entire modern history (see e.g. Sampson 1980, Newmeyer 1980).

The major semiotic descriptions within the Systemic Functional tradition have so far rarely concerned themselves with the principles of description. Despite different assumptions and points of departure, the common thread (excepting van Leeuwen 1999) is that each presumes a theoretical category that was originally developed for language (or more specifically English), such as metafunction, rank and/or strata. This is problematic if we wish to build descriptions that bring out the specific functionality of each semiotic resource. By simply assuming categories, we run the risk of homogenising descriptions, making everything look like the first resource to be comprehensively described (i.e. English) and thus watering down the specific functionality of each resource.

Notwithstanding the issue of assuming categories from English to other resources, the lack of consensus as to *which* categories to assume (metafunction, rank or strata) further emphasises the need for a detailed discussion of descriptive principles. For the description of language, a useful starting point is Halliday's (1992b) distinction between theoretical and descriptive categories (see also Caffarel et al. 2004b). Theoretical categories are those that are part of the general linguistic theory and by definition are general to all language – e.g. 'system', 'stratum', 'class', 'function' etc. Descriptive categories on the other hand are in principle language-specific, they are at a lower level of abstraction and cannot be assumed for all languages, such as 'clause', 'preposition', 'Subject', 'material process', 'Theme' etc. Descriptive categories are language specific instances of more theoretical categories. So, for example Halliday highlights that 'while "system" itself is a theoretical category, each instance of a system, such as "mood", is a descriptive category. Similarly, "option" (or "feature") in a system is a theoretical category, while each particular instance of an option, like "indicative" or "declarative", is descriptive.' (1992b: 3). When it comes to broader semiotic theory and description, the same principles hold. Although Systemic Functional theory maintains a series of theoretical categories that can potentially be used to describe various semiotic resources, their actual applicability or form in any description must be justified for every system. Given these issues surrounding the transfer of categories from English to other resources, this thesis argues as a starting point that we cannot simply assume phenomena such as metafunction, rank and strata will occur for every resource. These categories were originally developed for English through the bundling of systems (discussed below) and there has yet to be a detailed justification for their use in other resources. This is not to say that they won't occur, but that for such categories to be used in the description of a semiotic resource, they need to be justified internal to the system being studied.

This takes us to the crux of the issue. If we cannot assume metafunction, rank or strata what is the basis for constructing descriptions? Martin (2013) argues that each of these categories can be derived from the more fundamental theoretical primitive of axis, i.e. the interaction of the paradigmatic and syntagmatic axes (also less formally known as system and structure and formalised in system networks) (see Section 2.1.4 above). These axes are the foundational principles in Systemic Functional theory that evolved from Saussure (1916), Hjelmslev (1943) and Firth (1957, 1968). If metafunction, rank and strata can be derived from axis, a powerful method opens upon which the architecture of various semiotic resources can be developed and justified. As such, this argument is of crucial importance for this thesis. This section will therefore illustrate this argument by showing how each of the categories of metafunction, rank and strata can be derived, beginning with metafunction.

The justification for metafunctions is based on two types of evidence: first, the relative paradigmatic in(ter)dependence of systems (originally developed through the bundling of clausal systems in English, Halliday 1967a, b, 1968, 1969, 1970b), and second, the types of syntagmatic structure (Halliday 1979, Martin 1983). In terms of paradigmatic systems, the basis for suggesting distinct functional components hinges on systems being more or less independent of each other. If choices in a bundle of systems are shown to be heavily dependent on other choices in the bundle, and this entire bundle is relatively independent of another bundle of systems, evidence exists for distinct functional components. In relation to the English clause, this plays out in the distinction between TRANSITIVITY and MOOD. TRANSITIVITY and MOOD are relatively independent of each other, which means that any choice in TRANSITIVITY has relatively free choice of MOOD, with only a few constraints. This is shown in Table 2.5.[15]

[15] As we are developing a systemic argument, Matthiessen's (1995) description of nuclear transitivity is being followed here – it being the most fully developed paradigmatic account. In this description, the four least delicate process types are material, mental, verbal and relational. Existentials and behaviourals, considered to be at primary delicacy in Halliday and Matthiessen (2014), are taken in Matthiessen's model as more delicate subtypes of relational and material, respectively.

MOOD / TRANSITIVITY	**indicative**	**imperative**
material	*She built him a house.*	*Build him a house.*
mental	*It pleased the staff.*	*Please the staff.*
relational	*She is a good leader.*	*Be a good leader.*
verbal	*She praised the man.*	*Praise the man.*

Table 2.5. MOOD vs TRANSITIVITY in English.

The relative independence of MOOD choices with TRANSITIVITY indicates their potential to be part of distinct functional components that are the basis of metafunctions. In contrast, comparing the relation between MOOD and MODALITY shows that the MODALITY system is entirely dependent on choices within the MOOD system (see discussion in Martin 2013: 52ff). In particular, choices in MODALITY can only be made if indicative and not imperative is chosen in MOOD (the asterisk * indicates that an example is not possible).

(2:3) *She may build him a house* indicative + modality (+material)

(2:4) **may build him a house* imperative + modality (+material)

As well as this, like MOOD, modality can occur for all TRANSITIVITY types:

(2:5) *It may please the staff* mental + modality

(2:6) *She may be a good leader* relational + modality

(2:7) *She may praise the man* verbal + modality

Paradigmatically, MOOD and MODALITY are thus interdependent, suggesting their organisation within same functional component. In addition, they are both independent of TRANSITIVITY, suggesting that they form a distinct functional component.

The second branch of axial evidence for metafunction involves the type of syntagmatic structure used to realise systemic choices. This draws on Halliday's (1979) suggestion that distinct functional components tend to be realised by different modes of meaning (see Section 2.1.2 above). For MOOD and MODALITY, this in particular concerns their similarity of prosodic structure (Halliday 1979: 66-67). Beginning with MOOD, distinctions between relatively indelicate types of MOOD (imperative, declarative, interrogative etc.) in English revolve around the presence or absence and ordering of the functions Subject and Finite (though this is by no means the case for all languages, see for example Quiroz 2008 for Spanish, Caffarel 2006 for French, and Teruya et al. 2007 for a cross-linguistic mood typology). Indicative clauses have both a Subject and a Finite, whereas imperative clauses typically have neither. The subtypes of indicative, declarative and interrogative differ in terms of their ordering of the two functions: Subject before Finite for declarative, and Finite before Subject for interrogative[16], as in (with Subject and Finite in bold):

(2:8)	*Bring your sister.*	imperative (no Subject or Finite)
(2:9)	***Are you** bringing your sister?*	interrogative (Finite before Subject).
(2:10)	***You are** bringing your sister.*	declarative (Subject before Finite).

The Subject and Finite are crucial realisational structures for MOOD types in English. Looking syntagmatically these functions also present a prosody of PERSON, NUMBER, GENDER, and TENSE or MODALITY throughout the rest of the clause (more commonly known as 'agreement' or 'concord'). For example, the Subject and Finite must agree in terms of NUMBER and PERSON:

(2:11)	***Am I** bringing your sister?*	first person, singular.
(2:12)	***Are you** bringing your sister?*	second person, singular
(2:13)	***Is he/she** bringing your sister?*	third person, singular
(2:14)	***Are we/you/they** bringing your sister?*	first/second/third person, plural

[16] This simplifies the case somewhat by not taking into account the more delicate option of a wh-interrogative that has the Wh- element conflated with the Subject (e.g. *Who had seen you?*), and thus having the Subject preceding the Finite. This case, however, does not affect the argument being developed.

And in addition, the Subject and Finite must agree with their counterparts in the Moodtag in terms of PERSON, NUMBER, GENDER and either TENSE or MODALITY.

(2.15) ***I am*** *bringing your sister,* ***aren't I****?*	first person, singular, present
(2.16) ***You are*** *bringing your sister,* ***aren't you****?*	second person, singular, present
(2.17) ***He is*** *bringing your sister,* ***isn't he****?*	third person, singular, masculine, present
(2.18) ***She is*** *bringing your sister,* ***isn't she****?*	third person, singular, feminine, present
(2.19) ***She will*** *bring your sister,* ***won't she****?*	third person, singular, feminine, future
(2.20) ***She was*** *bringing your sister,* ***wasn't she?***	third person, singular, feminine, past
(2.21) ***She must*** *bring your sister,* ***mustn't she?***	third person, singular, feminine, modal: high
(2.22) ***She can*** *bring your sister,* ***can't she?***	third person, singular, feminine, modal: low

This agreement between the central interpersonal functions of Subject and Finite, and the Moodtag (also considered an interpersonal system, Martin 2013:59) can be interpreted as a structural prosody encompassing the breadth of the clause. The choice of PERSON, NUMBER etc. affects multiple elements cutting across the entire clause. Similarly, MODALITY also displays a prosodic structure (Martin 2004, 2008). This can be most clearly seen when discussing metaphors of modality. (7) illustrates an example given by Halliday (1979: 66):

(2.23) *I wonder if perhaps it might be measles, might it d'you think?*

In this example, the same modality choice is realised a number of times through different expressions: *I wonder, perhaps, might, might* and *d'you think*. As Halliday suggests, each could work effectively on its own, but working together the effect is cumulative, reasserting the speaker's angle on the statement and giving a prosody that colours the entire clause.

Following Martin's (1983, 2013) argumentation, as both MOOD and MODALITY have both a large degree of paradigmatic interdependence and reflect the same prosodic structure, they can be considered part of the same functional component: the interpersonal metafunction. Further, as they both are relatively independent of TRANSITIVITY, and TRANSITIVITY tends to have its own distinctive structure (i.e. a particulate structure, Halliday 1979), MOOD and MODALITY can be treated as forming a different functional component to that of TRANSITIVITY. It is in this sense that metafunction can be said to be derived from axis.

Similar arguments can be made for rank and strata. Like distinct metafunctions, ranks and strata arise from distinct bundlings of systems. The factor that distinguishes metafunctions, ranks and strata is the relation between these bundles of systems. Beginning with rank, we can see the distinct systems by considering nominal groups in English in relation to the clause. An English clause may have a Subject, such as *Those houses* in *those houses are being renovated*. In this case, the Subject is realised by a nominal group (*those houses*). This nominal group can be expanded to produce, for example, *Those two blue federation houses*. The addition of every word in the nominal group requires a distinct choice. That is, for example, to insert *those* (Deictic), requires a paradigmatic choice that is independent of the choice to insert *two* (a Numerative). We can have one without the other, (*those blue federation houses* or *two blue federation houses*), both (*those two blue federation houses*) or neither (*blue federation houses*). The same independence holds for both *blue* (an Epithet) and *federation* (a Classifier).[17] The system for nominal groups can thus be shown as in 2.18 (simplified for discussion).

[17] *houses* in this case is a Thing, and, aside from certain specific situations, is always inserted.

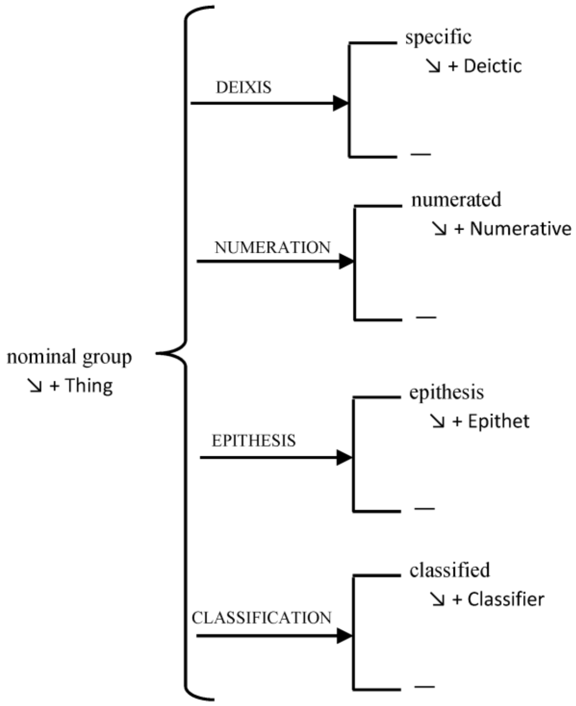


Figure 2.18 Simplified network of nominal groups in English

In the clause given above, the entire nominal group occurs within the Subject of the clause. This is shown by the following examples where the nominal group remains together despite the Subject moving around:

(2.24) ***Those two blue federation houses*** *are being renovated*.

(2.25) *Are* ***those two blue federation houses*** *being renovated?*

In these examples, the expanded nominal group remains together as the clause shifts from being a declarative (2.24, Subject first) to an interrogative (2.25, Subject second). The question that arises is where does the nominal group network fit in relation to the clause network? As the nominal group above occurs with the Subject, one option is to wire the nominal group network into the clause network for all clauses with a Subject, i.e. the nominal group network would be entered at the clause rank when choosing indicative (including declarative and interrogative clauses) rather than imperative.

The issue with this approach is that nominal groups can occur in other elements of the clause. For example in addition to realising the Subject, they can realise the Complement:

(2.26) *We renovated* ***those two blue federation houses***.

Or an element within a prepositional phrase realising an Adjunct[18]:

(2.27) *We grew up in **those two blue federation houses**.*

If we place the nominal group network at the same level as the clause, then we need to repeat the network at every possible point that it could occur. A more elegant solution that captures the generalisation of nominal groups across multiple elements in the clause is to set up a separate network at a level below. This second network (the rank of group) then becomes a constituent of the functions at the higher rank of clause. That is, each selection at the clause level will be realised by a particular configuration of clausal functions, and each clausal function will be in turn realised by a particular selection in the group rank network. This allows the nominal group to be generalised across all its possible points of realisation in the clause, and accounts for the constituency that is seen where clausal functions tend to be realised by whole groups. It is in this way that ranks can be derived from systemic bundlings on the paradigmatic axis and structural realisations on the syntagmatic axis.

Finally, stratal distinctions also involve distinct systemic bundles. However the relation between these systems is not one of constituency, as for rank, but of abstraction. To see this, we can consider the grammatical system of MOOD in relation to the discourse semantic system of SPEECH FUNCTION in English. These systems both organise the distinct roles played in dialogue, however they do so from different perspectives. These two perspectives, MOOD from lexicogrammar, SPEECH FUNCTION from discourse semantics, do not map directly onto each other. Rather, any particular choice in one system may coincide with a number of different choices in the other system. For example, we can consider the possible realisations of the SPEECH FUNCTION choice of question, which involves demanding information. The most direct way of asking for information is to use a wh-interrogative within the MOOD system:

(2.28) – *Where shall we put it?*

- *Over there.*

This achieves its speech functional goal of receiving information from the addressee. However depending on how we wish to position themselves in relation to other speakers, we

[18] More strictly, the nominal group realises a Minor Complement within the prepositional phrase.

can alternatively use other grammatical choices to fulfil the same speech function. For example, we could use a declarative:

(2.29) - *And we should put it….?*

- *Over there.*

Or a polar interrogative, giving two choices:

(2.30) - *Should we put it over here or over there?*

- *Over there*.

Or we could even use an imperative to make clear what we want our responder to do:

(2.31) - *Tell me where to put it*.

- *Over there*.

In each case, there is a distinct grammatical choice (2.28 - interrogative:wh; 2.29 – declarative; 2.30 – interrogative:polar; 2.31 – imperative). However they each fulfil the same choice in speech function; they all demand (and receive) information. This is in contrast to the different ways in which we can demand an action (issue a command). A command can be direct, through an imperative:

(2.32) - *Move it over there.*

Or it could be indirect by using a polar interrogative (known as an interpersonal grammatical metaphor):

(2.33) - *Can you move it over there?*

Or even a declarative:

(2.34) - *I'd like it moved over there*.

What these examples show is that there are a series of choices of SPEECH FUNCTION and MOOD that, although related, do not map onto each other precisely one to one. That is, there are two distinct networks of systems. Unlike those for the rank scale, however, the two systems are not related through constituency; there is not a structural function such as Subject in the network of SPEECH FUNCTION that every MOOD choice sits within. Rather, the SPEECH

FUNCTION system is at a more abstract level than the MOOD system. The result of this is that discourse semantic choices can often be realised by a range of elements within the grammar. We can see this by looking a choice within the discourse semantic system of ENGAGEMENT in English (Martin and White 2005; building on Halliday's (1982) description of modality metaphors). This system organises the varying ways in which English allows a speaker to mark that other possibilities than what is being said may be present. Crucially for our argument, the choices in this system at the stratum of discourse semantics can be realised across a large range of lexicogrammatical elements in different ranks. The following examples all show the discourse semantic choice of *entertain* (specifying that other possibilities may be available other than that mentioned; Martin and White 2005), realised across different grammatical environments. First, it can be realised by modality in the verbal group:

(2.35) *Cronulla **may** win next year*

Alternatively, it may be realised by a Comment Adjunct at the rank of clause:

(2.36) ***Possibly**, Cronulla will win next year*

Or by a projecting clause complex:

(2.37) ***I think** Cronulla will win next year*

Or a clause with a discontinuous Subject:

(2.38) ***It is possible** Cronulla will win next year*

The distribution of possible realisations shows that the discourse semantic choice of entertain is not tied to any lexicogrammatical unit. Rather, it cuts across units and affords many possible realisations. Distinct strata, therefore, maintain distinct systemic bundlings. However unlike ranks, these bundling are not related through constituency, but through abstraction. It is in this way that strata can be derived from axis.

The theoretical categories of metafunction, rank and strata can thus all be derived from axis. Each rely on having their own sets of systems (paradigmatic choices) and their own structures (syntagmatic realisations); however the relation between each system-structure cycle determines whether they constitute ranks, metafunctions or strata. The system-structure cycles of rank are related through constituency; those of strata are related through abstraction;

and those of metafunction are related through relative independence and distinct structural types. Table 2.6 outlines the axial evidence needed for a distinction in metafunction, rank and strata.

<table>
<tr><th></th><th>Axial justification</th></tr>
<tr><td>metafunction</td><td>Each metafunction displays
relative paradigmatic independence to other metafunctions
a distinct type of structure, e.g.
interpersonal: prosodic structure;
ideational: particulate structure
textual: periodic structure
relative paradigmatic interdependence within metafunctions</td></tr>
<tr><td>rank</td><td>Each rank displays distinct paradigmatic options and syntagmatic structures
System-structure cycles are related through constituency</td></tr>
<tr><td>strata</td><td>Each stratum displays distinct paradigmatic options and syntagmatic structures
System- structure cycles are related through abstraction</td></tr>
</table>

Table 2.6 Axial justifications for metafunction, rank and strata

This axial reasoning allows broader theoretical categories in Systemic Functional to be developed and justified from the same starting point. Moreover, it offers a common basis upon which semiotic descriptions can build, allowing each semiotic resource to reveal its functionality. From this basis we can begin to test, rather than assume, claims such as the pervasiveness of metafunctionality across semiosis, and potentially develop new, previously unseen categories. Accordingly, the description of mathematics in this thesis will build its architecture from the interaction of the paradigmatic and syntagmatic axes. It will propose a metafunctional distribution of meaning across multiple levels (ranks, strata and other types), with each category justified by having their own system-structure cycle. By working to justify these categories, we will see that mathematics organises its architecture in a significantly different manner to English.

Before moving on to the description, the next section gives a final note on the data used for this thesis.

2.5 Data used in the study

This thesis is primarily descriptive. It develops models of mathematics, image and language at various strata in order to understand their role in building the technical knowledge of physics. As such, the text analysis presented throughout each chapter is used in the service of the broader models developed throughout each chapter. The data used reflects this. As the thesis is most readily inspired by educational concerns, the corpus is entirely gathered from educational sources (in Bernstein's 1990 terms, the field of reproduction). Accordingly, it does not consider the field of research (the field of production). Although it is probable that much of what is discussed in this thesis could be used to analyse research, it remains to be seen just how valid it remains outside an educational context.

The corpus has been developed from three broad sources: textbooks, classroom discourse and student work.[19] Drawing upon these three sources creates a breadth of data that allows a perspective on the differing ways mathematics, language and image are used to build knowledge in each context. The data focuses on classical mechanics and quantum physics (broadly defined) as they appear to be the fields that most commonly use all three resources. Moreover, the two fields between them occur across all stages of schooling from primary (elementary) school to first year undergraduate university physics. Although these fields constitute the main corpus, the model was periodically corroborated with data from other areas including astrophysics, relativity and elecromagnetism.

[19] Primarily from New South Wales, Australia. The full details of the corpus are given in Appendix C.

The textbooks include excerpts from classical and quantum physics that range from primary (elementary) school through junior and senior high school to first year undergraduate university.[20] The primary school (~ ages 6-12) and junior high (secondary) school (ages ~12-16) textbooks are general science textbooks that focus on the areas of motion and forces (broadly, the field of classical mechanics in physics). The senior high (secondary) school (~16 -18) and undergraduate textbooks are designed for stand-alone physics subjects across both classical and quantum physics. Second, the classroom discourse arises from excerpts of videos from two sources. The first includes sixteen classes from a final year high school physics course in a high achieving school in New South Wales, Australia, that focus on a topic called *Quanta to Quarks* (dealing with quantum physics, Board of Studies 2009). The second is a quantum physics lecture series of twelve lectures in the second semester of an undergraduate university physics course, also in New South Wales. Finally, the student work incorporates the final exams from both the high school and university courses (covering multiple topics alongside those of quantum physics) from twenty-seven students (seven from high school, twenty from university), ranging from low to high achieving. The corpus used in this thesis is summarised in Table 2.7.

[20] I greatly thank Qingli Zhao and Shi Wen Chen for allowing me to use their corpora of primary school (Qingli) junior high school (Qingli and Shi Wen) and senior high school (Shi Wen) textbooks, and the university lecturer and classroom teacher who very generously let me record their classes.

	Data
Primary School (~6-12 years old)	• Excerpts from eight general science textbooks. ○ Focused forces and motion (classical mechanics)
Junior High School (~12-16 years old)	• Excerpts from five general science textbooks. ○ Focused on forces and motion (classical mechanics)

Senior High School (~16-18 years old)	• Excerpts from five physics textbooks.
	  ○ Four focusing on forces and motion (classical mechanics)
	  ○ One focused on quantum physics
	• Excerpts from video and audio of one unit in a final year high school classroom.
	  ○ Includes sixteen classes focused on quantum physics
	• Marked final exam student responses focused on quantum physics, classical mechanics, special relativity and electromagnetism
	  ○ Seven exams ranging from high to low achieving responses

1st Year Undergraduate University	• Excerpts from one physics textbook
	  ○ Focused on quantum physics
	• Excerpts from video and audio of one unit in an undergraduate lecture series.
	  ○ Includes twelve lectures on quantum physics
	• Marked final exam student responses focused on quantum physics, fluid physics and electromagnetism.
	  ○ Twenty exams ranging from high to low achieving responses

Table 2.7 Corpus used in this thesis. Full details in Appendix C

This corpus presents a large and varied data set that involves a high usage of image and mathematics. The high use of mathematics and images allows an insight into the possible variation in each of these resources in physics, which is crucial for developing the descriptive models in Chapters 3-5. In addition, the range of data from primary school to university allows the development of knowledge in physics to be tracked, and the evolving role each resource plays in construing this knowledge. The corpus thus offers a rich pool of data that underpins both the descriptive and knowledge-building goals of this thesis.

The more specific segments of the corpus used for various components of the model will be detailed in each chapter.

2.6 Knowledge and multisemiosis in physics

The three traditions traced in this chapter have significantly grown our understanding of scientific knowledge and the resources used to construe it. Each chapter builds upon these traditions in order to expand the models and further understand how physics organises its knowledge and discourse. Chapter 3 develops O'Halloran's (2005) description of mathematics to construct a fully systematised grammar of mathematics based on axial principles that brings out mathematics specific functionalities. Chapter 4 considers the interaction of mathematics and language through the Systemic Functional model of genre and utilises Legitimation Code Theory's dimension of Semantics to trace the role of mathematics in building physics knowledge from primary school to university. Chapter 5 interprets mathematics and images in terms of field and contrasts them with the field-based description of scientific English to reveal the specific types of meaning each resource offers for physics. Finally, Chapter 6 brings together the threads that arise in the preceding chapters and calls for the development of a genuine semiotic typology. We now turn to the descriptive component of the thesis that looks at mathematical symbolism as a meaning-making resource in its own right.

CHAPTER 3

A Grammar of Mathematical Symbolism

In science, mathematics is pervasive. It permeates many disciplines, often underpinning theoretical and descriptive architecture. This is apparent in no discipline more so than physics (Parodi 2012), and is highlighted throughout studies of scientific discourse informed by SFL and social semiotic theory. Huddleston et al. (1968: 684), in their early report on the grammatical features of scientific English, suggest the distinctive role of mathematics to be one of the most obvious differences between scientific texts and most registers of English. The more recent work of O'Halloran (e.g. 2005) and Lemke (e.g. 2003) have demonstrated that aside from its sheer quantity in use in scientific text, mathematics also provides unique avenues for meaning-making that greatly expand the potential of science (see Chapter 2).

As students move through schooling, mathematics increasingly plays a critical role in the high stakes reading needed to build their knowledge of physics and the high-stakes writing of assessment. However the uncommon sense nature of mathematics creates a potential barrier for students accessing physics. As with the language of science in general, physics discourse is considerably removed from the everyday language students use and from the language of other academic disciplines (Halliday and Martin 1993). Without effective instruction, many students cannot access this highly valued language, and will struggle for success through schooling. As discussed in Chapter 1, in response to this general problem, a program of educational intervention has been developed, informed by the Systemic Functional model of language (cf. Rose and Martin 2012, Martin and Doran 2015e). Crucially, this program is buttressed by rich descriptions of the English language from the perspectives of lexicogrammar (Halliday and Matthiessen 2014), discourse semantics (Martin 1992a), genre (Martin and Rose 2008) and various registers of academic language (e.g. Halliday and Martin 1993, Martin and Veel 1998 for science). These descriptions provide the basis for the knowledge about language and text structure that is crucial in Sydney School genre pedagogy (Rose and Martin 2012). Without these elaborate descriptions and their recontexualisation for educational purposes, teachers, and consequently their students, would be left with only a common sense understanding of the linguistic resources necessary to read and write the highly valued texts that construe academic knowledge.

As it is for language, so it is for mathematics. If we seek to develop an explicit pedagogy that encompasses multimodal meaning making, we must have rich descriptions of the various semiotic resources used in the wide array of academic disciplines comparable to those we have for language. With regard to mathematics, this is particularly pertinent; mathematical symbolism construes extraordinarily uncommon sense knowledge and, as we will see, is organised in a manner which is considerably distinct to English. It is, however, remarkably consistent in its meaning making patterns, and can be described through relatively few systems. This chapter takes up this task by developing a comprehensive description of the grammar of algebraic mathematical symbolism used in school and university physics. This will be consolidated in the following chapter through a description of the bimodal genres that involve both English and mathematics.

By developing these descriptions, we can interpret the progression of mathematics in physics from primary school to university physics through shifts in both genre and grammar. These shifts organise the meanings made at various stages in the knowledge building, allowing physics to build relatively condensed and abstract models of its object of study, at the same time as maintaining contact with the real world. As an organising principle, Chapter 5 will interpret the various changes in terms of the register variable field, and will bring in images to round out the picture. By developing descriptions of mathematical symbolism at the levels of grammar, genre and register, the specific roles mathematics play in knowledge building can be more effectively compared and contrasted to scientific English as described over the last sixty years of Systemic Functional work. To appropriate Rose's (2001: 2) justification for his description of Pitjantjatjara:

> My purpose for undertaking this description was not simply to document the features of [mathematical symbolism] for the benefit of the academy, but to provide a comprehensive account of its resources for meaning that can be systematically related to functional accounts of these resources in spoken and written English [and other languages and resources]... It is my hope that this will help inform [mathematics and science] pedagogy.

Although the justification for developing the descriptions and the data used both come from the field of education, the descriptions here are not, as it were, educational. That is, the descriptions here are not those of a teachers' grammar. The descriptions developed here are first and foremost semiotic descriptions that aim to do the semiotic system under study justice

(Halliday 1992b, Caffarel et al. 2004b). In these terms, although the description seeks to be appliable, it is not yet applied. Much of this chapter is thus more akin to work within Systemic Functional language description (e.g. Caffarel et al. 2004a, Martin and Doran 2015b) than it is to work with more specific educational concerns. By building a description that is neutral to any particular applied research goal, it is hoped it can be adapted for any purpose, educational or otherwise. The description is thus situated within the field of semiotic description and typology (a field inclusive of language description and typology) (Martin 2011a).

Mathematical symbolism is a semiotic resource that is organised in a substantially different manner to English. The grammar developed in this chapter brings out these differences while also providing certain points of contact and similarity between the two. The description primarily builds upon O'Halloran's (2005) description of mathematics, aiming to build a formalised systemic and structural account of the grammar.

3.1 Principles of description

Semiotic description in the Systemic Functional or Social Semiotic tradition has taken many forms. As discussed in Chapter 2, a range of different assumptions have been used as the basis for description, but there has been insufficient discussion of their motivation and of the criteria for justifying distinctions. Accordingly, in the absence of an agreed upon methodology for semiotic description, this chapter takes three principles as the overarching goals to guide the development. First, the description must in some way bring out the specific functionality of the resource under study. This involves accounting for the possible variation within the resource and proposing varying degrees of generalisation so as to push beyond a simple inventory of discrete possibilities. Second, the description must be able to be compared with descriptions of other resources (such as gesture, image, English, Pitjantjatjara, Tagalog etc.), and in doing so show similarities and differences in organisation. Third, the description must be based upon explicit methods of argumentation that allow it to be compared and judged in relation to competing descriptions of the same resource. These three broad principles will guide this chapter, which will thus take a step toward building a methodology and theoretical basis upon which descriptions can be developed, compared and argued over.

To develop an adequate semiotic typology that allows for description, comparison and argumentation, each semiotic resource must be described on their own terms. This means first and foremost that categories developed for language cannot be transferred unquestioningly into the description of other resources (see Caffarel et al. 2004b for similar cautions in relation to language description). Any category proposed must be justifiable internal to the system being studied.

The basis for the description in this chapter will be the interaction of the paradigmatic and syntagmatic axes. Following Martin (2013), these axial relations are taken as the theoretical primitive from which larger macrotheoretical categories such as metafunction, rank and strata can be derived. Recapping the more detailed argument developed in Chapter 2 (Section 2.4.4), we will focus briefly on the axial basis for metafunction. Broadly, metafunction can be derived from two distinct strands: the relative paradigmatic in(ter)dependence of systems, and the type of syntagmatic structure (for a more fully developed discussion of the criteria for determining metafunction see Martin 1983). In terms of paradigmatic systems, evidence for suggesting distinct functional components hinges on systems being more or less independent of each other. If choices in a bundle of systems are shown to be relatively independent of another bundle of systems, there exists evidence for distinct functional components. Structurally, Halliday (1979) suggests that distinct functional components tend to be realised by distinct structural modes of meaning. For English, logical meaning is related to iterative univariate structures, experiential meaning to multivariate structures, interpersonal meaning to prosodic structures and textual meaning to periodic structures. Thus, if different bundles of systems tend to be realised by different types of structure, there is evidence for metafunctions. Axial argumentation such as this will be used throughout the various components of the grammar of mathematical symbolism.

3.2 Architecture of the grammar of mathematical symbolism

Developing a description with axis as a theoretical primitive leads to an architecture of mathematics that is significantly different to that of English (or indeed any other language described in Systemic Functional terms to date). The three broad areas of difference revolve around the predominant types of structure coordinating the architecture (see Section 3.4.1-3.4.5), the metafunctional organisation (Section 3.4.7) and the levels (rank and nesting) at which choices are made (Section 3.4.6). Each of these will be briefly forecast before moving onto the description proper. Questions regarding stratification in mathematics will not be pursued in detail here.

Very broadly, the structure of mathematical symbolism can be characterised as having a very large univariate component. This means that throughout the grammar most choices can be made iteratively. Although there are some multivariate (non-iterative) components, these are restricted to smaller segments of the overall system (see Section 3.4.3). This predominantly univariate structure determines significant aspects of both the metafunctional and hierarchical organisation of the system.

Looking metafunctionally, the most elaborate areas of the grammar resemble those of the logical component in a language like English (Section 3.4.7.1) In addition, there are two other components with distinct systems and structures. One that can be compared to the textual component in English, and another that will be termed the operational component (Sections 3.4.7.2 - 3.4.7.3). Notably, there are no groups of systems or structures that resemble those comprising the interpersonal metafunction in language (Section 3.4.7.4). If we are taking metafunctions as emergent phenomena from axis and not as primitives themselves, it follows that without any distinct interpersonal systems or structures, there is no evidence for an interpersonal component in the grammar of mathematical symbolism. This pushes one step further O'Halloran's (1999, 2005) insight that in the evolution of mathematical symbolism from language, interpersonal meaning has been dramatically contracted; this chapter in fact suggests it does not appear at all.

Looking in terms of the hierarchical level organisation of mathematical symbolism, we again see that mathematics is organised differently to languages such as English. There is a small two-level rank-scale built out of the operational component, organising a small set of systems on both levels. However the main hierarchical organisation has to do with obligatory univariate nesting (Section 3.4.4), derived from the logical component. The hierarchy of mathematics is thus an interaction between univariate-based nesting and a multivariate-based rank scale.

Each of these main features of mathematical symbolism poses challenges for Systemic Functional descriptive procedures, and so will be built up and justified in detail as the description is developed.

3.3 Boundaries on the description

Before moving to the description, a final note must be made of the limitations of the grammar. First, the mathematics considered in the grammar is only the algebraic symbolism used in physics in schooling and first year university (for details of the corpus see Appendix C). This means that the description does not consider other registers of mathematics such as calculus or geometrical symbolism. The second restriction is that it only looks at the internal organisation of the mathematical system; it does not consider the interaction of mathematical symbolism with English or any other resources, such as in *Let* $x = 2$. This allows the mathematics to be seen as a system in its own right, and provides a firmer basis to consider what happens when it does interact with language.

While the methodology and theoretical apparatus to deal with non-linguistic semiotic systems is still being developed, it is appropriate to consider such a restricted grammar. This is in part because at this stage, as discussed above, it is not clear whether all the various types of mathematical symbolism can be considered one semiotic resource or a family of resources, nor is there any clear understanding as to where exactly to draw the line between mathematical symbolism and written language (e.g. O'Halloran's 2005 discussion of semiotic adoption), or indeed how to draw boundaries between any semiotic resources (Bateman 2011). An axial approach to semiotic description offers a path to make some progress with these and many other questions.

Due to the constraints on the grammar detailed above, the description laid out in this chapter is offered as a step toward building a more comprehensive description that accounts for all mathematical symbolism and the broader field of semiosis in general.

3.4 Grammar of mathematics

The bulk of the chapter will be devoted to building the systems and structures needed to account for the variation in mathematics. The description will begin with individual symbols and their complexing relations into expressions (Section 3.4.1). Following this, we will look at larger mathematics statements such as equations (Section 3.4.2), taking into account their organisation of information flow, and their further complexing. We will then move to consider the internal variation of symbols themselves (Section 3.4.3) before considering the impact the univariate nature of mathematics has on the hierarchy of units in mathematics

(Section 3.4.4). The final component of the grammar will consider the different elements that can occur within symbols (Section 3.4.5). Once these descriptions have been completed, they will form the basis of a discussion of the rank and nesting hierarchies in mathematics (Section 3.4.6) and the metafunctional organisation of the grammar (Section 3.4.7).

3.4.1 Expressions as symbol complexes

To begin our discussion, consider the following equation:

(3:1) $F = 0.5 \times 3$

The equation is made up of an equals sign, =, surrounded on either side by two *expressions*: F and 0.5×3 (following the terminology of O'Halloran 2005 and Huddleston et al. 1968). Each expression in turn consists of *symbols*[21]: the expression on the left contains a single symbol F, whereas the expression on the right consists of two symbols, 0.5 and 3, related by the multiplication operator ×. The organisation of the statement into expressions and symbols is shown in Table 3.1

F	=	0.5	×	3
expression		expression		
symbol		symbol		symbol

Table 3.1 Expressions and symbols

The units expression and symbol are presented at this stage so that we have some basic terminology to discuss mathematical statements. Further into the discussion, we will formally derive symbols in relation to axis. We will also show that expression as a unit is not needed in the formalised grammar; however at this stage it provides a useful term for each side of the statement. To begin, we will build up from the smaller of these units, the symbol.

[21] O'Halloran terms this unit component (1996, 2005) or more recently element (2007b). The term symbol is preferred here as a more register neutral term than component as, in physics, components refer to a single dimension within a multi-dimensional vector, signified by a symbol with a subscript, such as *Fy*. Element will be used for a separate but closely related unit, see Section 3.4.5.

Equation (3:1) above shows that symbols can occur within expressions either on their own or in relation to other symbols. If they occur with other symbols, the relations between them, known as operators, can be of various types: multiplication ×, division ÷, addition +, subtraction -, roots √ and powers shown by a superscript, e.g. x^2 (the system for these relations will be detailed in the following section). The relations set up occur between two symbols or groups of symbols. As O'Halloran (2005) points out, there is no limit to the number of relations and symbols that can occur within an expression. This means that symbols in expressions are indefinitely iterative. The expression on the right hand side of (3:2), for example, shows four symbols (3, z, 9 and 2) related by three operators (×, -, +).

(3:2) $y = 3 \times z - 9 + 2$

Continuing O'Halloran's argument, in expressions there is no need to postulate a central entity that functions analogously to the Medium in an English clause (that can be contrasted to an Agent or Range). Each operator must necessarily have a symbol or group of symbols on both sides of it. As they relate two symbols or groups of symbols, in mathematics these operators are known as binary operators (in contrast to unary operators which will be introduced in Section 3.4.3).

The possibility for indefinite repetition of symbols without any central entity indicates that the grouping of symbols into expressions is best modelled as a univariate structure (Halliday 1965). This means that each symbol has the same function as all others; what differs is the specific dependency relation among symbols in any particular expression. Expressions can thus be viewed as symbol complexes. This is similar to one dimension of Halliday's description of the English nominal group (1985), whereby the group is seen as a complex of words. Unlike the English nominal group, however, there is no evidence for a multivariate interpretation as well (along the lines of the Epithet, Classifier, Thing etc. distinctions). Rather, the grouping of symbols in expressions can be entirely explained univariately.

3.4.1.1 System of EXPRESSION TYPE

Throughout this chapter, the systems that organise the description will progressively be built, complementing the structural descriptions. These will form the basis of the broader conclusions about the organisation of mathematics discussed later in the chapter. As the description is being built piece-by-piece, some systems will necessarily be simplified until further areas of the grammar have been filled out. Any changes to previous systems will be specified as we go. To begin, we will focus on the system of EXPRESSION TYPE, which organises the number of symbols within each expression.

As any number of symbols can occur within expressions, the system of EXPRESSION TYPE is modelled as a recursive system. The first choice then, is between an expression involving only a single symbol, termed [simple], or those that include multiple symbols, termed [complex]. As mentioned above, if more than one symbol occurs, with every new choice of symbol, there must also be a choice of binary operator (O'Halloran's operative processes) - such as ×, -, + etc. This choice occurs within the system of BINARY OPERATION, with the two most indelicate choices being between [arithmetic] and [exponentiation], described in more detail below. Choosing [complex] means that the system of EXPRESSION TYPE is entered again, to determine whether a third symbol is to be chosen. This recursion can go on indefinitely, resulting in the system network shown in Figure 3.1.

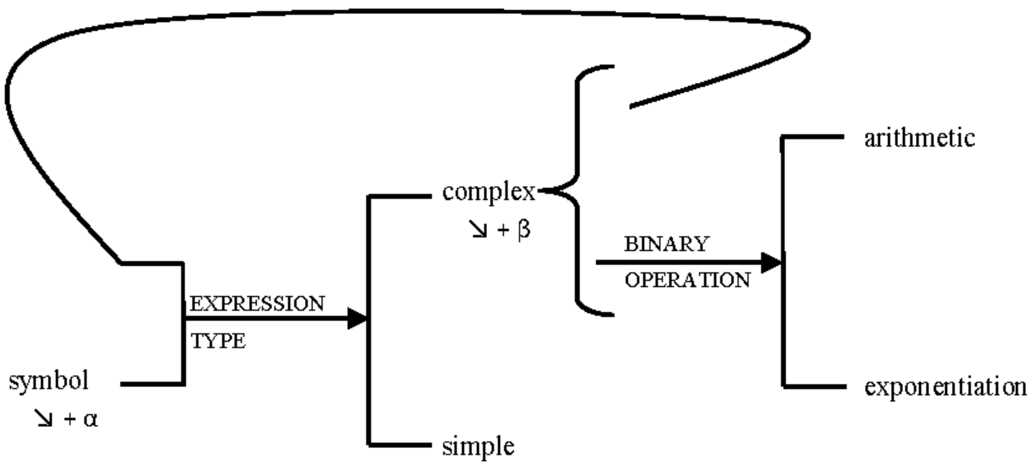


Figure 3.1 Simplified system of EXPRESSION TYPE

The network shows that when choosing the feature [complex] an extra symbol is inserted, denoted by β. As well as this, a choice from the BINARY OPERATION system must be made and the EXPRESSION TYPE system is entered again.[22] The wire from [complex] to EXPRESSION

[22] Right facing square brackets, '[', indicate alternative, either/or choices. Right-facing braces, '{', indicate simultaneous choices. Left-facing brackets, '}', indicate a disjunctive entry condition. System network formalism described in detail in Appendix A.

TYPE indicates a recursive loop, with the possibility for indefinite iteration. Structurally, each symbol is realised by the insertion of a function, the first of which is labelled α, the following β, then γ etc.[23] This produces a univariate structure where the same function is repeated indefinitely. The use of the Greek alphabet, α, β etc. is not used to suggest a hypotactic relation, as in Halliday's description of English (1965). Rather it is used to distinguish the complexing of symbols with other forms of complexing that will use 1, 2, etc. and will be described further into the chapter in Section 3.4.2.2.

3.4.1.2 System of BINARY OPERATION

The system of BINARY OPERATION sets up the choices of operators between symbols, such as the arithmetic operations ×, -, +, ÷ and others grouped under the feature [exponentiation] that include powers, roots and logarithms. Binary operations necessarily link two symbols. Operations that involve only one symbol, such as the trigonometric functions sine, cosine and tangent, factorials ! and absolute value |…|, are classed as unary operators and will be dealt with in section 3.4.3. Binary operations are chosen each time [complex] is selected in the system of EXPRESSION TYPE, as shown by Figure 3.1. The set of operations accounted for in this description is shown in Table 3.2. This table shows the terminal realisations of the network of BINARY OPERATIONS.

[23] The network only shows the insertion of β with the choice of [complex]. More strictly, it should specify that the realisation of [complex] is the insertion of function indicated by the Greek letter following the previous insertion. This has not been spelt out in the network purely for ease of reading.

Operation	**Symbol**	**Example**
addition	$+$	$x+4$
subtraction	$-$	$x-4$
addition and subtraction	$\pm$	$x \pm 4$
division	$\div$ or — or $/$	$x \div 4$ $\frac{x}{4}$ $x/4$
multiplication	$\times$ or elided	$4 \times x$ $4x$
power	superscript	x^4
root	√	$\sqrt[4]{x}$
logarithm	log	$\log_4 x$

Table 3.2 Binary operations

The network developed in this section will describe a set of generalisations based on agnation patterns. Agnation patterns allow for instances to be grouped according to their similarities and differences with other instances, based on what can or cannot be said under certain conditions (or possibly more appropriately, what is acceptable or not under these conditions) (for a detailed discussion of agnation, see Davidse 1998). As an example of agnation patterns in English, we can consider MODALITY in relation to MOOD and TRANSITIVITY (as we did in Chapter 2, Section 2.4.4). MODALITY choices can occur for all TRANSITIVITY types:

(3:3) *It may please the staff* mental + modality

(3:4) *Roxburgh may play Estragon* relational + modality

(3:5) *She may praise the man* verbal + modality

But is more restricted in relation to MOOD types:

(3:6) *She may build him a house* indicative + modality

(3:7) **may build him a house* imperative + modality[24]

The restrictions on MODALITY in relation to MOOD but not TRANSITIVITY suggests that it is a subsystem of MOOD but simultaneous with TRANSITIVITY.

In mathematics, grammaticality judgements are more sharply defined than in language. This is due to the fact that mathematics is a designed system with strictly defined possibilities for variation. In addition, the realisations of BINARY OPERATIONS are a relatively small closed class, allowing many choices to be realised lexically.

3.4.1.2.1. Arithmetic operations

The primary distinction within BINARY OPERATION opposes [arithmetic] to [exponentiation] operations. [arithmetic] operations are those of ×, ÷, –, + and ±, and [exponentiation] includes powers, roots and logarithms.[25] Beginning with [arithmetic] operations, there are four basic relations: multiplication ×, division ÷, addition + and subtraction –. These operations each have different characteristics that can be grouped as two cross-classifying pairs. The first characteristic distinguishes those operations which are both associative and commutative (+ and ×), against those that are not (- and ÷). Being commutative indicates that the order of the symbols related by this operation does not matter; symbols are reversible. Both addition (+) and multiplication (×) are commutative for every value of *a* and *b*:

(3:8) addition as commutative:

$$a + b = b + a$$

e.g.

$$2 + 4 = 6$$

[24] Note that an asterisk * indicates that something is ungrammatical or unacceptable, i.e. it cannot be said.

[25] In mathematics, the term *exponentiation* is often reserved just for the power relation. However as I am not aware of a general term that covers powers, roots and logarithms, and as my argument for grouping them together involves the simple conversion between them and their interlocking definitions, I am appropriating [exponentiation] to cover all these operations. Obviously the term *exponentiation* in this sense is also distinct from the linguistic process of realising a grammatical category with a phonological or graphological exponent.

$$4 + 2 = 6$$

(3:9) multiplication as commutative:

$$a \times b = b \times a$$

e.g.

$$2 \times 4 = 8$$
$$4 \times 2 = 8$$

These equations show that with addition (+) and multiplication (×), the sequence of the symbols a and b can be reversed without affecting the result. i.e. $a + b$ is the same as $b + a$, and $a \times b$ is the same as $b \times a$. As well as being commutative, addition and subtraction are both associative. This indicates that when there are multiple operations of the same type, the order in which each operation is made does not matter. This can be shown through adding in brackets around different combinations of a, b and c:

(3:10) addition as associative:

$$(a + b) + c = a + (b + c)$$

e.g.

$$(2 + 4) + 6 = 12$$
$$2 + (4 + 6) = 12$$

(3:11) multiplication as associative

$$(a \times b) \times c = a \times (b \times c)$$

e.g.

$$(2 \times 4) \times 6 = 48$$
$$2 \times (4 \times 6) = 48$$

In this regard, multiplication and addition can be opposed to division and subtraction which are neither associative nor commutative:

(3:12) subtraction, non-commutative:

$$* \; a - b = b - a$$

e.g.

$$2 - 4 = -2$$
$$4 - 2 = 2$$

(3:13) division, non-commutative:

$$* \; a \div b = b \div a$$

e.g.

$$2 \div 4 = 0.5$$
$$4 \div 2 = 2$$

(3:14) Subtraction, non-associative:

$$* \; (a - b) - c = a - (b - c)$$

e.g.

$$(2 - 4) - 6 = -8$$
$$2 - (4 - 6) = 4$$

(3:15) division, non-associative:

$$* \; (a \div b) \div c = a \div (b \div c)$$

e.g.

$$(2 \div 4) \div 6 = \frac{1}{12}$$
$$2 \div (4 \div 6) = 3$$

To generalise this distinction, we will say that × (multiplication) and + (addition) hold the feature [associative], and ÷ (division) and – (subtraction) hold the feature [non-associative].[26] This system of ASSOCIATIVITY is cross-classified with another system that groups × (multiplication) with ÷ (division), and + (addition) with – (subtraction). This distinction is

[26] And so, in this system, the feature [associative] includes both associativity and commutativity.

intended to capture the regularly used conversion between division and multiplication, and between addition and subtraction in mathematical texts, whereby:

(3:16) $a \div b = a \times \frac{1}{b}$

and

(3:17) $a + b = a - (-b)$

These equations show that division can be converted to multiplication by dividing a with $\frac{1}{b}$. Similarly, addition can be converted to subtraction by subtracting (–*b*) from a. These conversions are regularly used in mathematical derivations and as such are an important relationship between the operations.

The equations above each shift b to its inverse. For equation 3:16, shifting division to multiplication, the inverse of b is $\frac{1}{b}$. This is known as its reciprocal or *multiplicative inverse*. In 3:17, shifting addition to subtraction, the inverse of *b* is –*b*, known as its *additive inverse*. The different forms of the inverse are a result of different variables known as identity elements. An identity element (*x*) is the number that when operated on another number (*y*) results in the same number (*y*). Using multiplication as an example, *x* is the identity element if: $y \times x = y$. For multiplication and division, this number is 1. Any number can be multiplied or divided by 1 and the result is the same number: $5 \times 1 = 5$ and $5 \div 1 = 5$. Multiplication and division, then, have an identity element of 1. Addition and subtraction, on the other hand, have an identity element of 0. When adding or subtracting from a number, it is 0 that produces the same number: $5 + 0 = 5$ and $5 - 0 = 5$. Due to their shared identity elements, multiplication can be converted to division, and addition can be converted to subtraction and vice versa.

To capture this distinction, a system termed INVERSE PAIR is proposed with two features: [multiplicative], which includes multiplication and division that have the multiplicative inverse, and [additive], which includes addition and subtraction that have the additive inverse. The cross-classification of the systems of INVERSE PAIR and ASSOCIATIVITY results in a paradigm that produces each of the basic arithmetic operations, shown in Table 3.3.

INVERSE PAIR / ASSOCIATIVITY	**multiplicative**	**additive**
associative	multiplication (×)	addition (+)
non-associative	division (÷)	subtraction (-)

Table 3.3. Paradigm of arithmetic operations

The basic system of arithmetic operations thus involves two simultaneous systems, shown in Figure 3.2.

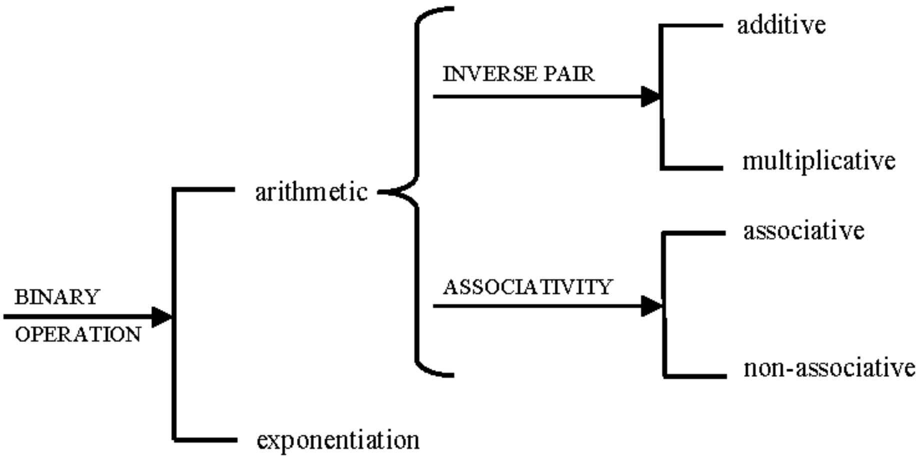


Figure 3.2 Options for arithmetic operations

Adding to these basic operations is the possibility for the operation ±, usually glossed as 'plus or minus'. This operation is an [additive] inverse operation, as 0 is its identity element, e.g: $5 \pm 0 = 5$. As well as this, it is both non-associative: *$a \pm (b \pm c) = (a \pm b) \pm c$; and non-commutative: * $a \pm b = b \pm a$. Therefore it takes the feature [non-associative]. This combination of [additive] inverse and [non-associative] is the same as subtraction (–). Thus a conjunctive entry condition between these features is proposed that produces a system distinguishing between [subtraction] (–) and [plus-minus] ($\pm$).

The final distinctions deal with alternative realisations of multiplication and division. Multiplication can be explicitly realised by $\times$, as in $5 \times b$, or more commonly, it can be elided, as in $5b$. There are three symbolic realisations of division, the obelus ÷, as in $5 \div b$, the vinculum —, as in $\frac{5}{b}$, and the forward slash /, as in $5/b$. The full network of arithmetic operations is shown in Figure 3.3.[27]

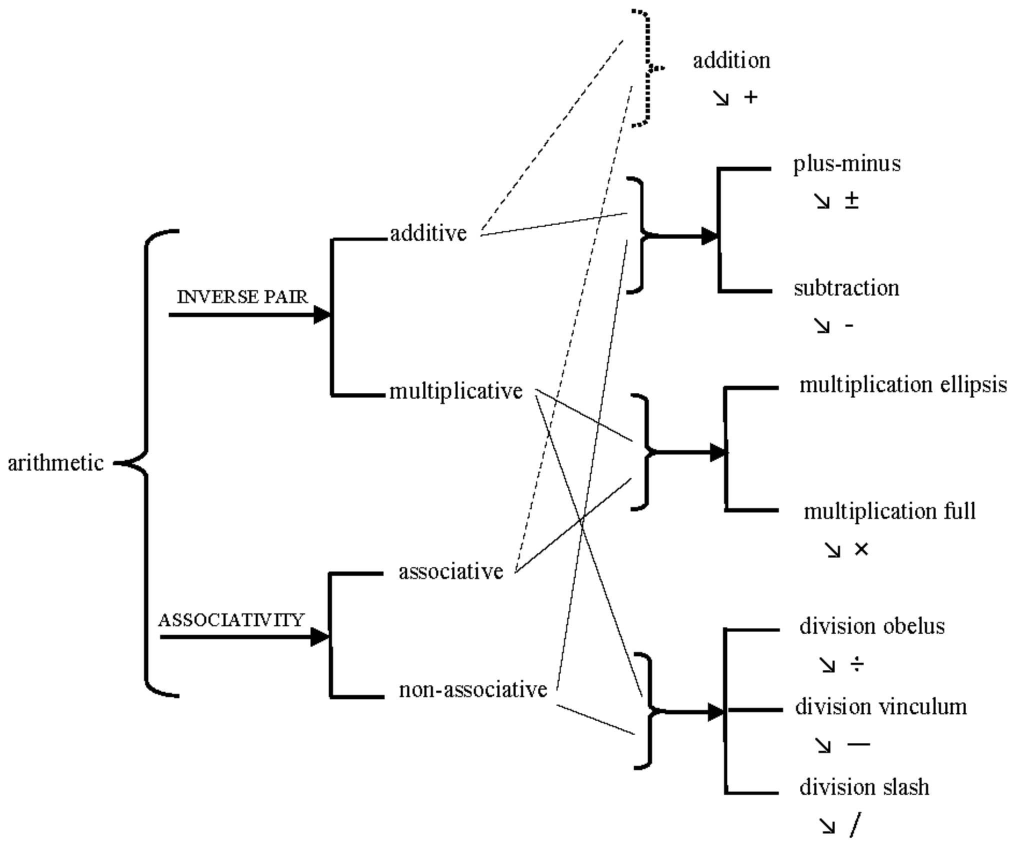


Figure 3.3 Network of arithmetic operations.

3.4.1.2.2 Exponentiation operations

As well as the arithmetic operations described above, there is a second group of binary operations occurring in high school physics that will be grouped here under [exponentiation]. [exponentiation] includes three distinct operations: roots such as $\sqrt[3]{x}$, powers such as x^3 and logarithms such as $\log_3 x$. The justification for grouping these three operations together derives from their mutual definition, and the fact that they each provide a different perspective on the same relation between three numbers or variables (see section 3.4.2.5).

[27] [addition] has been added with dotted lines and a dotted brace to indicate that the selection of [additive] and [non-associative] is realised by +, following the convention in Martin 2013:20. Formally, this is not required, however it is helpful in order show explicitly the realisation rule for these choices.

Powers, roots and logarithms are all mutually definable in that they each relate two of a set of three numbers, to equal the third. We can see this by taking a, b and c as variables, and using as examples the numbers $a = 2$, $b = 5$, $c = 32$. [power] relates a and b to equal c:

(3:18) $a^b = c$ [power]

e.g. $2^5 = 32$

[root] relates b and c to equal a:

(3:19) $\sqrt[b]{c} = a$ [root]

e.g. $\sqrt[5]{32} = 2$

[logarithm] relates a and c to equal b:

(3:20) $\log_a c = b$ [logarithm]

e.g. $\log_2 32 = 5$

From these we can show an equivalence relation between roots, powers and logarithms:

$a^b = c$ is equivalent to $\sqrt[b]{c} = a$ is equivalent to $\log_a c = b$

This means that if one relation holds true, each of the others also hold true. This is exemplified by drawing on the numerical examples used above:

$$2^5 = 32 \Leftrightarrow \sqrt[5]{32} = 2 \Leftrightarrow \log_2 32 = 5$$

These agnation patterns justify grouping powers, roots and logarithms under the single feature [exponentiation].[28] Adding these features to the system of arithmetic operations produces the full system of EXPRESSION TYPE in Figure 3.4.

The system in Figure 3.4 shows that expressions can include a single symbol or a symbol complex related through binary operations. As discussed previously, the recursive system allows for an indefinite number of symbols to occur in the expression. Consequently, symbol complexes are best modelled as univariate structures. Their univariate nature is not unique to symbol complexing, however, as we will see below. Rather, it is a motif that runs through the entire system of mathematics - grammar, register and genre. Aside from being a distinct typological feature in comparison to language, this property has broader implications for the functional organisation of mathematics and its use in physics. Before discussing this, however, we will turn to the organisation of expressions into larger statements.

[28] Regarding [power], O'Halloran (2005: 124) suggests that powers can be grouped as textual variations of multiplication as, for example $y^3 = y \times y \times y$. Grouping powers and multiplication together, however, does not account for powers involving transcendental numbers such as π, e.g. x^{π}. x^{π} cannot be broken down into relations of multiplication or division, as one cannot have π multiples of a number. For this reason, it is best to group powers and multiplication as separate features.

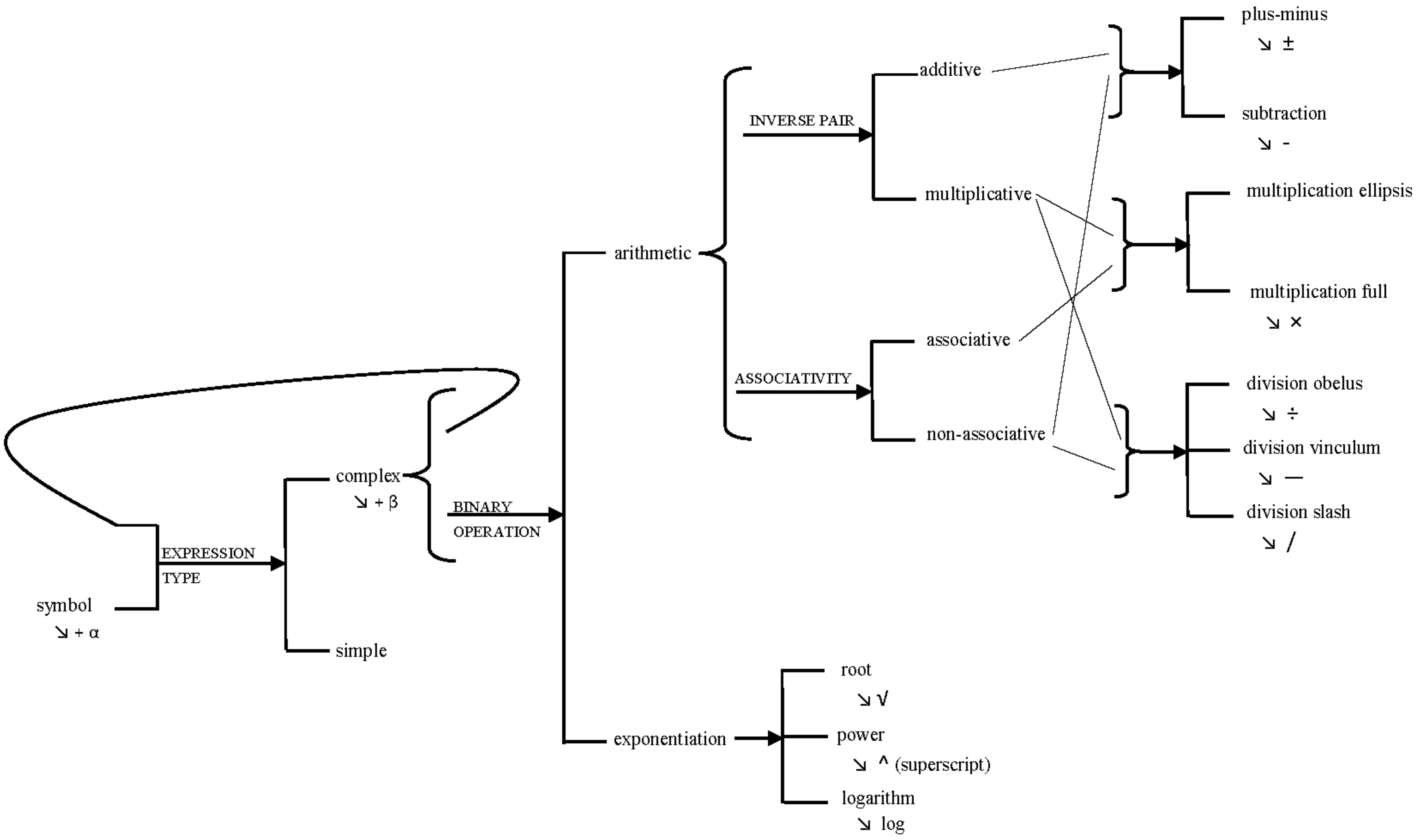


Figure 3.4. Full system of EXPRESSION TYPE

3.4.2 Mathematical statements

In written texts, mathematical symbolism is organised into *statements*, the most common of which are equations, such as $\Delta y = u_y t + \frac{1}{2} a_y t^2$. Statements involve at least two expressions, e.g. Δy and $u_y t + \frac{1}{2} a_y t^2$, linked by a relator (e.g. the equals sign =). This section will be concerned with mapping the possible options and structures of statements. It will focus in particular on how statements link expressions and how they coordinate their meanings within larger texts. One lesson learnt from systemic descriptions of language is the importance of looking at examples in real data. Large stretches of text can highlight patterns impossible to see in de-contextualised examples, which in turn can provide insights into structural relations. An example of this is Fries' (1981) study of thematic patterns in English text, which provides a discourse-based justification for Halliday's clausal Theme in English (1985). Fries' study shows that thematic progression correlates with the structure of unfolding discourse, and as such is not arbitrarily chosen. Without such an ecologically sensitive interpretation, we'd be left with English Theme defined purely by syntactic sequence (i.e. it comes first) and the notional glosses of 'point of departure' or 'what the clause is about'. A discourse-based justification makes clear what the role of the grammatical function Theme is in language.

3.4.2.1 Information organisation of statements

Considering the importance of text-based descriptions, our focus on mathematical statements will take as its point of departure Text 3.1, from a senior high school textbook.

Calculate how much weight a 50kg girl would lose if she migrated from the earth to a colony on the surface of Mars.

Answer

On the earth: On Mars:

$$\boldsymbol{W}_{\text{earth}} = m\boldsymbol{g}_{\text{earth}}$$
$$= 50 \times 9.8$$
$$= 490 \text{ N downwards}$$

$$\boldsymbol{W}_{\text{Mars}} = m\boldsymbol{g}_{\text{earth}}$$
$$= 50 \times 3.6$$
$$= 180 \text{ N downwards}$$

Loss of weight = 490 – 180 = 310 N. But there is no loss of mass!

Text 3.1 (a). de Jong et al. (1990: 249)

For our discussion, we will extract the mathematics and look at Text 3.1(b):

$$\boldsymbol{W}_{\text{earth}} = m\boldsymbol{g}_{\text{earth}}$$
$$= 50 \times 9.8$$
$$= 490 \text{ N downwards}$$

$$\boldsymbol{W}_{\text{Mars}} = m\boldsymbol{g}_{\text{earth}}$$
$$= 50 \times 3.6$$
$$= 180 \text{ N downwards}$$

Loss of weight = 490 – 180 = 310 N.

Text 3.1 (b). de Jong et al. (1990: 249)

The mathematical statements in this text are grouped into three sections, beginning with $\boldsymbol{W}_{\text{earth}}$, $\boldsymbol{W}_{\text{Mars}}$ and Loss of weight. To begin, we will focus on the two sections beginning with $\boldsymbol{W}_{\text{earth}}$ and $\boldsymbol{W}_{\text{Mars}}$. Each of these sections involving three lines, moving through a similar sequence (in Chapter 4, these will each be classed as a genre termed *quantification*). The first line includes a single symbol on the left (e.g. $\boldsymbol{W}_{\text{Mars}}$) and a symbol complex on the right(e.g. $m\boldsymbol{g}_{\text{earth}}$). The second line elides the symbol on the left of the equation, and replaces the right side with numbers (e.g. 50×3.6). The final line continues the ellipsis of the left side and concludes the right side with a single number (e.g. 180) as well as units (N, glossed as 'Newtons', which are the units of force) and a direction (downwards). The ellipsis of the left side of the equation in lines two and three is a common pattern in mathematics. Filling in the ellipsis, these sections become:

$$\begin{aligned} \boldsymbol{W}_{\text{earth}} &= m\boldsymbol{g}_{\text{earth}} \\ \boldsymbol{W}_{\text{earth}} &= 50 \times 9.8 \\ \boldsymbol{W}_{\text{earth}} &= 490 \text{ N downwards} \end{aligned}$$

$$\begin{aligned} \boldsymbol{W}_{\text{Mars}} &= m\boldsymbol{g}_{\text{earth}} \\ \boldsymbol{W}_{\text{Mars}} &= 50 \times 3.6 \\ \boldsymbol{W}_{\text{Mars}} &= 180 \text{ N downwards} \end{aligned}$$

In the revised version, the symbol on the left (glossed as *weight*) is repeated in each set of equations. Those on the right, on the other hand, are changing. Taking a quick glance at the statement in the final line "Loss of weight = 490 – 180 = 310 N", we see that the expression on the left also refers to *weight*, with the following expressions showing a progression of numbers. A similar progression occurs in Text 3.2, a different type of mathematical text that does not include numbers but only non-numerical symbols (we will term this text a *derivation* in the following chapter).

Alternative Form of Newton's Second Law

We have mentioned that the more momentum an object has, the greater will be the force required to stop it. But exactly how are force and momentum related?

Consider an alternative statement of Newton's Second Law (and the way in which Newton originally stated it) as follows:

The time rate of change of momentum is proportional to the resultant force and acts in the direction of the force.

To show the equality of these two different statements of Newton's Second Law, consider the following:

$$\vec{F} = m\vec{a}$$

$$\text{But } \vec{a} = \frac{\vec{v} - \vec{u}}{\Delta t}\text{ , therefore}$$

$$\vec{F} = \frac{m(\vec{v} - \vec{u})}{\Delta t}$$

$$= \frac{m\vec{v} - m\vec{u}}{\Delta t}$$

$$= \frac{\Delta m\vec{v}}{\Delta t}$$

Force is the time rate of change of momentum as stated by Newton!

Text 3.2 (a) Warren (2000:123)

Extracting the mathematics again produces Text 3.2(b):

$$\vec{F} = m\vec{a}$$

$$\vec{a} = \frac{\vec{v} - \vec{u}}{\Delta t}$$

$$\vec{F} = \frac{m(\vec{v} - \vec{u})}{\Delta t}$$

$$= \frac{m\vec{v} - m\vec{u}}{\Delta t}$$

$$= \frac{\Delta m\vec{v}}{\Delta t}$$

Text 3.2 (b) Warren (2000:123)

In this text, the first two statements shift both the left and right side of the equation. The left side shifts from $\vec{F}$ (glossed as *force*) to $\vec{a}$ (*acceleration*). The right side shifts from $m\vec{a}$ to $\frac{\vec{v}-\vec{u}}{\Delta t}$. Following this, the third statement settles on $\vec{F}$ for the left side. This is then kept stable through ellipsis in the rest of the text. The right side, however, continues to change.

The relative stability of the left-hand side in comparison to the right is a consistent feature across most texts. Added to this is the strong tendency for the left-side to contain only a single symbol (or at the very least contain fewer symbols than the right) and to be only very rarely a number (as opposed to the non-numerical forms above). These tendencies are in spite of the fact that it is perfectly grammatical to swap the left and right side, or to have more symbols or numbers on the left than the right. Equations (3:21-26) are all grammatical, and in some sense mean the same thing:

(3:21) $\boldsymbol{W}_{\text{earth}} = 50 \times 9.8$

(3:22) $50 \times 9.8 = \boldsymbol{W}_{\text{earth}}$

(3:23) $50 = \frac{\boldsymbol{W}_{\text{earth}}}{9.8}$

(3:24) $\frac{\boldsymbol{W}_{\text{earth}}}{9.8} = 50$

(3:25) $9.8 = \frac{\boldsymbol{W}_{\text{earth}}}{50}$

(3:26) $\frac{\boldsymbol{W}_{\text{earth}}}{50} = 9.8$

Despite the appearance of free variation in decontextualised examples, it is clear that in text, the probabilities of what will occur on the left and what on the right are significantly constrained. To account for this, we will follow O'Halloran's suggestion of the function Theme for the left side of a statement (2005: 124). The Theme in mathematics texts, as in language ones, tends toward relative stability. It holds in place the angle through which the statement is being viewed and signals the relevancy of the statement to its co-text by indicating the symbols to which the text is orienting. As such it locates the statement within its co-text. Accordingly, the Theme tends not to be something completely new within the text, but something that has been mentioned previously, either in the mathematics, language or other resources such as images. In Text 3.1(a) the three Themes all refer to weight ($\boldsymbol{W}_{earth}$, $\boldsymbol{W}_{Mars}$, and *Loss of weight*) which is what the question asks to be calculated. Moreover, W_{earth} and W_{Mars} distinguish themselves by their subscripts that refer back to the Circumstances of location at the beginning of their section: *On the earth* and *On Mars* respectively. As we can see, the mathematical Themes are not pulled out of thin-air; they are related to the previous co-text. The Themes are used to emphasise the angle of the field to which the statements are orienting. Labelling this function Theme emphasises the similarity with Theme in language, in regard to their relative stability and co-referentiality with previous co-text (c.f. Fries 1981 for English and Fang et al. 1995 for Chinese).

The right-hand side of the statements poses more challenges. It clearly performs a different function from Theme in that it expands the text and involves continual change. To distinguish it from Theme, we will use the term Articulation. More precisely, the Articulation includes the relator (such as =) and the following expression. The Theme-Articulation structure of one of the sections in Text 3.1 is thus:

$\boldsymbol{W}_{\text{earth}}$	$= m\boldsymbol{g}_{\text{earth}}$
Theme	Articulation
$(\boldsymbol{W}_{\text{earth}})$	$= 50 \times 9.8$
Theme	Articulation
$(\boldsymbol{W}_{\text{earth}})$	$= 490\ \text{N downwards}$
Theme	Articulation

The Articulation is an explicit elaboration of the Theme. Whereas the Theme orients the text to its field, the Articulation is more oriented toward genre staging. It shows the progression of a mathematical text from its beginning to its final result. The term Articulation is used to emphasise its difference with the English Rheme (Halliday and Matthiessen 2014). Rheme in English has a minimal effect on the textual patterns of English, aside from not being the Theme. The Articulation, on the other hand, plays a significant role in its own right. The role of the Articulation will be discussed in more detail in section 3.4.2.3 and in Chapter 4. However before this, we first need to introduce another aspect of mathematical statements, its univariate organisation.

3.4.2.2 Statements as expression complexes

The analysis of statements into Theme and Articulation allows an insight into the strong tendency for statements to organise their left and right side differently. However, as noted above, this is only a tendency. Aside from the thematic organisation, there is free variation as to what can be put on the left and right side; $\boldsymbol{W}_{\text{earth}} = 50 \times 9.8$ and $50 \times 9.8 = \boldsymbol{W}_{\text{earth}}$ are both grammatical statements. As well as this, there is a sense in which the two equations are in some sense the same. That is, the logical relations between the expressions and the symbols within it are the same. By analysing the statement purely as a Theme^Articulation structure, this similarity is not brought out; the only aspect shown is their difference. This section will be concerned with accounting for this similarity by developing a complementary structural configuration.

If taking a notionally metafunctional view of mathematics, the similarity between $F = ma$ and $ma = F$ is in their ideational meaning, as opposed to the textual meanings shown by distinct Theme and Articulation choices. To understand the ideational meanings of the statement, O'Halloran suggests a Token-Value structure for the statement (2005: 106), akin to relational identifying processes in English (Halliday and Matthiessen 2014). This Token-Value structure is independent of the Theme-Articulation structure, resulting in the analyses:

(3:27)

F	$=$	ma
Token		Value
Theme	Articulation	

(3:28)

ma	=	F
Value		Token
Theme	Articulation	

Providing a Token-Value structure complementary to the Theme-Articulation structure brings out both the similarities and differences between the above equations, however it raises a number of other issues. First, distinguishing Token from Value implies two orders of abstraction, a distinction difficult to justify in mathematics. More importantly, however, it raises the question of how to distinguish Token from Value. O'Halloran's analysis makes it clear that sequence is not criterial, via the following examples (2005: 106-107):

(3:29)

$\frac{a}{b}+\frac{c}{d}+\frac{e}{f}$	=	$\frac{adf+bcf+bde}{bdf}$
Value		Token

(3:30)

$\frac{adf+bcf+bde}{bdf}$	=	$\frac{a}{b}+\frac{c}{d}+\frac{e}{f}$
Token		Value

Her suggested probe for distinguishing Token from Value is to determine which expression would be the Subject in the active if = was translated into an English relational process, such as: $\frac{adf+bcf+bde}{bdf}$ *represents* $\frac{a}{b}+\frac{c}{d}+\frac{e}{f}$ (*Token is the Subject in the active* is a probe for English from Halliday and Matthiessen 2014, Martin et al. 2010). For mathematics, this probe suffers from two shortfalls. The first is the problem of arguing from a translation. Languages and semiotic resources organise their meanings in distinct ways. If we wish to bring out the specific organisation of each resource in their own terms, translation will tend to neutralise the specific patterns of that resource. In other words, by translating mathematics to English and arguing from English probes, mathematics will inevitably begin to look a lot like English. The second issue is more pragmatic. It is not clear why $\frac{adf+bcf+bde}{bdf}$ *represents* $\frac{a}{b}+\frac{c}{d}+\frac{e}{f}$ is preferred to $\frac{adf+bcf+bde}{bdf}$ *is represented by* $\frac{a}{b}+\frac{c}{d}+\frac{e}{f}$. Which of these forms is appropriate is not readily apparent, significantly undermining the probe. It is possible that the number of symbols in each expression could be used as a probe, however this begs the question as to what to do when there are the same number of symbols on each side.

Without a clear set of criteria for distinguishing Token and Value, it is thus not obvious which expression is Token and which is Value in each of the following statements:

(3:31) $PV = nRT$

(3:32) $nRT = PV$

(3:33) $\frac{PV}{n} = RT$

(3:34) $RT = \frac{PV}{n}$

(3:35) $T = \frac{PV}{nR}$

(3:36) $\frac{1}{nR} = \frac{T}{PV}$

A larger issue for a Token and Value analysis, however, is what to do with statements that involve more than two expressions, including examples like the following:

(3:37) $E_2 = E_1 = k$

(3:38) $\lambda = \frac{v}{f} = \frac{3\times10^8}{104.1\times10^6} = 2.9\text{ m}$

With more than two expressions, a simple binary analysis is difficult to apply without significant embedding. One possibility could be to consider these equations as being realised by iterative Token-Value structures. In this case, each Value aside from the last would be conflated with a Token. An analysis of equation 3:38 above, under these terms, would then be:

λ	$=$	$\frac{v}{f}$	$=$	$\frac{3 \times 10^8}{104.1 \times 10^6}$	$=$	2.9 m
Token		Value/Token		Value/Token		Value

The possibility for indefinite iteration of the Token-Value structure, however, indicates that it might be better to approach the statement from a different point of view.

A Token-Value distinction is based on a multivariate analysis (Halliday 1965), entailing that each expression performs a distinct function that isn't repeated. An alternative is to view the statement from a univariate perspective as involving a single function repeated indefinitely (see Section 2.1.2 for a more detailed discussion of the multivariate/univariate distinction). In this analysis, each expression would fulfil the same structural role, with the labelling simply being used to distinguish them in sequence. To distinguish this analysis from that for symbols within expressions, the numbers 1, 2 etc. will be used, ordered from left to right (following Halliday and Matthiessen's 2014 notation for parataxis). This produces the analyses:

(3:39)

F	$=$	ma
1		2

(3:40)

ma	$=$	F
1		2

(3:41)

E_2	$=$	E_1	$=$	k
1		2		3

(3:42)

λ	$=$	$\frac{v}{f}$	$=$	$\frac{3 \times 10^8}{104.1 \times 10^6}$	$=$	2.9 m
1		2		3		4

It must be stressed that assigning expressions 1 or 2 or any other number, does not indicate any sort of prominence with respect to the others. Although a taxis distinction is not being made in this description, the sequencing of expressions in statements is more akin to parataxis rather than hypotaxis, in that the expressions each have the same status. More accurately, we could assign each expression the same label, say X and use subscripts for sequencing (see Halliday 1965), as in:

(3:43)

λ	$=$	$\frac{v}{f}$	$=$	$\frac{3 \times 10^8}{104.1 \times 10^6}$	$=$	2.9 m
X_1		X_2		X_3		X_4

However for ease of tracking, and to distinguish this notation from symbols within expressions using α, β etc., we will use 1, 2, 3 etc.

A univariate analysis such as this indicates that each expression performs the same function and has the same status. This means that the ordering of the expressions is immaterial for the analysis. In this way, it brings out the similarity between $F = ma$ and $ma = F$. Both equations link two expressions with exactly the same structural function. Analysing these equations for both their univariate structure and their Theme^Articulation structure allows the following tiered structures:

(3:44)

F	$=$	ma
1		2
Theme	Articulation	

(3:45)

ma	=	*F*
1		2
Theme	Articulation	

Again we must stress that the labelling of 1 and 2 indicates nothing more than their sequence in the syntagm, not their structural relationship. The *F* in (3:44) plays the same function as the *F* in (3:45), which is the same function as *ma* in (3:44) and *ma* in (3:45). The meanings made by their sequence are accounted for in the Theme-Articulation structure discussed in the previous section.

Positing a univariate structure for statements provides an avenue for describing statements including more than two expressions. Univariate structures are, in principle, indefinitely iterative; they allow any number of expressions to be placed in sequence, making the description of extended statements a simple affair. Further to this, the issues surrounding how to justify which expression performs which function that occur with a Token-Value distinction are neutralised with labelling determined purely by sequence.

Viewing the statement through a univariate lens produces an analysis whereby statements are simply expression complexes. The minimal statement is thus an expression complex with two expressions, as in:

F	=	*ma*
1 expression		2 expression

This has affinities to the discussion in Section 3.4.1 above where expressions were considered symbols complexes. If statements are expression complexes and expressions are symbols complexes, we can say that statements are in essence highly elaborated symbol complexes. This has implications for the levels structure in mathematical symbolism and will be taken up in detail in Section 3.4.4.

Now that the basic univariate organisation of statements has been outlined, we need to address the question of what to do with the Theme-Articulation structure in extended statements with more than two expressions. Following this, we will consider the paradigmatic organisation of statement types.

3.4.2.3 Articulations revisited

The Theme-Articulation structure for statements is a multivariate structure whereby each expression in a two-expression statement performs a distinct function. This will be justified in more detail in the following chapter once genre has been introduced. Up to this point we have said that the choice of Theme is more oriented toward field, whereas the choice of Articulation is oriented toward genre. To focus on this, we will look again at Text 3.1, reproduced below:

> Calculate how much weight a 50kg girl would lose if she migrated from the earth to a colony on the surface of Mars.
>
> **Answer**
>
> On the earth:
>
> $$\begin{aligned} \boldsymbol{W}_{\text{earth}} &= m\boldsymbol{g}_{\text{earth}} \\ &= 50 \times 9.8 \\ &= 490\text{ N downwards} \end{aligned}$$
>
> On Mars:
>
> $$\begin{aligned} \boldsymbol{W}_{\text{Mars}} &= m\boldsymbol{g}_{\text{earth}} \\ &= 50 \times 3.6 \\ &= 180\text{ N downwards} \end{aligned}$$
>
> Loss of weight = 490 – 180 = 310 N. But there is no loss of mass!

Text 3.1 de Jong et al. (1990: 249)

The Themes of each set of equations hold the focus on the field stable, which angles in on the technicality *weight*. The Articulations in the first two sections under *On the earth* and *On Mars* show a steady progression from symbolic relations to their final numerical form - quantifying the weight of the girl on each planet. In the first two sections, each equation contains only two expressions so the Theme-Articulation analysis is relatively unproblematic. It is the final equation in the final line, however, that we wish to focus on now[29]:

(3:46) Loss of weight = 490 – 180 = 310 N

The Theme, *Loss of weight*, continues the angle on the field by the previous equations. Building upon this, the following two expressions show a distinct progression similar to the previous sequence of equations. The second expression, $490 - 180$, gives two numbers taken from the results of the previous sections (490 from the final line of the *On the earth* section, 180 from *On Mars*). The third expression, 310 N, gives the final result of the calculation of $490 - 180$. The ordering of the two expressions is important. The entire statement realises a short quantification genre (discussed in the following chapter), of which the final expression correlates with the stage that is the nucleus of the quantification, known as the Numerical Result. The previous expression, $490 - 180$, realises another, optional, stage called Substitution. As with all genre staging, Numerical Results and Substitutions are realised by a distinct pattern of grammatical features.[30] Furthermore, the order of the staging is strict: if they occur, Substitutions will always precede Numerical Results. Looked at from the point of view of genre, then, the ordering of the second and third expressions is not arbitrary. It is tightly determined by the generic staging.

Looked at from within the grammar, there is a sense in which the third expression follows on from and is in some way logogenetically dependent on the second expression. The 310 N is calculated from the subtraction of 180 from 490 shown in the second expression. Although in principle, the expressions can be swapped (accounted for by the univariate organisation described above), in practice the ordering has definite meaning for the logogenetic unfolding of the text - and is thus tightly constrained. For this reason, the analysis proposed here will not take the entire set of expressions and relators following the Theme as a single function (akin to the Rheme in English); rather each expression and relator pair will be given a distinct function. As there is the potential for an indefinite number of expressions in sequence, there is an issue for labelling. To surmount this, we will

---

[29] *Loss of weight* in the Theme of the statement is a single symbol for the purpose of the grammar of mathematics. Why a linguistic nominal group is being used rather than the usual single Roman or Greek letter is a matter for a more inter-modal study.

[30] As to this point we have only considered the grammar and not the possibility of register, discourse semantics or any other strata, we are necessarily skipping any stages of realisation that these strata would mediate.

call the functions Articulation$_1$, Articulation$_2$, Articulation$_3$ etc[31]. The analysis would thus become:

(3:47)

Loss of weight	$= 490 - 180$	$= 310$ N
Theme	Articulation$_1$	Articulation$_2$

(3:48)

E_2	$= E_1$	$= k$
Theme	Articulation$_1$	Articulation$_2$

(3:49)

λ	$= \dfrac{v}{f}$	$= \dfrac{3 \times 10^8}{104.1 \times 10^6}$	$= 2.9$ m
Theme	Articulation$_1$	Articulation$_2$	Articulation$_3$

By allowing the non-thematic sections of the statement to unfold as different functions, further justifications can be made for distinguishing Articulations from the Rheme of English. First, on purely structural grounds, within a single clause in English, there can be only one Rheme, and the Rheme includes everything that is not the Theme. Second, the position of groups within the Rheme has no effect for the thematic structure; the position of New, most often contained with the Rheme, is organised by separate system within the information unit (Halliday and Matthiessen 2014, Halliday and Greaves 2008). In mathematics, on the other hand, the ordering of Articulations does have meaning for this strand of structure. Thus, whereas the Theme is labelled as such to emphasise the similarity to Theme in English and other languages, Articulation is labelled to emphasise its difference with Rheme.

[31] This is similar to the labelling convention for higher levels above MacroTheme in the hierarchy or periodicity for English, being labelled MacroTheme^{i}, MacroTheme^{ii}, MacroTheme^{iii} etc. (Martin 1992b, Martin and Rose 2007). This is not however to suggest a periodic structure in the mathematical statements.

As noted above the Theme-Articulation is a multivariate structure. Each function performs a distinct role and occurs only once. The possibility for indefinite iteration of expressions, with all except the thematic expression being labelled Articulation, calls into question this characterisation. In one sense, this is purely an issue of labelling. If each subsequent expression was labelled something completely different to all others, the structure would look more comfortably multivariate; this nomenclature however would become very unwieldy very quickly. Unlike the 1, 2 labelling of the univariate structure, the numbering of labels is intended to show a distinction in the status of the Articulations; they do not all perform the same function. In a second sense, however, the potential for iteration and the fact that they do all correlate with genre, suggests that the system is not strictly multivariate. Halliday, in introducing hypotaxis and parataxis (1965), suggests that due to its ability for iteration coupled with its ordering of status in dependency relations, hypotaxis sits in an intermediate position between the prototypically univariate structure of parataxis, and strictly multivariate structures. It is this position that I will take with the ordering of Articulations. At various points in this description, we will see tensions between a multivariate and univariate interpretation, with the line being drawn determined by the concerns of the specific area being described. More generally, however, this raises the issue of how clearly we can distinguish multivariate from univariate structures, and whether we should instead consider it as cline between two poles.

The previous three sections have focused on statements from the point of view of its syntagmatic structure. It was shown that it can be successfully modelled as a univariate structure, with some multivariate features arising from its interaction with genre and field. The following section will focus on the paradigmatic organisation of statements, in particular developing the system determining different types of relator and the recursion implied by a univariate analysis.

3.4.2.4 Systemic organisation of statements

To this point, the illustrative texts have only used equations with the equals sign =. This is by far the most common relator in the data, but it is only one of a dozen distinct types. Throughout the corpus, a number of other relators are used in statements. For example text 3.3 from a senior high-school textbook uses the proportional sign $\propto$ in three statements to encapsulate the technical meaning of 'proportionality' built up in the verbal co-text. The progression eventually concludes with an equation using = :

Newton's Second Law

As mentioned earlier, this law is one of the most important laws in physics and relates force, mass and acceleration. You know from your own experience that if you apply the same force to a 'heavy' object such as a car and 'lighter' object such as a pushbike, the bike moves off more easily, that is, it accelerates more.
Experiments over hundreds of years prove that, for a constant force, *the acceleration is in fact inversely proportional to the mass:*

that is, $a \propto \frac{1}{m}$

For a constant force, this means that as the mass increases, the acceleration decreases. For example, doubling the mass of an object results in a halving of the acceleration.

Similarly, experience tells us that the more force we apply to an object (that is, the 'harder' we push), the greater the acceleration of that object. Experiments prove that for a constant mass, *the acceleration is proportional to the force:*

that is, $\vec{a} \propto \vec{F}$

For the same mass, increasing the force increases the acceleration in proportion. For example doubling the force doubles the acceleration.

Combining these relationships, we have:

$$\vec{a} \propto \frac{\vec{F}}{m}$$

Newton's Second Law can hence be stated as follows:

The acceleration of an object is directly proportional to the unbalanced force acting on it and is inversely proportional to its mass.

Rearranging this relationship we can write: $\vec{F} = km\vec{a}$ where *k* is the constant of proportionality.

By a suitable choice of units (see below), *k* can be made equal to 1, so this equation reduces to:

$$\boxed{\vec{F} = m\vec{a}}$$

It must be stressed that $\vec{F}$ is the *resultant* (or *net* or *unbalanced*) force acting and Newton's Second Law is sometimes written as $\sum \vec{F} = m\vec{a}$ to emphasise the resultant nature of the force.

Text 3.3 Warren (2000: 114-115)

The first year university student exam response in Text 3.4, on the other hand, uses < and > (glossed as *smaller-than* and *greater-than*) as the relators in statements within each response's linguistic Circumstance of location. These are used to specify the conditions under which the following equation or linguistic response applies:

a) i) for $r < a$, $E = 0$

ii) for $a < r < b$, $E = \frac{q}{4\pi\varepsilon_0\kappa r^2}$

iii) for $r > b$, $E = 0$

for $r < a$, there is an enclosed charge, therefore, according to guasses law, no net flux in the sphere and hence no electric field.

for $a < r < b$, $$E = \frac{\text{flux}}{\text{Area}} = \frac{\frac{q}{\kappa\varepsilon_0}}{4\pi r^2}$$
$$= \frac{q}{4\pi\varepsilon_0\kappa r^2}$$

for $r > b$, there is no net charge on the outside of the sphere, hence no electric field.

Text 3.4 University student response

Clearly any description of the mathematics used in these texts must account for relators other than =. It is possible these relators pattern differently across registers, and play particular roles in the interaction with language and other modalities. Indeed Text 3.4 above shows a distinct separation of functions between the inequalities > and <, and the equals =. < and > were used to signpost phases of the text as part of linguistic Circumstances of location fronted to become marked Theme. Statements using the = sign, on the other hand, provided the news for each phase.

This section will thus be concerned with building a system that accounts for the different statements used in texts and the relators that realise them. Table 3.4 outlines the statement types and relators that will be described.

Statement type	**Relator**	**Example**
equation	$=$	$x = y$
identity	$\equiv$	$x \equiv y$
similar	$\approx$	$x \approx y$
order of magnitude	$\sim$	$x{\sim}y$
proportionality	$\propto$	$x \propto y$
inequation	$\neq$	$x \neq y$
greater than	$>$	$x > y$
smaller than	$<$	$x < y$
greater than or equal	$\geq$	$x \geq y$
smaller than or equal	$\leq$	$x \leq y$
much greater than	$\gg$	$x \gg y$
much smaller than	$\ll$	$x \gg y$

Table 3.4 Statement Types

As discussed above, each statement includes an expression labelled 1 and another labelled 2, with a relator linking them. To account for their sequencing, we will now consider the Relator to be a function (and label it with initial capitals appropriately), and sequence it with the expressions. Following Halliday's conventions for complexing (Halliday and Matthiessen 2014), analysis of statements will show the Relator as a superscript between each expression. As well as this, for convenience, we can replace the word Relator with the character standing for the statement types. That is, we can analyse $F \propto a$ by inserting a superscript ∝ between the 1 and 2, as in $1^{\propto}2$.

Different statement types are most obviously distinguished by their choice of Relator: =, >, ~, ≡ etc. A focus entirely on Relators, however, gives only a partial picture. Each Relator sets up a relation between two expressions and in doing so, coordinates the potential choices that can occur between those expressions. That is, by choosing a particular Relator, the entire expression complex is affected. We can see this, for example, by distinguishing between = and >. Statements involving = can reverse the ordering of their expressions, changing only Theme-Articulation structure: $F = ma$ is the same as $ma = F$. For > (glossed as *greater than*), however, this cannot occur. $a > b$ is not the same as $b > a$. To translate into English, the first statement is saying *a is greater than b* and the second is saying *b is greater than a*. We can show this distinction purely within the mathematical system by inserting numbers: taking a as 7 and b as 5, we can say $7 > 5$, but not * $5 > 7$. Relators are thus only the most obvious manifestations of a set of characteristics of a statement; they are the phenotypic marking associated with a number of other less explicit reactances.

To develop the system, we can begin with the distinction made above – namely between statements that can reverse their expressions (referred to as [symmetric] and including =, ~, ≠,∝, ≡ and ≈), and statements that order their expressions in terms of magnitude (referred to as [magnitudinal] and including >, <, ≥, ≤, ≪ and ≫). This primary system is shown by Figure 3.5.

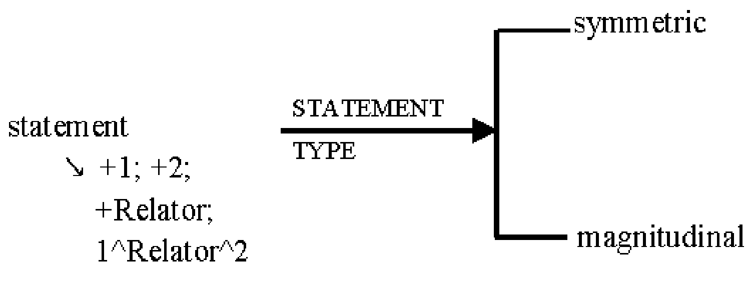


Figure 3.5 Primary delicacy of STATEMENT TYPE

We will begin by focusing on [symmetric] statements. The first distinction within symmetric statements sees [inequation], realised by the Relator $\neq$ and glossed as *not equal to*, set against all other [symmetric] statements. The key characteristic distinguishing [inequation] from the others is its lack of reflexivity. In mathematics, the property of reflexivity indicates that the same expression can occur on both sides of the Relator. Thus, inequations may not have the same expression on both sides, while all other symmetric statements may:

(3:50) $*x \neq x$

(3:51) $x = x$

(3:52) $x \sim x$

(3:53) $x \equiv x$

(3:54) $x \propto x$

(3:55) $x \approx x$

Statements that may have the same expression on both sides will be called [reflexive]. Within [reflexive] we can distinguish [proportionality] from a set of Relators that include ~, ≈, =, ≡ that we will call [alike]. Notionally, [alike] statements all show that the expressions on either side are in some way similar; they are either exactly equal as in = and ≡, or approximations as in ~, ≈. On the other hand, [proportionality], realised by the Relator ∝, does not necessarily indicate this. Rather, it shows that a change in the variables in one expression will necessarily be accompanied by a change by those in the other expression. As it deals with change, it would be odd for [proportionality] to include expressions that do not contain a variable symbol, (i.e. to only include a numeral such as 4, 7 etc. or a constant such as π; see Section 3.4.5 for the distinction between numerals, constants and variables). This means that $x \propto y$ is acceptable, whereas $x \propto 4$ is odd at best. [alike] statements do not have this restriction and regularly contain only constants or numbers in their expressions.

The final four [symmetric] statement types, all within [alike], can be distinguished between those that show exact equality, = and ≡, and those that show an approximation, ~ and ≈.

Mathematically, [exacts] are transitive whereas [approximations] are not. In mathematics the property of transitivity indicates that if, for example, $a = b$ and $b = c$, it is necessarily the case that $a = c$. For [approximations] this is not necessarily the case: if $a \approx b$ (glossed as *a is approximately equal to b*) and $b \approx c$, it is not necessarily the case that $a \approx c$; a series of small differences can make a big enough difference that two variables are no longer approximately equal. [exact] includes [equations] realised by =, and [identity] realised by ≡. Identities (≡) are rarely used, however when they are, it tends to indicate that the relation is a definition of some sort and is technical in the field. An example from university course notes shows a definition of κ and λ:

(3:56) $$\kappa \equiv \frac{1}{\lambda} = R_\infty Z^2 (\frac{1}{{n^2}_f} - \frac{1}{{n^2}_i})$$

[approximation] includes two Relators: ~ and ≈. The use of these Relators is by no means fixed, and appears to be somewhat idiosyncratic in hand-written work. A broad distinction can be made, however, between ~ which tends to shows that the two expressions are within one order of magnitude with each other (within around ten times the other), while ≈ generally suggests closer similarity.

The full network for [symmetric] statements is shown in Figure 3.6.

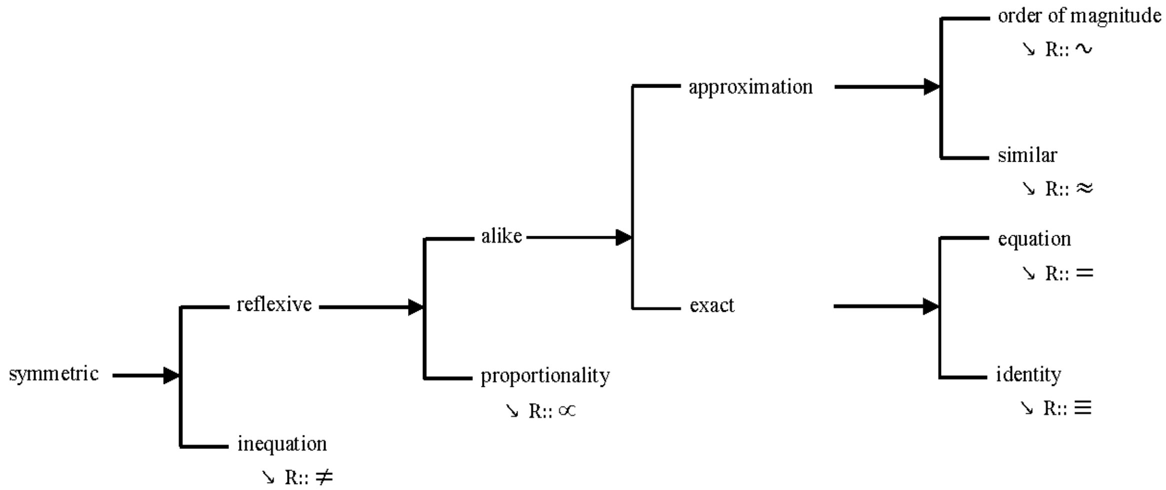


Figure 3.6 Network of symmetric statements

Opposed to [symmetric] statements is the category [magnitudinal]. These statements order their expressions in terms of their magnitude. Two sets of contrasts characterise [magnitudinal] statements. The first indicates whether the left side of the Relator is greater-than (>) or smaller-than (<) the right. The second indicates whether the two expressions have the potential to be equal or not. If they may not be equal, they are known as [strict] (e.g. >), whereas if they may be equal, they are [non-strict] (e.g. ≥). This sets up two simultaneous systems: [greater-than]/[smaller-than] and [strict]/[not-strict]. Within [strict] statements, there is a further distinction between whether the Relator indicates a large difference or not. Table 3.5 shows a paradigm of magnitudinal statements with their Relators and English glosses.

<table>
<tr><th colspan="2"></th><th>greater-than</th><th>smaller-than</th></tr>
<tr><td rowspan="2">strict</td><td>large difference</td><td>$\gg$
much greater than</td><td>$\ll$
much smaller than</td></tr>
<tr><td>not large difference</td><td>$>$
greater than</td><td>$<$
smaller than</td></tr>
<tr><td colspan="2">not-strict</td><td>$\geq$
greater than or equal to</td><td>$\leq$
smaller than or equal to</td></tr>
</table>

Table 3.5 Paradigm of magnitudinal statements

It was noted above that the feature distinguishing between [symmetric] and [magnitudinal] statements is that [symmetric] statements can reverse their expressions without changing their Relator, whereas [magnitudinal] statements cannot. This characteristic does not preclude [magnitudinal] statements from allowing a shift in Theme, however. Changing Theme in [magnitudinal] statements by swapping the expression is coupled with a shift in the GREATER-SMALLER system - via swapping from [greater-than] to [smaller-than] or vice versa. Equations (3:57-62) show the shift in Theme and the concomitant change in Relator in [magnitudinal] statements.

(3:57) $F > 4$

(3:58) $4 < F$

(3:59) $F \geq 4$

(3:60) $4 \leq F$

(3:61) $F \gg 4$

(3:62) $4 \ll F$

The system distinguishing between [greater-than] and [smaller-than] is thus a system geared toward shifts in Theme. This is not dissimilar to the systems that distinguish ‘like-type’ and ‘please-type’ mental processes in English (Halliday and Matthiessen 2014: 247-248); the ideational meanings can be held the same while the thematic structure varies. Figure 3.7 shows the options within [magnitudinal] statements.

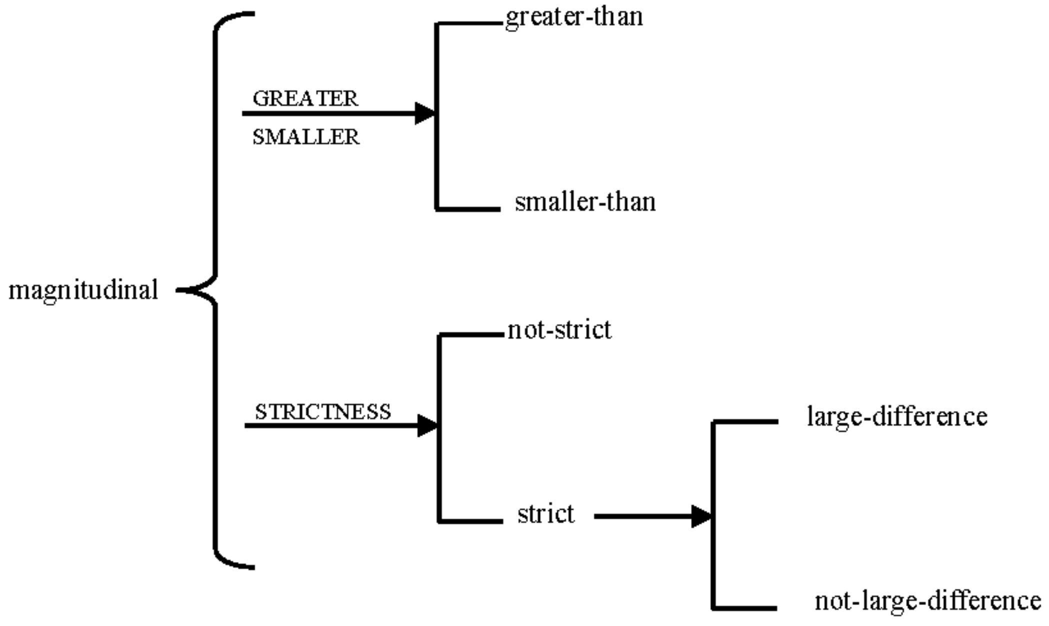


Figure 3.7 Network of magnitudinal statements

Like [symmetric] statements, [magnitudinal] statements can be indefinitely iterative. Indeed it is possibly more common for [magnitudinal] Relators to occur in three-expression statements than [symmetric], as they often specify the boundaries within which an equation holds. An example from Text 3.4 above shows this:

(3:63) for $a < r < b$, $E = \frac{q}{4\pi\varepsilon_0\kappa r^2}$

$a < r < b$ shows the upper and lower limits within which the equation $E = \frac{q}{4\pi\varepsilon_0\kappa r^2}$ holds: above $r = a$ and below $r = b$.

The recursion of expressions is thus simultaneous with the entire system of STATEMENT-TYPE. Each time a new expression is added, a new Relator is coupled with it. This Relator and expression pair is essentially a new Articulation, so we can characterise the extension of statements are rearticulations. This label also captures the extension of statements as indicating genre progression, precisely what Articulations tend to correlate with. This recursive system is modelled as Figure 3.8.

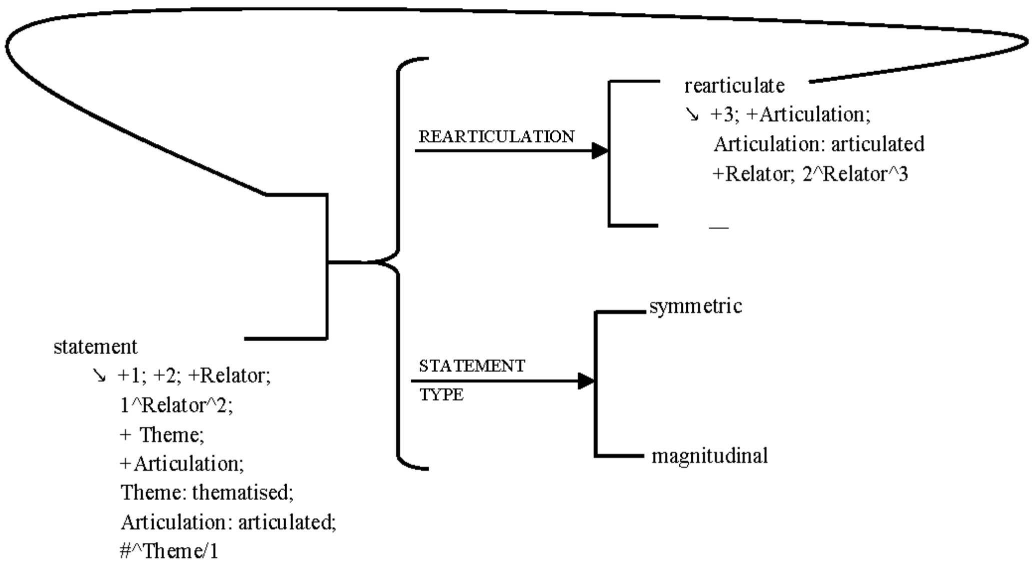


Figure 3.8 Rearticulation of statements

Note that in order to model the paradigmatic choice of which symbols realise the Theme and which realise the Articulations, Theme preselects the feature [thematised] in the level below and Articulation preselects [articulated]. These choices will be modelled within the symbol network further into the chapter.

We can now put together the network for statements as it currently stands, shown as Figure 3.9.

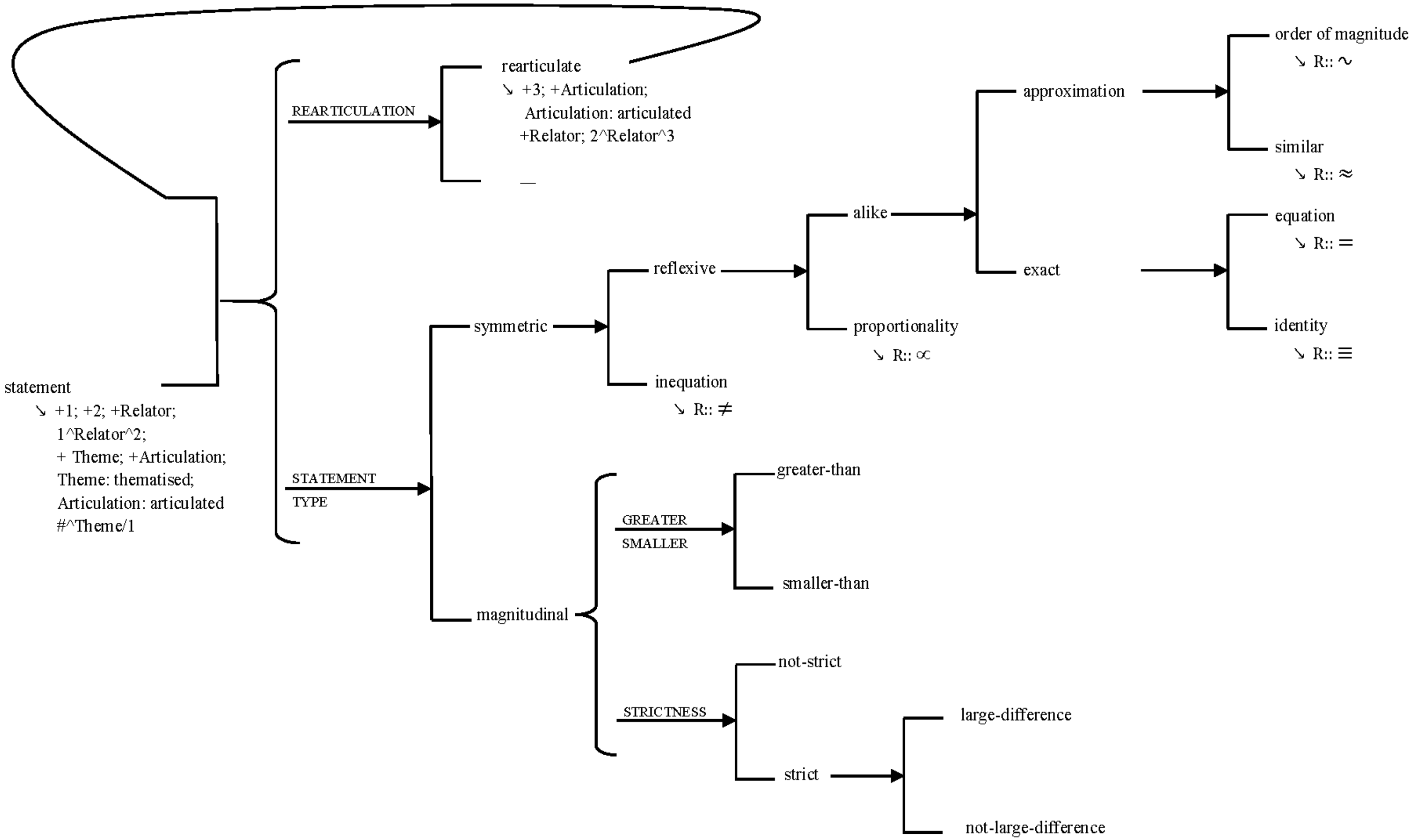


Figure 3.9 Partial network of statement

3.4.2.5 Relations across symbols in statements

It was discussed earlier in the chapter that part of the justification for developing a univariate structural analysis was in order to account for the similarity of statements that have their expressions swapped around:

(3:64) $F = ma$

(3:65) $ma = F$

By positing a univariate structure, the two expressions could be shown to hold the same function. Aside from the Theme/Articulation distinction, the univariate structure indicated the two statements were the same. It is fruitful to ask whether we can go one step further and suggest the similarity of statements that change the Theme not simply by swapping the expressions, but also by reorganising the symbols *within* expressions. That is, to show the similarity in the grammar among:

(3:66) $F = ma$

(3:67) $m = \frac{F}{a}$

(3:68) $a = \frac{F}{m}$

(3:69) $ma = F$

(3:70) $\frac{F}{m} = a$

(3:71) $\frac{F}{a} = m$

In each of these statements, the symbols remain the same but their relations within each expression differ. Across the entire statement, however, the relations between the symbols do in some way hold stable, in the sense that each can be reorganised to produce the other. O'Halloran (2005: 108) observes that despite the rearrangement of the above equations the relationships between each symbol are preserved. This is an important insight, one that is worth pursuing. It is also, however, one that puts forward challenges for the description. If we want to allow for free choice of Theme, (i.e. we want to allow the expression functioning as Theme to have any symbol or any number of symbols in it), then we must allow for equations like (33:66-71) to be similar in some way. To do this, we can return to a characterisation of statements suggested in Section 3.4.2.2: statements are expression complexes and expressions are symbol complexes. Thus we can ultimately view statements as symbol complexes. Looked at from the point of view of the symbol (from 'below', as it were), the statement is simply a set of relationships between symbols. In this view, equations (3:66-71) keep the overall symbol complex the same, but simply change the thematic focus.

Explicitly generating this similarity, however, is a significant challenge. It is clear that although we wish to maintain the relations between each symbol across the statement, the binary operations (+, -, ÷ etc.) *within* expressions are different. Thus, in order to maintain a hold on the similarity, a system must be put forward that shows logical relations between symbols remaining constant when both an equation is being rearranged and the binary operations within expressions are being changed. The key test as to whether these relations remain constant between two equations is whether one equation can be rearranged to become the other. Taking the equations above, we can describe the relations between *F*, *m* and *a* through two distinct features:

- The pairs *F* and *m*, and *F* and *a,* are both *proportional*[32]: as one in the pair increases, so does the other at the same rate.
- *m* and *a* are *inversely proportional*: as one increases, the other decreases at the same rate.

With just two relations, proportional and inversely proportional, we can account for the similarity of equations (3:66-71). Across these equations, the thematic structure and the binary operators within expressions change, but the proportional/inversely proportional relations remain the same. Structurally, the relations appear to be similar to the covariate structure indicated by Lemke (1985) for thematic systems (distinct from thematic structure discussed above) and Martin (1992a) for IDENTIFICATION in discourse semantics (see Section 2.1.2 for an introduction to covariate structures). They show a set of relations that each relate two symbols at a time, whereby a change in one necessarily shows a change in the other.

[32] The use of proportional here is distinct from, though related to, the proportionality Relator: $\propto$

Each covariate relation is in some way tied to or constrains the possible binary relations between symbols in expressions. For example, taking the relations shown between *F*, *m* and *a* above, the fact that *F* and *m*, and *F* and *a* are proportional, while *m* and *a* are inversely proportional, restricts the possibilities for how these symbols are related in equations. Using only these symbols, the only equations that can show these relations are (3:66-71). Any other configuration, such as $m = \frac{a}{F}$ or $F = \frac{a}{m}$, does not conserve these relations. Utilising the rules of mathematics, these two equations cannot be 'rearranged' to produce any of the equations in (3:66-71)

Notably, the two covariate relations we have specified so far, *proportional* and *inversely* proportional, necessitate that only division or multiplication are used in the equations between these three symbols, e.g. $F = ma$ and $a = \frac{F}{m}$. If addition or subtraction are used, they form different relations between the symbols. For example if we had the equations (3:72-73) below, neither could be rearranged to produce (3:66-71) above.

(3:72) $F = m + a$

(3:73) $m = F - a$

This tells us that in some sense the covariate relations between the symbols in these equations are different to those in (3:66-71). Changing equation (3:72) or (3:72) to one of (3:66-71) does not preserve the configuration of relations. There are, however, similarities that we can consider. In both (3:66-71) and (3:72-73) when either *a* or *m* increases, *F* also increases. Similarly, in both sets of equations, when *a* increases, *m* decreases. This is a generalisation we will pick up below. The difference between the two is the rate at which each symbol increases or decreases in relation to the others. As mentioned above, the relation between *F* and *a*, for example, in (3:66-71) (e.g. $F = ma$) is that of *proportionality*. This means that as *a* increase, *F* increases at the same rate; the increase of one will be a constant multiple of the increase of the other. We can see this in (3:74-76). By keeping *m* constant as 2, and increasing *a* in intervals of 5, in the equation $F = ma$, *F* increases in intervals of ten. That is, *F* increases at two times the increase of *a*.

(3:74) If $a = 5$, $F = 2 \times 5 = 10$

(3:75) If $a = 10$, $F = 2 \times 10 = 20$

(3:76) If $a = 15$, $F = 2 \times 15 = 30$

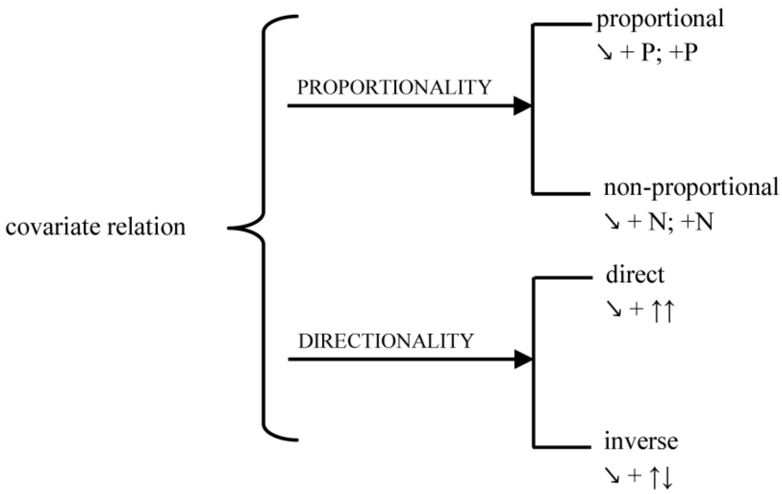
Looking in terms of grammatical reactances, symbols that are proportional can be related by the Relator $\propto$, e.g. $F \propto a$. In contrast, the relation between F and a in $F = m + a$ is not-proportional. Although they are similar in the sense that if one increases, so will the other, the increase of each is not a constant multiple of the other. Rather, the increase is precisely *the same* number. For example if a increases by 5, F will also increase by 5, as shown in (3:77-79) (again keeping m equal to 2).

(3:77) If $a = 5$, $F = 2 + 5 = 7$

(3:78) If $a = 10$, $F = 2 + 10 = 12$

(3:79) If $a = 15$, $F = 2 + 15 = 17$

We can thus make a distinction between relations that show proportionality (shown between F and a in equations (3:66-71) and (3:74-76)) and those that do not show proportionality (shown between F and a in (3:72-73) and (3:77-79)). We can also cross-classify these relations by whether both symbols increase together, or whether as one increases the other decreases. If they increase together, they will be considered to have a *direct* relation. For example, as a increases in $F = ma$, so will F. Thus, they are *directly* related. As we have seen, they are also proportional (they increase at the same rate); their relation is thus *directly proportional*. On the other hand, in the same equation, the relation between m and a, although proportional, is such that as one increases, the other decreases. We will term this relation an *inverse* relation. In $F = ma$, therefore, m and a are *inversely proportional*. The direct and inverse distinction also occurs for non-proportional relations. In $F = m + a$, F and a increase together but are not proportional, and so their relation will be termed *directly non-proportional*. In the same equation (possibly best seen in the form $m = F - a$), however, as a increases, m decreases but again they are not proportional. Thus the relation between a and m in this case is termed *inversely non-proportional*. We can thus set up a simple network showing the cross-classification of these relations, in 3.10.



3.10 Systems of PROPORTIONALITY and DIRECTIONALITY

To formalise these covariate structures, each symbol involved in a proportional relation will realise a function labelled P. Those realising a non-proportional relation will be indicated by an N. Direct relations will be indicated by two upward point arrows, ↑↑, while inverse relations will be one up, one down, ↑↓. Thus, in $F = ma$, the directly proportional relation between *F* and a is indicated by P↑↑P, while the inversely proportional relation between a and *m* is indicated by P↑↓P. In $F = m + a$, the direct non-proportional relation between *F* and a will be shown by N↑↑N, while the inverse non-proportional relation between *m* and a will be shown by N↑↓N.

The nature of the covariate relations in mathematics is that in a single equation, each symbol is related to every other symbol. As well as this, however, each symbol is related to every possible symbol complex (their possibility determined by whether the equation can be rearranged to show that symbol complex), and each possible symbol complex is related to every other possible symbol complex. This quickly produces a large set of relations in only a small amount of space. Taking $F = ma$, for example, *F* is related to both *m* and a individually (as we have seen), but it is also related to *ma* as a single symbol complex. In this case, as *F* increases, so does *ma* at the same rate: *F* and *ma* are directly proportional. Similarly, this equation implies that *m* is related to each of a and *F*, as well as the complex of *F* and a (shown by $\frac{F}{a}$), and a is related to each of *F* and *m* and the complex of *F* and *m* ($\frac{F}{m}$). The full set of covariate relations indicated by the equation $F = ma$ is shown in Table 3.6.

$F \leftrightarrow a$	P↑↑P	directly proportional
$F \leftrightarrow m$	P↑↑P	directly proportional
$F \leftrightarrow ma$	P↑↑P	directly proportional
$m \leftrightarrow a$	P↓↑P	inversely proportional
$m \leftrightarrow \frac{F}{a}$	P↑↑P	directly proportional
$a \leftrightarrow \frac{F}{m}$	P↑↑P	directly proportional

Table 3.6 Covariate relations shown by $F = ma$.

In the scheme of things, $F = ma$ is a relatively simple equation. As the table shows, it sets up only six covariate relations. With every addition of a symbol and its corresponding binary relation, however, the number of covariate relations increases dramatically. Adding one symbol to the above equation to produce $F = ma + c$, for example, results in the fifteen covariate relations shown by Table 3.7.

$F \leftrightarrow ma + c$	P↑↑P	directly proportional
$F \leftrightarrow ma$	N↑↑N	directly non-proportional
$F \leftrightarrow c$	N↑↑N	directly non-proportional
$F \leftrightarrow m$	N↑↑N	directly non-proportional
$F \leftrightarrow a$	N↑↑N	directly non-proportional
$c \leftrightarrow F - ma$	P↑↑P	directly proportional
$c \leftrightarrow ma$	N↑↓N	inversely non-proportional
$c \leftrightarrow m$	N↑↓N	inversely non-proportional
$c \leftrightarrow a$	N↑↓N	inversely non-proportional
$a \leftrightarrow \frac{F - c}{m}$	P↑↑P	directly proportional
$a \leftrightarrow F - c$	P↑↑P	directly proportional
$a \leftrightarrow m$	P↑↓P	inversely proportional
$m \leftrightarrow \frac{F - c}{a}$	P↑↑P	directly proportional
$m \leftrightarrow F - c$	P↑↑P	directly proportional
$ma \leftrightarrow F - c$	P↑↑P	directly proportional

Table 3.7 Covariate relations shown by $F = ma + c$.

The sheer complexity of these relations is the basis for much of the power of mathematics. In Chapter 4, we will see how these relations are used to develop new and previously unspecified equations, which in turn allow the development of further technical meaning. In Chapter 5, we will consider these relations as the basis of field specific ‘implication complexes’ realised through mathematics. In both cases, these relations will be shown to have great power for the knowledge base of physics. Looking purely grammatically, however, specifying these relations allows us to show the similarity between the following equations (3:80-89) and account for the fact that anyone sufficiently trained in mathematics can easily rearrange any of them into any of the others.

(3:80) $F = ma + c$

(3:81) $ma + c = F$

(3:82) $c = F - ma$

(3:83) $F - ma = c$

(3:84) $a = \frac{F-c}{m}$

(3:85) $\frac{F-c}{m} = a$

(3:86) $m = \frac{F-c}{a}$

(3:87) $\frac{F-c}{a} = m$

(3:88) $ma = F - c$

(3:89) $F - c = ma$

Despite both the thematic structures at the level of statement and the binary operations at the level of expression changing in each equation, the covariate relations shown by Table 3.7 remain the same. The system of covariate relations thus cuts across the extraordinary complexity in variation available in mathematical statements, to preserve the relations between symbols.

So far we have only considered covariate relations that interact with the arithmetic binary relations of addition (+), subtraction (-), multiplication (×) and division (÷). We can also set up relations shown by the exponentiation operations of roots, e.g. $\sqrt[y]{x}$, powers x^y and logarithms $\log_x y$. Whereas the covariate relations presented above link two symbols or symbol complexes, those related to exponentiation link three. As was discussed above in section 3.4.1.2.2, the three exponentiation relations are all mutually definable as they provide a different angle on the relation between three symbols. This was shown by taking *a*, *b* and *c* as variables, and using as examples the numbers $a = 2$, $b = 5$, $c = 32$.
In this case, [power] relates *a* and *b* to equal *c*:

(3:90) $a^b = c$ [power]

e.g. $2^5 = 32$

[root] relates *b* and *c* to equal *a*:

(3:91) $\sqrt[b]{c} = a$ [root]

e.g. $\sqrt[5]{32} = 2$

[logarithm] relates *a* and *c* to equal *b*:

(3:92) $\log_a c = b$ [logarithm]

e.g. $\log_2 32 = 5$

From these we showed an equivalence relation between roots, powers and logarithms:

$a^b = c$ is equivalent to $\sqrt[b]{c} = a$ is equivalent to $\log_a c = b$

As the three exponentiation operations relate the same variables, each can be rearranged to become the other. This sets up three functions in a covariate relation, that we will call the Base, Exponent and Power. These three functions are related such that:

$$Base^{Exponent} = Power$$

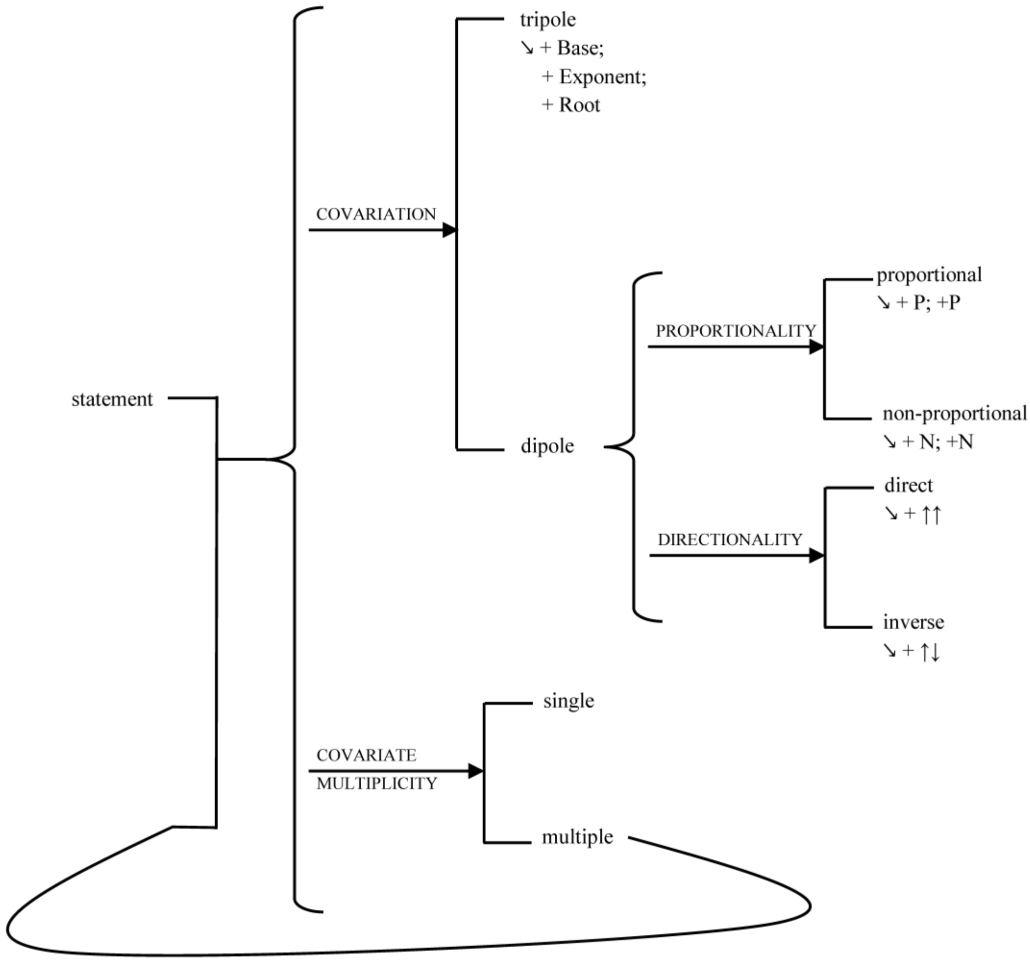
is equivalent to:

$$\sqrt[Exponent]{Power} = Base$$

is equivalent to:

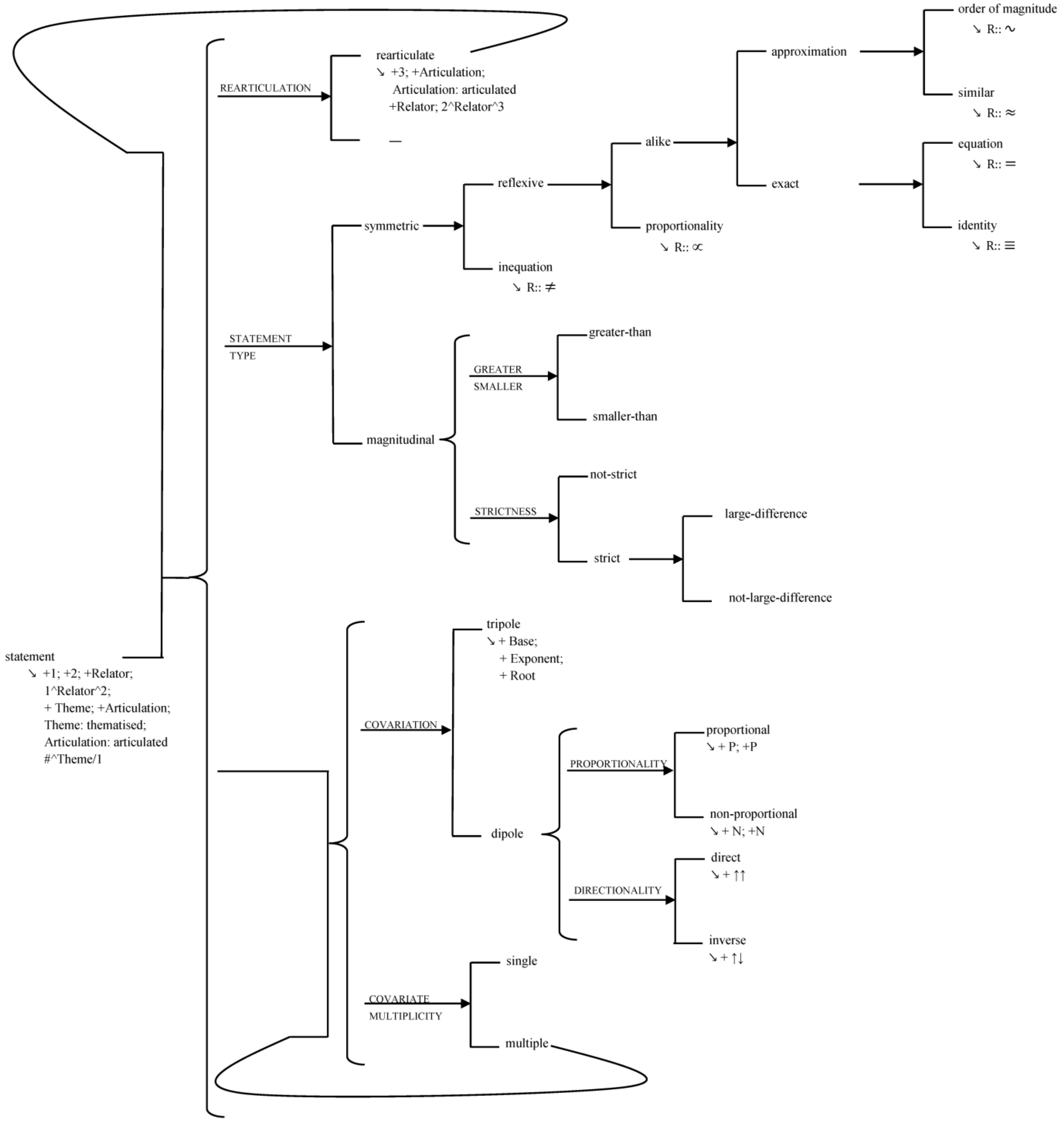
$$\log_{Base} Power = Exponent$$

We can now add these to the system of covariate relations developed above to complete the description of this section. We will group those relations involving two variables as types of *dipole* relation, while those involving three variables will be termed *tripole*. As there is the potential for multiple covariate relations to occur if more than one symbol is selected in an expression, we will once again insert a recursive loop. The network for covariate relations is shown in 3.11.



3.11 Network of covariate relations

This network is only a partial formalism of covariate relations. It does not specify the realisational and preselection relations between covariate relations and binary relations in the level below, nor does it account for the possibility of Relators other than =, nor the relations involved in the unary operations that will be specified in the following section. Formally modelling of each of these poses significant descriptive challenges. These arise from the interaction between covariate, univariate and multivariate structures and the levels derived from them. As such, they will be left for future development of the grammar. Nonetheless, with this system, we can now present the full system for the level of statement as 3.12.



3.12 Network of statement

3.4.3 Multivariate structure of symbols

The description to this point has considered the complexing relations between symbols that have built major features of the architecture of mathematical symbolism. It has revealed the way symbols complex into expressions, how expressions in turn complex into statements, and how these statements can be rearranged to produce distinct thematic structures. Returning to the example texts, we see there is still some variation that has not been accounted for in the description. An example of this is the use of subscripts to distinguish between different types of *E* (glossed as *energy*) at beginning of Text 3.5, a mathematical text written by the teacher on the white board in a high school physics class:

$$\Delta E_{emitted} = E_i - E_f \qquad\qquad E_i = \text{initial}$$

$$E_f = \text{final}$$

$$\begin{aligned} \Delta E_{3\to 2} &= -1.5 - -3.4 \\ &= 1.9 \text{ eV} \\ &= 1.9 \times 1.6 \times 10^{-19} \\ &= 3.04 \times 10^{-19} \quad \text{Joules} \end{aligned}$$

Text 3.5 High School Classroom Whiteboard.

This text is concerned with calculating the energy *E* when an electron moves between two levels in a hydrogen atom. To do this, it distinguishes between four different instances of energy: E_i and E_f, glossed as the initial and final energy during a transition, $E_{emitted}$, the energy emitted in a transition between two levels, and $E_{3\to 2}$, the energy emitted specifically in the transition between level 3 and level 2. The subscripts indicate different instances of the same technical symbol. As well as the subscripts, in the first and the third line, the use of the Greek character Δ modifies *E*. Δ is usually glossed as *change*, so that $\Delta E_{3\to 2}$ would be read as *the change in energy from 3 to 2*. This character is related to other modifications such as the trigonometric functions sin and cos shown in bold in Text 3.6, from a senior high school textbook:

A car of weight 20,000 N rests on a hill inclined at 30 degrees to the horizontal. Find the component of the car's weight:

(a) perpendicular; and
(b) parallel to the plane of the hill.

SOLUTION

From 3.49 [not shown YJD] we see that:

(a) The component of weight perpendicular to the plane is given by:

$$\begin{aligned} F_{perp} &= W\,\mathbf{cos}\,\theta \\ &= 20\,000\ \ \mathbf{cos}\,30 = 17\,300\ \mathrm{N} \end{aligned}$$

(b) Similarly, the component of weight parallel to the plane is given by:

$$\begin{aligned} F_{perp} &= W\,\mathbf{sin}\,\theta \\ &= 20\,000\ \ \mathbf{sin}\,30 = 10\,000\ \mathrm{N} \end{aligned}$$

Text 3.6 Warren (2000:117)

Each of the characters above will be classed provisionally as different types of modifiers. There are numerous other modifiers that occur throughout the texts under study. Indeed, they form a valuable component of the discourse, performing a host of different functions within the texts. The justification for grouping the various modifications together is twofold. First, these characters cannot sit on their own in an expression. That is, each of (3:93-95) is ungrammatical:

(3:93) $*\Delta = y$

(3:94) $*\mathrm{sin} = \mathrm{y}$

(3:95) $* \quad _2 = y$

Second, binary operations cannot hold between the modifier and the head; each expression (3:96-98) is thus unacceptable:

(3:96) $*\Delta \times y$

(3:97) $*\sin \times \mathrm{y}$

(3:98) $*y_{\times 2}$

These two characteristics distinguish modifiers from the symbols dealt with so far such as x, y, 2, π etc. This sets up two distinct functions: those that can sit on their own in an expression and can be related to other symbols through binary operations, and those that cannot. Beginning with the subscript relation, e.g. E_i, we will call E a Quantity, and the subscript a Specifier. Thus we would analyse E_i as Quantity^Specifier. Quantity will be used for any symbol that can enter into a binary operation whether or not there is a Specifier. As the Quantity^Specifier structure contains two distinct functions that are not recursive, this relation is a multivariate structure. The multivariate nature of these functions is in contrast to most of the grammar described so far and has an impact on the overall architecture of mathematics. This, however, will be dealt with in Section 3.4.6 below. Before this, the different options for modifications will be mapped.

We began with the Specifier above as it is the odd one out within the system to be developed. Specifiers can only modify a single symbol, such as E_i . They cannot modify symbols complexes, (i.e. $*(5E)_i$ is ungrammatical), nor can they modify symbols involving other modifications: $*(\Delta x)_i$. This is in contrast to other modifiers, such as sin and Δ, which can modify whole complexes, e.g. $\sin(\frac{n\pi x}{L})$ and $\Delta(m\vec{v})$. As well as this, Specifiers can only modify pronumerical symbols, not numbers (i.e. they can modify E but not 5). This final feature is important as it necessitates a distinction between different elements that realise the Quantity, and will be dealt with in Section 3.4.5. For the reasons specified above, Specifiers will be split from the rest of the system as an optional feature.

The other modifiers will be grouped as unary operations. Unary operations are similar to binary operations (+, -, ×) in that their insertion usually changes the value of the expression they are in. Unlike binary operations, however, unary operations only necessitate one symbol.

The symbol that is being operated on we will call the Argument. The unary operator will take the function Operation. Thus, the modification of the number 3 with the unary operation sin to produce $\sin 3$ will be analysed as Operation^Argument. We have mentioned previously that a key feature of unary operations is that, although they can take a single symbol, they can also take a complex of symbols. That is, the Argument can be realised by a symbol complex. This means that $\sin(\frac{n\pi x}{L})$ would also be analysed as Operation^Argument. We can thus now further specify the distinction between unary and binary operations. Unary operations take a single Argument that could be realised by any number of symbols in a complex; binary operations, on the other hand, take two arguments (using the term informally), labelled α and β, each of which could be realised by a single symbol or a complex of symbols. One final feature of unary operations is that they cannot modify all elements within an expression. Looking at the symbols in $\boldsymbol{F} = 1 \times 10^{-2}$ N west, they can modify the $\boldsymbol{F}$, 1, 10 and -2, but they cannot modify the 'N' or 'west'. The 'N' indicates the units of $\boldsymbol{F}$ (force) and 'west' indicates the direction the force is moving in. We will term the symbols that can be modified by unary operations values.

It is possible for a Quantity with a Specifier (a symbol with a subscript, such as E_i) to be modified as a whole by a unary operation. For this reason, an expression such as $\sin E_i$ would have two function structures: one accounting for the Operation^Argument structure, and the other the Quantity^Specifier structure:

(3:99)

sin	E_i
Operation	Argument
	Quantity^Specifier

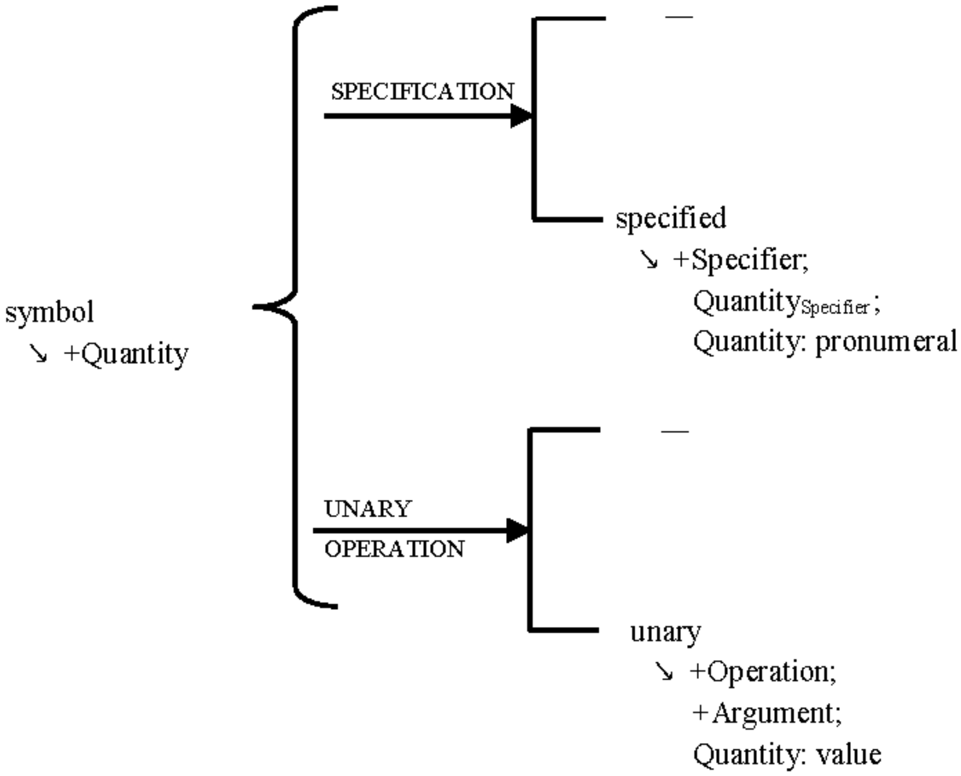
We have now introduced all of the functions needed to account for the grammar of mathematics. It is worth pausing for a moment to show an example of a statement fully analysed for its functional structure (excepting covariate structures). To capture each of the functions, a constructed example will be used: $y = \cos\frac{E_i}{mvr} \approx 0.5$.

(3:100) $y = \cos \frac{E_i}{mvr} \approx 0.5$

y	$=$	$\cos$	$\frac{E_i}{mvr}$	$\approx$	0.5
1	$^{=}2$			$^{\approx}3$	
Theme	Articulation$_1$			Articulation$_3$	
		Operation	Argument		
Quantity			$\frac{\alpha(\text{Quantity\^{}Specifier})}{\beta(\alpha/\text{Quantity}^{\times}\beta/\text{Quantity}^{\times}\gamma/\text{Quantity})}$		Quantity

At the levels of statement and symbol, there are both univariate and multivariate structures. At the level of statement, the univariate structure is shown by $1\,^{=}2\,^{\approx}3$ and the multivariate by Theme^Articulation$_1$^Articulation$_{2.}$ At the level of symbol, the univariate structure is shown within the second expression by $\frac{\alpha}{\beta(\alpha^{\times}\beta^{\times}\gamma)}$ and the multivariate by the Quantities, Specifier and the Operation^Argument sequence.

Continuing the description, we have so far distinguished between Specifiers that modify Quantities and are shown by subscripts, and Operations that modify Arguments. To capture the fact that the structures Quantity^Specifier and Operation^Argument can enter into binary operations of multiplication, addition, division etc. we will now use the term *symbol* as the class label that includes the affixation of the modifiers onto the elements that realise the Quantity. That is, for example, *symbol* is used to capture the entirety of $\sin x_1$. This is similar to the label *word* in English being used to capture the grouping of root morphemes with prefixes or suffixes. Reasoning thus, symbols are realised by Quantities plus optional modifiers. From this, we can map the initial distinction between Operation^Argument and Quantity^Specifier structures as types of symbol, as in the system network in 3.13.



3.13 Primary delicacy of symbol

This network shows that modifications are optional; expressions can simply have a single Quantity as the symbol, as in $x = 0.3$. Alternatively, expressions can take a unary operation, a Specifier or both: $x_1 = 0.3$, $\sin x = 0.3$, $\sin x_1 = 0.3$.

As mentioned previously, there are a number of different types of unary operations. In fact, within the entire field of mathematics, the number of unary operations is enormous, far too many to be accounted for in this thesis. Compared to the vast array of operators used across all mathematics, the different types used in the physics texts under analysis is relatively modest. Table 3.8 outlines the set of unary operations included in this description.

Unary type	Unary	Example
change	Δ	Δx
factorial	!	$x!$
absolute value	$\|\ldots\|$	$\|x\|$
summation	Σ	$\sum x$
sine	sin	$\sin x$
cosine	cos	$\cos x$
tangent	tan	$\tan x$
positive	+	$+x$
negative	$-$	$-x$
generic	most common: $f(\ldots)$	$f(x)$

Table 3.8 Unary operation types

The first distinction is between unary operators that come after the Argument (suffixual), those that come before (prefixual) and those that occur on both sides (circumfixual). There is only one type of each of suffixual and circumfixual unaries and so these can be generated first. The suffixual operator is the factorial, shown by !, as in 5!. Factorials indicate that each positive whole number (integers) between 0 and the number in the Argument are multiplied together. For example $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$. The circumfixual operator is known as absolute value and is denoted by |…|, as in $|x|$. Whatever the sign of the Argument, whether positive (above zero) or negative (below zero), the absolute value shifts the sign to positive. For example, both $|5|$ and $|-5|$ are equal to 5: $|5| = 5$; $|-5| = 5$. All other unary operators are prefixual, coming before the Argument.

The prefixual operators include three distinct types: the trigonometric operations of sine, cosine and tangent (e.g. $\sin x$, $\cos x$, $\tan x$); the change operator shown by Δ (e.g. Δx) and the summation operator Σ (e.g. $\sum x$). The trigonometric operators can be grouped under a single feature through their agnation patterns.[33] Given a right-angled triangle with three sides

[33] This description will not generate the inverse trigonometric operations (e.g. $\sin^{-1} x$), the reciprocals (e.g. $\csc x$) nor the hyperbolic operations (e.g. $\sinh x$), however these could be easily generated as three simultaneous systems to the cosine/sine/tangent system.

termed opposite, adjacent and hypotenuse, each of the trigonometric operators are equal to a relation between two of them:

(3:101) $\sin\theta = \frac{opposite}{hypotenuse}$

(3:102) $\cos\theta = \frac{adjacent}{hypotenuse}$

(3:103) $\tan\theta = \frac{opposite}{adjacent}$

More succinctly, the three operators can be related in a single equation through:

(3:104) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

The change operator Δ gives the numerical difference between two instances of the Argument. This is usually formalised as $\Delta x = x_2 - x_1$. For example, if $x_2 = 5$ and $x_1 = 3$ the change in *x* would be shown through: $\Delta x = x_2 - x_1 = 5 - 3 = 2$. As Δ necessitates a symbol that can change, it cannot modify numerals (e.g. 5) nor can it modify pronumerals known as constants (symbols that don't change such as π). The elements it can modify are known as variables.

Finally, the summation operator Σ indicates the sum of all different instances of the Argument. For example if there are three instances of x: $x_1 = 3$, $x_2 = 5$ and $x_3 = 7$, the sum of all *x* is shown by: $\sum x = x_1 + x_2 + x_3 = 3 + 5 + 7 = 15$.

Each of the unary operators so far can be equated with a specific set of binary relations. For example, a change in *x* indicated by Δx is equal to the difference between *x* at two specific points, or $\Delta x = x_2 - x_1$. Indeed most unary operators across the field of mathematics appear to encode sets of relations such as this to greater or lesser specificity. This fact in part explains their existence. They are used to encode sets of relations in a relatively economical way, not dissimilar to the function of technicality in language (Halliday and Martin 1993, similar to Lemke's thematic condensation 1990: 96). In Legitimation Code Theory terms (Maton 2014), the distillation of these relations indicates relatively strong semantic density (Maton and Doran in press 2016a,b), allowing physics to efficiently describe quite complex relations.[34] The distillation of univariate relations into multivariate operators will be taken up again in Chapter 4 in relation to their role in knowledge building.

There is one set of unary operators, however, that do not encode a specific set of relations. These have been termed [generic] operations and are most commonly indicated by *f(...)*, such as $f(x)$ (read as *f of x*) in $f(x) = \frac{1}{x}$, though others can be used - e.g. $g(x)$ (read as *g of x*). These operators do not encode any specific set of relations within the broader grammar of mathematics, but rather work as general operators whose meaning shifts instantially with each text. More field-specific operators can be found, such as $\Psi(x,t)$ in $\Psi(x,t) = Ae^{i(kx-\omega t)}$ used in quantum physics (from Young and Freedman 2014: 1333). However the relations these operators encode are constrained by field, not the grammar, and generally allow a larger set of relations for different situations in comparison to the entirely grammaticalised operators such as Δ.

The final pair of unary operators concern signs distinguishing positive (+) and negative ($-$). These operators most obviously occur in statements that appear at first to have two binary operations in sequence, or a binary operation that links only a single symbol. An example of this is shown twice in an equation from Text 3.5 above: $\Delta E_{3\to 2} = -1.5 - -3.4$. Here there is a negative sign before 1.5 not linking it to anything else, and two negative signs between 1.5 and 3.4. Although having the same form, these negative signs are not the binary operators of subtraction. Rather, they are unary operations that distinguish between positive and negative. This is justified by the fact that they can come before a single symbol without following another, as shown above. The positive sign (+) can do the same as the negative, however it is much less common, owing to the fact that positive is default for numbers, and so does not need a sign in the unmarked case. Unary operators such as these can only occur for positive and negative; there are no correlates for multiplication $\times$ or division $\div$.

From the discussion above, we can now set out the options for unary operations as in Figure 3.14.

[34] More specifically, they indicate stronger *discursive* semantic density (Maton 2014 chapter 9). That is, the relations being encoded have nothing to do with the object of study, but rather relations entirely internal to the system of mathematics. It is only once the unary operations have been placed within the field of physics (or another field) and operate on technicality within that field that the operators add meanings relating to the object of study (strengthening the *ontic* semantic density).

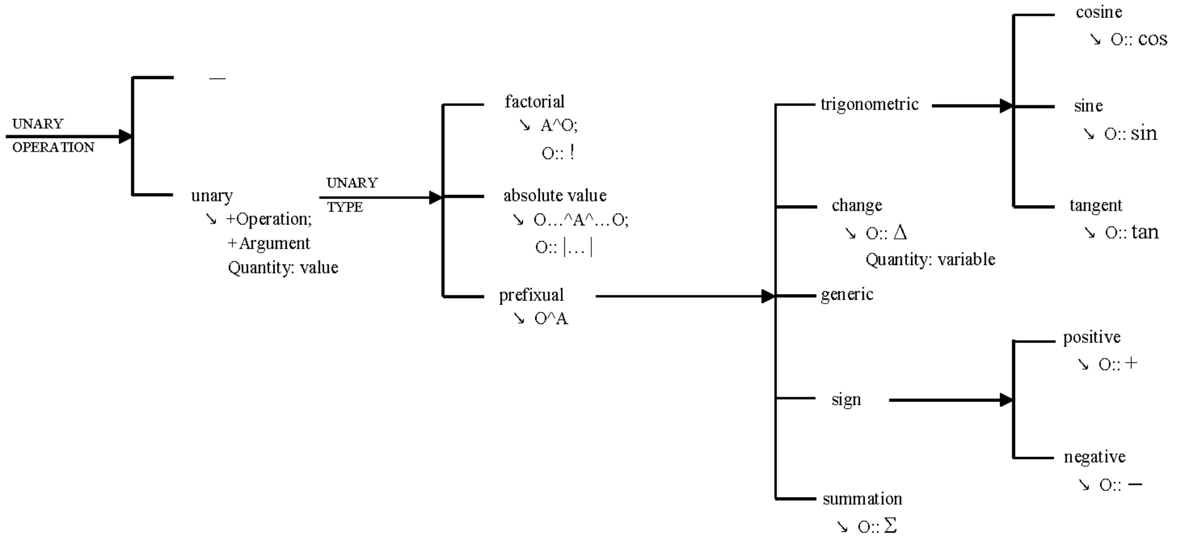


Figure 3.14 System of UNARY OPERATION

It was mentioned previously that unaries can operate on Quantities with a Specifier. That is, for example, it is acceptable for an expression to include $\sin E_i$. It is also possible for unaries to operate on other unaries, e.g. to produce something like $\sin \Delta x$. This possibility for unaries to be repeated indicates a recursive system. However recursive unaries such as this are rare. In particular, it is unusual for recursive operators that repeat the same choice, e.g. $\sin(\sin x)$ at least in the data under study. Although this study has not made any quantitative measurements, it appears reasonable to suggest that the probabilities of choosing a second operator over not choosing another operator would be far less than the 1:9 suggested by Halliday for the skewed probability in his bimodal hypothesis for grammatical systems of English (Halliday and James 1993). Nonetheless, recursive unaries are grammatical. This creates an issue, as specifying a recursive system without probabilities or stop rules suggests that unary operations are better described as a univariate structure rather than as the multivariate structure suggested above. The relatively rare instantiation of recursive unaries, however, means that in the vast majority of situations there only two distinct functions appearing; functions that have considerably different agnation patterns. For this reason it seems preferable to stand by the multivariate analysis for unary operations, while accepting the possibility for recursion, albeit rare.

3.4.3.1 The system of symbol

Although the choice of repeated unary operations is rare, it does happen. Thus the network of unaries must include a recursive loop. The system network for symbols including unary operations and Specifiers is shown in Figure 3.15.

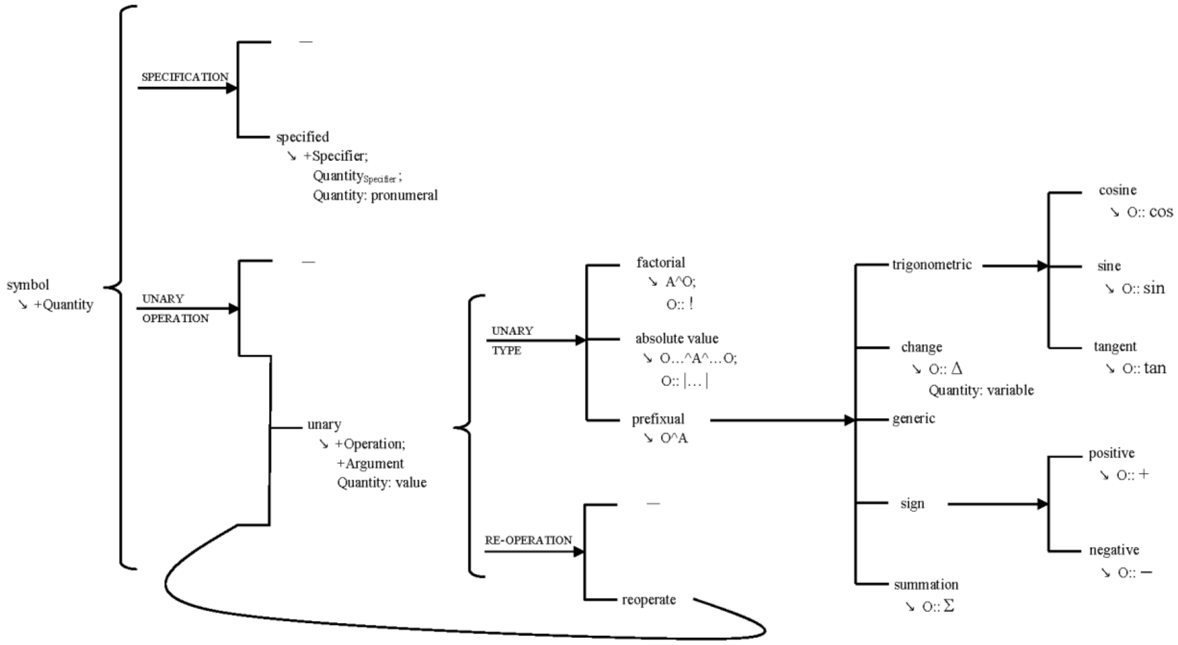


Figure 3.15 Symbol modifications

This network is concerned with the modification of Quantities that realise symbols. It indicates that a symbol can include any number of unaries and/or a Specifier, or it can occur as an unmodified quantity. In Section 3.4.1 we saw that symbols can also complex with any number of other symbols to form large symbol complexes. This complexing involves linking symbols through binary operations. Symbols with any unary operation or Specifier can be complexed with any other symbol through these binary operations. Crucially for our paradigmatic description, the entry condition for choosing between symbol complexing or not, and the modifications given by unaries and Specifiers is the symbol. Thus the two sets of systems are simultaneous with each other at the same level; they each form systems of primary delicacy. The system of EXPRESSION TYPE, determining a symbol complex or not, looks outwards to the external relations between symbols; the systems of SPECIFICATION and UNARY OPERATION look inwards to the internal structure of symbols. As each of these systems are simultaneous at the level of symbol, they can be placed in the same system network. In this network, we can also include the choice of symbols that realise Theme and Articulation, as well as Theme ellipsis. Thus, Figure 3.16 presents the entire network for the level of symbol.

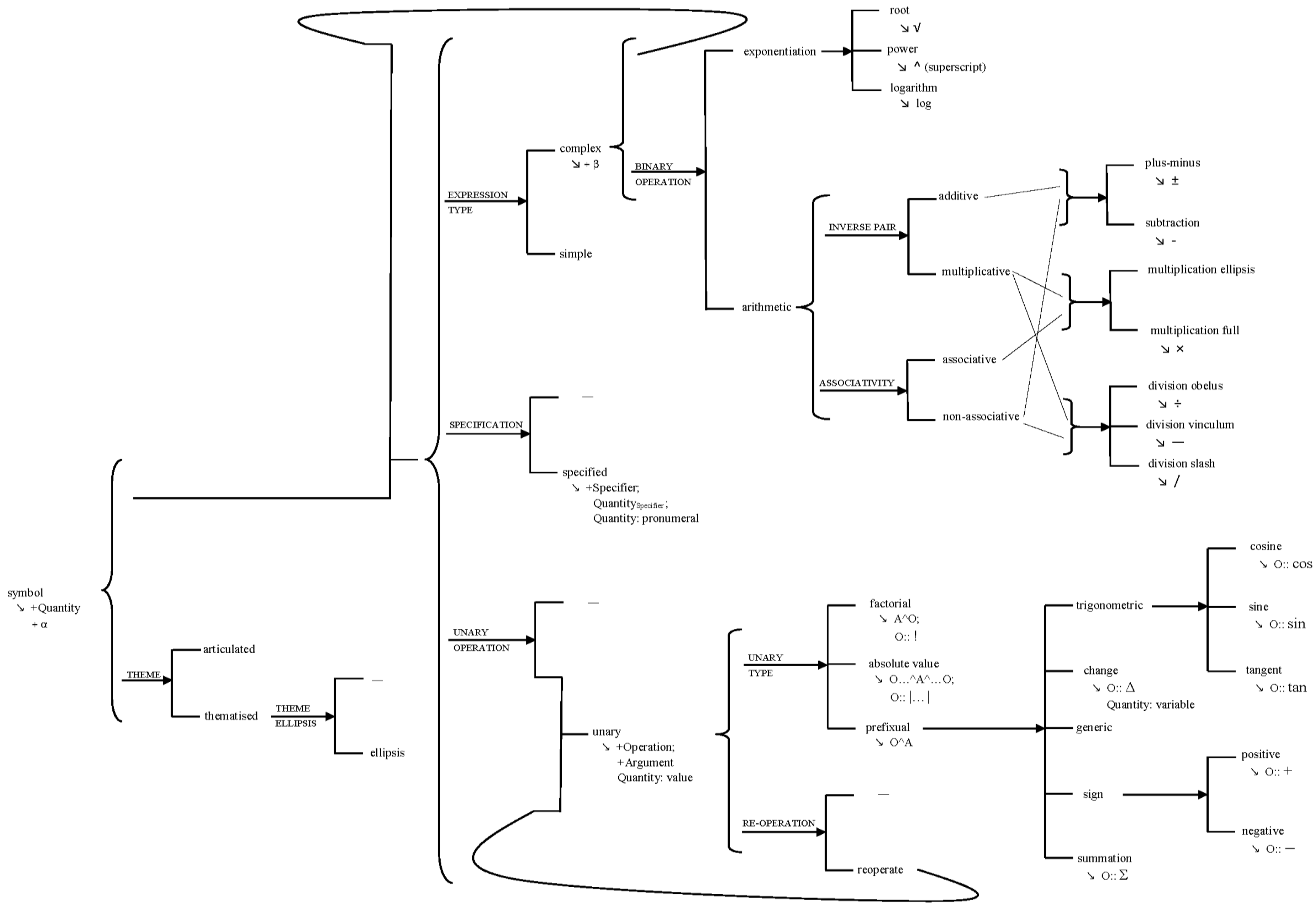


Figure 3.16 Network of symbol

This symbol network complements the statement network given in Figure 3.12 above. These two networks account for all the variation from the level of symbol up. They describe the internal structure of symbols, their complexing into expressions and the complexing of expressions into statements. These networks represent two levels in the architecture of mathematics. As these make up the bulk of the variation in mathematics, it is pertinent that we consider the relation between them. To this we now turn.

3.4.4 Layering in mathematics

The discussion so far has focused in detail on variation within statements, expressions and symbols and has developed structural and systemic models for each. Although there is still one more area of the grammar to cover, it is worth taking a step back and viewing the grammar as it stands. In particular, we can focus on the interaction between statements, expressions and symbols and characterise the hierarchy of levels that they suggest. Two levels of networks have been used to describe mathematics so far: statement and symbol. Systemically, expressions have arisen within the symbol network as complexes of symbols, and within the statement network as parts of the statement. In one sense then, the relationship between statements, expressions and symbols is straightforward: statements contain expressions and expressions contain symbols. This rather simple characterisation, however, clouds the organising principles of these levels and their interaction.

In English (and to this point every language described in the Systemic Functional tradition, see Caffarel et al. 2004a), the scale of units – e.g. morpheme, word, group/phrase and clause – are organised hierarchically in a rank scale. Structurally, this rank-scale is organised in terms of constituency, i.e. a relation of parts to wholes. A clause contains one or more groups, groups contain one or more words and words contain one or more morphemes (Halliday 1961, 1965, Huddleston 1965, Halliday and Matthiessen 2014). Paradigmatically, there is a tendency for preselection from higher units to lower units. For example options in the clause network tend to preselect options in the group/phrase network etc. (Matthiessen 1995).

The rank scale proposed by Halliday for English involves multivariate structures associated with the experiential component of the ideational metafunction (Halliday 1965, 1979). For mathematics, however, this chapter has argued that the overarching structural organisation relating statements, expressions and symbols is not multivariate, but univariate. Although there is internal variation within symbols that is best described multivariately, this internal variation has no bearing on a symbols' relation with its higher levels (it does, however, impact on lower levels, to be discussed in Section 3.4.6). In the description, statements are complexes of expressions, and expressions are complexes of symbols. This hierarchy of levels in mathematics is not one of multivariate constituency, but of univariate complexing (or interdependency). In this sense, the hierarchical scale in mathematics is more like the layering that occurs in larger clause complexes than it is like constituency within a clause.

On the other hand, like the constituency-based rank scale of English and unlike clause complexing, mathematics has obligatory levels with distinct sets of choices. Any mathematical statement makes choices at both the level of statement and the level of symbol. Mathematics thus has a scale with obligatory levels. Following from the previous paragraph, however, these obligatory levels are not a multivariately based rank scale, but one based on univariate layering. We thus have an obligatory set of levels based on univariate layering. This hierarchy we will call a nesting scale. As shown above, only two networks are needed to account for the variation from the level of symbol up. As there are only two networks, only two nesting levels are needed: symbol and statement; the level of expression is not needed.

The nesting scale comes about through choices made on two levels. At the level of symbol, symbols can complex with other symbols through binary operations such as ×, ÷, – etc. These symbol complexes can in turn complex to form statements. The relations of binary operations, however, are not available for the complexes that form statements. At this level, Relators such as =, > etc. are used. Thus, there are two mutually exclusive sets of relations that form different sized units. From this, two distinct levels are justified. The term nesting is used purely to distinguish levels based on a univariate structure from those based on a multivariate structure.

The choice of relations at two univariate levels is comparable to those within and between verbal groups in English verbal group complexes. Within the verbal group, there is a serial tense system, built on a hypotactic univariate structure (Halliday and Matthiessen 2014). For example[35]:

[35] Notation follows Martin et al. 2010. Superscript 0 indicates present tense, - indicates past tense, + indicates future tense; perf. indicates perfective aspect, imp. indicates imperfective aspect.

has	been	running
v-s	have…v-en	be…v-ing
α^0	β^-	γ^0

As well as this, whole verbal groups can complex. The choice of relations between verbal groups, however, is not the same as those within the verbal group. Within the verbal group, the univariate relations build serial tense: *had been going to run*. The relations between verbal groups, on the other hand, build phase, *began to run*, conation, *try to run*, or modulation, *tend to run*. The choices of different types of verbal group complexing are relatively independent of tense choices within the verbal group. This means that serial tense choices can occur within verbal group complexes, such as in *will have tried to have been running*. With two sets of choices linking two units, two layered levels occur based on the univariate structure:

will	have	tried	to have	been	running
α			β		
will	v-0	have…v-en	v-0	have…v-en	be…v-ing
α^+	β^0	γ^-	$\alpha^{perf.}$	β^-	γ^0

With two sets of choices relating two different units (tense between auxiliary and finite verbs in the verbal group, and phase/conation/modulation between verbal groups in the verbal group complex) two levels occur. This is distinct from the layering that occurs in clause complexing. In English, the choice of both projection and expansion is available at all layers. For example both (3:105) and (3:106) are grammatical. (3:106) has projection in the outer layer, with expansion in the inner layer:

(3:105) Halliday said that grammatical categories are not theoretical and that they are ineffable.

In contrast, (3:106) has expansion as the outer layer with projection in the lower layers:

(3:106) Halliday said that grammatical categories are not theoretical and he said that they are ineffable.

Thus the nature of layering in mathematics and English verbal group complexing is distinct from English clause complexing. In mathematics and verbal group complexing, there are distinct sets of choices at different levels, whereas this is not the case for clause complexing. In contrast to mathematics, however, in the English verbal group the choice of the highest layer (verbal group complex) is not obligatory. That is, a single verbal group can occur on its own within a clause. Thus, there is no obligatory univariate nesting that occurs. Rather, it is an optional set of levels. In mathematics, however, this chapter has argued that two expressions must be complexed into a statement. This necessitates an obligatory univariate nesting scale. The three different forms of layering can be distinguished as in Table 3.9.

	Univariate layering	Distinct choices at each level	Choices at highest layer obligatory
English clause complex	**Yes**	**No**	**No**
English verbal group/ verbal group complex	**Yes**	**Yes**	**No**
Mathematics symbol/ statement	**Yes**	**Yes**	**Yes**

Table 3.9 Types of layering

As Table 3.9 shows, it is only because mathematics involves univariate layering, distinct choices at each level *and* an obligatory choice at the highest layer, that a nesting scale is needed. Such a scale would not be needed if any of the above criteria were not met.

As mentioned above, only two nestings are needed: statement and symbol. The network of symbol accounts for both the internal structure of symbols and their complexing into expressions. The network of statement includes the complexing relations between expressions and the Theme-Articulation structure. It is possible this higher level network could be called expression rather than statement; however since two expressions must necessarily complex into a statement and this complex has its own Theme-Articulation variation, the label *statement* is preferred. Regardless of what they are called, only two nesting levels are needed. If we name them *statement* and *symbol*, the term *expression* no longer has any formal meaning in terms of the paradigmatic networks. Informally, however, it will continue to be used to refer to symbol complexes and to either side of a statement. The nesting hierarchy of symbols and statements can thus be represented diagrammatically as in Figure 3.17.

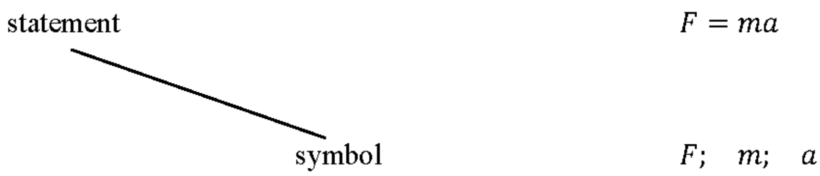


Figure 3.17 Nesting scale of mathematics

The nestings of statement and symbol broadly correspond to the ranks of statement and component in O'Halloran's (2005) grammar. As described above, there is no specific level corresponding to O'Halloran's expression, nor is there any equivalent level to O'Halloran's clause rank (introduced in Chapter 2 Section 2.4.2). Under the description developed in this chapter, O'Halloran's clause (e.g. $F = ma$) is a minimal statement with only two expressions.

As described in Chapter 2, O'Halloran also shows the high degree to which optional layering can take place. Working with a rank-scale, O'Halloran describes this optional layering as rankshift. As the description being built in this chapter uses univariate nesting, we will deploy the term *layering* for O'Halloran's rankshift. Thus, *nestings* indicate the obligatory levels, while *layering* indicates the optional levels. Nesting is to layering as rank is to embedding. This is summarised in Table 3.10.

levels	univariate-based	multivariate-based
obligatory	*nesting*	*rank*
optional	*layering*	*embedding*

Table 3.10 Types of levels

It must be noted that optional layering is not available at all levels. Indeed it is only within the nesting of symbol that layering can occur. Statements have no possibility of optional layering. That is, statements cannot occur within statements. Any insertion of a Relator such as = necessarily happens at the same level as every other Relator in the statement. Symbol complexes, on the other hand, can have quite deep and complicated optional layering, as shown in (3:107-108). Square brackets have been added to (3:108) to show the different *optional* layers within the expression:

(3:107) $$K + U = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

(3:108) $$[K + U] = [[\frac{1}{2}]m[v^2]] - [\frac{[GMm]}{r}]$$

On the left side of the equation, there is only a single optional layer. On the right hand side there are three optional layers. These optional layers occur at the obligatory nesting of symbol. Both the optional layering and the obligatory nesting are shown with their different relations in the Table 3.11.

Obligatory Nesting			
	Statement	**Relator**	**Expressions Involved**
statement	$K + U = \frac{1}{2}mv^2 - \frac{GMm}{r}$	$=$ equals	• $K + U$ • $\frac{1}{2}mv^2 - \frac{GMm}{r}$

		Expression			**Expression**		
		$K + U$			$\frac{1}{2}mv^2 - \frac{GMm}{r}$		
	Optional Layering	**Symbol Complex**	**Operation**	**Symbols Involved**	**Symbol Complex**	**Operation**	**Symbols Involved**
symbol	Layer 1	$[K + U]$	$+$ addition	• K • U	$[[\frac{1}{2}]m[v^2]] - [\frac{[GMm]}{r}]$	- subtraction	• $[\frac{1}{2}]m[v^2]$ • $\frac{[GMm]}{r}$
	Layer 2				$[[\frac{1}{2}]m[v^2]]$	× (elided) multiplication	• $[\frac{1}{2}]$ • m • $[v^2]$
					$[\frac{[GMm]}{r}]$	— division	• $[GMm]$ • r
	Layer 3				$[\frac{1}{2}]$	— division	• 1 • 2
					$[v^2]$	power (superscript)	• v • 2
					$[GMm]$	× (elided) multiplication	• G • M • m

Table 3.11 Obligatory nesting and optional layering in an equation

When looked at from the point of view of language, the univariate nesting scale that occurs in mathematics is 'exotic'. This is because, in Systemic Functional descriptions of language, the obligatory hierarchies are organised around multivariate rank-scales. It remains to be seen whether nesting scales are a broader feature of certain types of semiotic system. It may be the case, for example, that the broader family of symbolism including chemical symbolism, linguistic symbolism and formal logic symbolism are organised around nesting scales such as this (discussed in Chapter 6). We will see in Chapter 5 that when viewed through field, the univariate organisation of mathematics has a significant impact on the structuring of knowledge in physics. If it turns out that other symbolic systems are indeed organised univariately like mathematics, this field-based perspective could provide an explanation for their uptake and evolution alongside language.

The nesting scale is not the only hierarchy needed to account for mathematical symbolism. The following section will describe a network for types of element, which work at a level below symbol. This network derives from the pre-selection of the type of Quantity needed by various unary types. From this, it will become clear that a small rank scale based on a multivariate structure is needed to account for the variation in types of symbol. This will mean the architecture of mathematics involves two interacting hierarchies based on different structures, with the level of symbol facing both ways.

3.4.5 System of ELEMENT TYPE

Section 3.4.3 showed that not any element can take any unary operation. For example Specifiers cannot occur on numerals, $*2_1$ but can occur on elements taken from the Roman or Greek alphabet, known as pronumerals: E_1. To account for this, we will distinguish between different types of element. Elements realise Quantities, and thus occur at a lower level than symbol. Elements themselves do not have any internal structure, but rather are justified through preselection from higher levels. In this way, they are similar to morphemes in relation to words in English; they form the lowest level of the description and make up the constituents of symbols. We can use $\sin x$ to distinguish between the levels of symbol and element, through the analysis shown in (3:109).

(3:109)

	sin	x
symbol class	unary	
symbol structure	Operation	Argument
		Quantity
element class		pronumeral

Under this analysis, the entire symbol $\sin x$ is of the class [unary] at the level of symbol, realised by the Operation^Argument. The Argument is conflated with the Quantity, which is realised by the element class [pronumeral]. The 'sin' does not need an analysis at a lower level as it has been lexicalised at the level of symbol. There is no possible variation *within* sin that necessitates its own system. This section will be concerned with developing the system of different types of elements, before moving to a consideration of the relationship between the level of symbol and element in the following section.

The first distinction is between elements known as [units] and all others, termed [values]. The distinction can be seen in the equation $F_{perp} = 20\ 000\ \sin 30 = 10\ 000\ \mathrm{N}$ from Text 3.6 above. The final N (glossed as *Newtons*) is unlike the other numbers and pronumerals. It is a unit of measurement; in this case, the unit of Force (F in the equation). Each physical quantity will have its own unit or set of units, for example in the standard units of physics (know as SI units) mass is measured in kilograms (kg), length in metres (m), time in seconds (S). These units can complex through binary operations of multiplication, division, etc. just as other symbols can. For example acceleration is measured in m/s^2 (metres per second squared), and the universal gravitation constant (G) is measured in Nm^2/kg^2 (Newton metres squared per kilogram squared). They cannot, however, occur within a unary operation, e.g. neither $*\sin \mathrm{N}$ nor $*\Delta \mathrm{m}$ occur. Units are usually not written in italics (whereas pronumerals are), and predominantly occur after the final numerical solution of a quantification.[36] An example of this is shown in an excerpt from Text 3.6, with the unit N shown in bold:

[36] Often units will be accompanied with a direction, such as *downwards* in $W_{Mars} = 180\ \mathrm{N}$ downwards, from Text 3.1. This depends on whether the variable being measured is a vector or a scalar (discussed below). Directions are not taken as part of the grammar of mathematics described here as they cannot complex with binary operations, nor can they vary their position like all other symbols, and they are almost always realised by language, not a specific mathematical symbol. They appear best described as a linguistic element emergent from the intermodality between mathematics and language.

$$F_{perp} = W \sin\theta$$
$$= 20\,000\ \sin 30 = 10\,000\ \mathbf{N}$$

All other elements are classed as [values]. The basic distinction within [values] is between [numerals]: 1, 2, 7.3, 1049274 etc. and [pronumerals]: x, y, π, κ etc. Grammatically, they can be distinguished through preselection from specification at the level of symbol. Pronumerals can take a Specifier: E_1 whereas numerals cannot: *2_1. This distinction is important for the discussion of genre in the following chapter; two genres known as quantifications and derivations are differentiated by whether the final Articulation within the final statement involves numerals (quantification) or pronumerals (derivation).

Within [pronumerals], we can distinguish [variables] from [constants]. [variables] are pronumerals that could potentially have a number of different numerical values, whereas constants are those that cannot. An example of a [constant] is π which has an unwavering value of 3.1415.. In contrast, *F* and *W* in the equations above are considered variables. This distinction is presaged by the fact that the unary operator change, Δ, necessarily requires a variable: Δ*F* is acceptable, whereas *Δπ is odd at best. Finally, within variables are [scalars] and [vectors].[37] Notionally, [vectors] are number with a direction, whereas scalars do not include a direction. For example, if a force was to occur, it necessarily occurs in a particular direction (up, down, left, right or somewhere in between), thus it is a vector. Mass, on the other hand, is a directionless scalar; it does not occur in any direction, but is simply a physical quantity. The direction for vectors are often specified after the units in a quantification, such as *downwards* in $W_{Mars} = 180$ N downwards (though see footnote 16). Vectors are often indicated through being bold: e.g. ***F*** or through an arrow placed above it, e.g. $\vec{F}$.

Various distinctions could be made within [numerals], however this will be the furthest step in delicacy taken in this description. The system for element is thus shown in Figure 3.18.

[37] Particular types of vector known as Unit vectors, such as those used in certain coordinate systems (e.g. $\hat{r}, \hat{\varphi}$ and $\hat{\theta}$ for spherical coordinates) are not included in this grammar as they are more commonly used in vector calculus, which is beyond first year university physics. If they were to be included, however, the system distinguishing vectors and scalars would be simultaneous with the system distinguishing constants and variables.

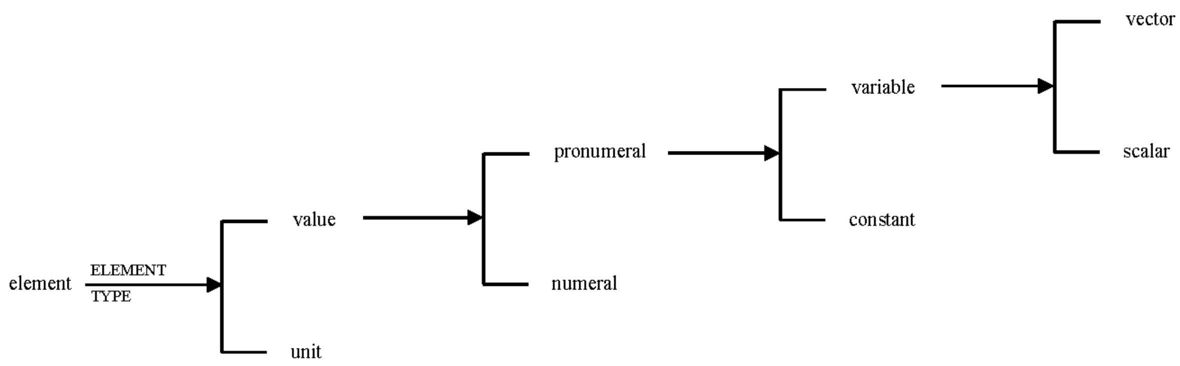


Figure 3.18 System of ELEMENT TYPE

3.4.6 Levels in the grammar of mathematics

Section 3.4.4 suggested the relationship between symbols and statements involves two levels on a nesting scale. This was based on the fact that symbols complex into statements through a univariate structure. Two networks were developed for these levels. The statement network showed complexing involving Relators such as = and >. The symbol level indicated complexing that utilised binary operators such as +, -, × etc. With each potential complexing relation only available at specific levels (Relators between expressions and binary operators between symbols), two nesting levels were justified. The previous sections have shown that as well as the univariate complexing, there is also internal structure within symbols that is best described multivariately. This multivariate structure preselects distinct types of element to be placed within each symbol. Accordingly another network of element types was proposed, accounting for the possible choices of element. This element network does not have the same relation to statements as symbols do. There is no univariate complexing of elements that make up symbols - we cannot say sin(*y*2) where *y*2 does not indicate some binary relation such as multiplication or power. Elements do not complex into symbols in the way symbols complex into statements. Rather, a more fruitful avenue is to view elements as constituents of symbols. This is similar to viewing morphemes as constituents of words and word groups as constituents of clauses in English. This comes about through the internal multivariate structure of symbols. Different components of symbols perform different functions, with some of these components preselecting certain types of element at the level below. Thus, due to the multivariate nature of symbols and their constituency relation with elements, the development of an element network sets up a rank scale of the type more commonly associated with language. This rank scale has the symbol as the highest level and the element as the lowest, as pictured in Figure 3.19.

Figure 3.19 Rank scale of mathematics

In this diagram, $\sin x$ sits at the rank of symbol, with x arising from the rank of element. As discussed in Section 3.4.3 above the sin is lexicalised at the rank of symbol and so does not need to be accounted for at the rank of element. Combining this rank scale with the nesting scale, we see that the symbol plays two roles. It is the lowest level of the nesting scale below statements as well as being the highest level of the rank scale above elements. It is both a rank and a nesting. With the symbol facing both above and below, the hierarchy of levels in mathematics can be viewed as an interaction between a nesting and rank scale, represented in Figure 3.20.

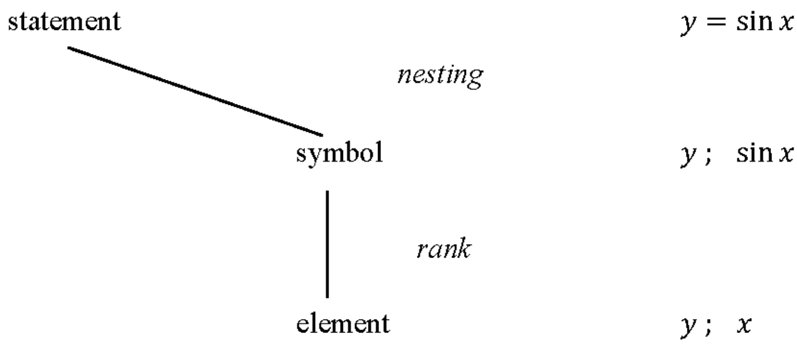


Figure 3.20 Levels in mathematics

We can now combine the rank scale with the obligatory nesting scale and its optional layering to represent a single statement. Table 3.12 shows each level within the equation $v_y = v \sin \theta - at$.

Obligatory Nesting	Rank	Statement	Relator	Expressions Involved
statement		$v_y = v \sin\theta - at$	= equals	• v_y • $v \sin\theta - at$

Obligatory Nesting	Optional Layering	Expression: Symbol Complex	Expression: Operation	Expression: Symbols Involved	Expression: Symbol Complex	Expression: Operation	Expression: Symbols Involved
		v_y			$v \sin\theta - at$		
symbol	Layer 1				$[[v \sin\theta] - [at]]$	- subtraction	• $[v \sin\theta]$ • $[at]$
	Layer 2				$[v \sin\theta]$	× (elided) multiplication	• v • $\sin\theta$
					$[at]$	× (elided) multiplication	• a • t

Rank	Symbol	Operation	Element Involved
element	v_y	specification	v
	v		v
	$\sin\theta$	sine	θ
	a		a
	t		t

Table 3.12 Obligatory nesting, optional layering and rank in an equation

Although only containing three levels, the janus-faced nature of mathematical symbols means the hierarchy of mathematics is somewhat more complicated than for any language described systemically to date. There is an interaction between obligatory nestings and ranks, with the potential for further optional layering involving symbol complexes. As discussed previously, an architecture such as this might be viewed as exotic from the perspective of language. To this point, descriptions of languages have not necessitated an obligatory nesting scale, let alone one which coexists with a rank scale. It remains to be seen whether interacting scales such as these are a feature of many families of semiotic resources, or whether they are a feature of only mathematics. What is clear, however, is that the two scales are possible and can occur simultaneously.

This discussion of the levels within the grammar of mathematics concludes the description proper. Each relevant network and their structural organisation has been introduced, with the possible variation at each level comprehensively described. It is now time to take a step back and view the grammar as a whole. It is here that we can approach the questions of how to interpret mathematics in terms of theoretical concepts such as metafunction. It is from this perspective that we will see the most striking feature of mathematics in relation to the Systemic Functional model of language.

3.4.7 Metafunction in the grammar of mathematics

The description put forward in this chapter has attempted to treat the grammar of mathematics on its own terms. To do this, it has taken the axial relations of system and structure as the primary basis upon which semiotic description holds. This has meant that broader phenomena such as metafunction, strata and rank have not been assumed at the outset. The challenge set forth was to independently justify these phenomena. Accordingly, this approach has the potential to produce different architectures for mathematics than for English. However if we wish to understand the specific functionality of any distinct semiotic system, such an approach is necessary. The discussion above on the levels in mathematics exemplifies this. In building a level scale built on distinct patterns of systems and structures rather than assuming a rank scale, a unique set of levels have been shown. A two level rank scale complements a two level nesting scale, with the level of symbol situated in both. This is in contrast to O'Halloran's (2005) hierarchy that posits a four level rank scale. The difference has come about through distinct methodologies and motivations. O'Halloran proposes a language-based view of mathematics which involves a rank scale; this description takes an axial view of mathematics and builds a distinct set of levels.

As it is for rank, so it is for metafunction. Following the same principles that determined the distinct level hierarchy of mathematics, this section will be concerned with building a model of metafunction in mathematics. As stressed throughout, metafunctions are not assumed, but must be justified. Evidence for metafunctionality is drawn from two sources: i) relative paradigmatic independence or interdependence and ii) structural similarity or dissimilarity (see Chapter 2 Section 2.4.4). If an area of the grammar has both the potential for relatively independent variation with other areas and a distinct type of structural realisation, evidence of a metafunctional component exists.

From this basis, we will see that the architecture of mathematics is dominated by the ideational metafunction. In particular, the logical component permeates the grammar and builds the nesting scale on which most sets of choices exist. The other component within the ideational metafunction we will call the operational component. In comparison to the logical component, this component is relatively small and is in some ways subservient to the logical. It is nonetheless responsible for the development of the rank scale. Textual variation comes about at both levels of the nesting scale, organising the information flow. Most strikingly, however, there appears to be no evidence for an independently motivated interpersonal component. Each of these observations will be considered in turn, before turning to a discussion of the overall functionality of mathematics that this analysis suggests.

3.4.7.1 The logical component

The predominant structural organisation of the grammar is univariate. Statements are built from a univariate complex of expressions that are indefinitely iterative. And symbols can indefinitely complex with other symbols. On the other hand, the Theme-Articulation distinction is based on a multivariate structure. However, the tension between this structure and the univariate organisation results in indefinitely iterative Articulations. The univariate structure is dominant at this level. This is reflected in the paradigmatic organisation of the statement where all choices are potentially recursive (see Figure 3.12). Each new expression necessitates a new choice in the type of Relator and vice versa. At the level of symbol below, recursion is also dominant. Symbols can and do complex into intricate expressions.

Furthermore, even unary operations, which display a more multivariate structure, are potentially recursive. Indeed it is through the obligatory complexing of expressions into statements (that is, the complexing of symbol complexes) that the nesting scale arises. The recursive and univariate organisation of the mathematics is pervasive.

The structural similarity of each of these systems, being univariate, suggests each could be part of a similar functional component. This is augmented by the fact that in mathematics the protypically recursive systems are in general independent of those that produce the multivariate structures. The choice of the number of expressions in a statement is independent of the choice of their organisation in terms of Theme and Articulation; the sequence of expressions does not determine the choice of Relator.[38] At the level of symbol, the system of EXPRESSION TYPE that determines the complexing relationships *between* symbols is simultaneous and thus independent of the choices *within* symbols that are multivariate. That is, any unary operation such as sin, cos, Δ etc. can occur with any binary operation, such as ×, ÷, + etc. The system of EXPRESSION TYPE is also closely intertwined with the system of COVARIATION. The covariate relations, in conjunction with the system of STATEMENT TYPE, determine the possible types of expression. This leads to the potentially indefinitely iterative nature of the COVARIATION system, whereby any number of covariate relations may occur. Due to this close interaction of COVARIATION and EXPRESSION TYPE and the fact that the COVARIATION system is indefinitely recursive, leading to a structure more closely related to univariate structures than multivariate structures, COVARIATION can also be considered part of this component (see Section 2.1.2 in Chapter 2 where univariate and covariate structures are grouped together as interdependent structures that are opposed to multivariate structures).

All areas of the grammar that are organised through an interdependency structure (univariate plus covariate) are almost entirely independent of those organised multivariately. That is, there is a large group of systems that has structural similarity and paradigmatic independence from other systems. Thus these systems fulfil the criteria for being grouped into a distinct functional component.

Given the fact that this component has recursive systems as one of its hallmarks and is primarily organised through a univariate structure, this component appears most similar to the logical component within English (Halliday 1979). We can thus responsibly classify these

[38] An exception being the swapping of [greater-than] (e.g. >) with [smaller-than] (e.g. <) in certain cases. See section 3.4.3.

systems as being part of the logical metafunction. The systems that constitute the logical component are shown in Table 3.13.

Metafunction / Level	Logical
statement	STATEMENT TYPE REARTICULATION COVARIATION COVARIATE MULTIPLICITY
symbol	EXPRESSION TYPE

Table 3.13 Logical metafunction in the grammar of mathematics

The logical metafunction dominates the grammar, colouring most other systems. As discussed above, this is seen through the potentially recursive unary operations (classed as operational below) and the iterative Articulations (classed as textual below) brought about by tension with the potentially iterative expressions. The logical metafunction is also responsible for the nesting scale in mathematics. As we will see, however, the rank scale comes about through the other component of the ideational metafunction, the operational component.

3.4.7.2 The operational component

Simultaneous to the logical system of EXPRESSION TYPE at the level of symbol are the systems of UNARY OPERATION and SPECIFICATION. Both of these systems are realised through a multivariate structure. The system of SPECIFICATION distinguishes between the functions of Quantity and Specifier, while the system of UNARY OPERATION gives both Operation and Argument. As these systems are paradigmatically independent of the systems in the logical component and are realised through a distinct structure, a case holds for these systems to form their own functional component. This component is entirely responsible for the rank scale in mathematics. It is through preselections within both SPECIFICATION and UNARY OPERATION that the development of a system at this lower level is justified. Moreover, the multivariate structure resulting from these systems provides the constituency relation between symbols and elements that distinguish this scale from the nesting scale derived from the logical component. As the distinctions within the rank of element are preselected from systems within a single component, and there are no simultaneous systems within this rank, the entire set of choices within the rank of elements can be classed as part of the same component.

The multivariate structures in this component are similar to those within the experiential metafunction in English. Notionally the experiential metafunction is concerned with construing our experience of the outside world (Halliday 1979). In English, for example, the experiential system of TRANSITIVITY divides the clause into the material, mental and relational clauses (Matthiessen 1995), broadly construing the realms of doing, thinking and being. Although in an axial description the notional "meanings" of categories are not privileged, they are helpful when labelling. At the level of symbol, it is difficult to reconcile the choices of UNARY OPERATION or SPECIFICATION as in some way construing our outside world. In this sense, the label 'experiential' is somewhat awkward. This component is more concerned with operations on elements in symbols than with construing the experiential world. Thus, to more easily capture this nature, this component will be called the *operational* component. The systems included in the operational component are shown in Table 3.14.

Metafunction / **Level**	**Operational**
symbol	UNARY OPERATION SPECIFICATION
element	ELEMENT TYPE

Table 3.14 Operational metafunction in the grammar of mathematics

At this point it is pertinent to note the ineffability of semiotic categories (Halliday 1984). The component labelled above as operational is justified as a distinct component through its paradigmatic and syntagmatic organisation, not through its notional meaning. By labelling this component 'operational' we emphasise the differences between this component and the experiential component in English. These differences include the considerably distinct sets of choices that each component includes: the operational component of mathematics does not include choices for TRANSITIVITY, CIRCUMSTANTIATION, CLASSIFICATION, EPITHESIS, QUALIFICATION, EVENT TYPE or ASPECT as it does in the grammar of English (Halliday and Matthiessen 2014: 87). Conversely, the experiential component of English does not include choices of UNARY TYPE or SPECIFICATION as occurs in the operational component of mathematics. Indeed, aside from their multivariate structure, there is little in common between the two components. Thus, distinct labelling is appropriate. It remains to be seen whether the systems captured under the experiential component in English and those in the operational component in mathematics are in some sense part of the same component in the broader scheme of semiosis (or indeed whether metafunction is a useful category in the broader description of semiosis). To determine this, detailed axially motivated descriptions of inter- and multi-semiosis would need to be developed. What this will uncover, or indeed what this would look like, is at this stage unclear.

This aside, the operational component will be grouped with logical component as parts of the ideational metafunction. This allows the mathematical system to be characterised as one built primarily by the ideational metafunction. They are both organised syntagmatically through a particulate structure and notionally allow mathematics to represent the world.

In section 3.4.3 it was shown that each unary operation can be equated with a set of logical relations. For example, the change operator Δ, when operating on a variable, is defined as $\Delta x = x_2 - x_1$. The unary operation Δ thus distils the logical relation $x_2 - x_1$.[39] Indeed all unary operator distil a set of logical relations that can be applied to a range of symbols.

This suggests that the multivariate component of mathematics has developed as a grammaticalisation of large sets of logical relations. As mathematics progresses through higher levels of schooling, more unary operations are introduced, distilling ever increasing sets of logical relations. Thus ontogenetically speaking, it appears that the operational component develops out of the logical. Indeed, as students move into calculus, increasingly multivariate structures are built upon growing sets of logical relations. The distillation of logical relations into the multivariate structure, is similar to the development of technicality in English. The operational component both condenses and changes the nature of the logical

[39] More strictly, the unary operation Δ distils a dummy relation of subtraction between two instances of the same symbol (shown through different specification subscripts). This relation can be applied to any symbol, so $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$, $\Delta z = z_2 - z_1$ etc.

relations (Halliday and Martin 1993: 33). Although the operational component of the grammar grows through the years, it is based on, and in some sense in service of, the logical metafunction. This provides another justification for both components to be aspects of the same, ideational metafunction, fulfilling complementary roles. The final set of variation considered relates mathematics to the textual metafunction.

3.4.7.3 The textual component

In addition to the variation involved in the logical and operational components, there is a small set of choices that organise the information flow of the text. These systems include the choice of Theme, its possible ellipsis and the ordering of Articulations. Each of these choices are independent of choices in the logical and operational components.[40] The Theme can be any expression involving any complex of symbols or unary operations. Similarly any set of symbols can be elided if they are Theme.

The Theme-Articulation structure was described multivariately, but with the potential for indefinitely iterative Articulations under pressure from the logical component. This structural configuration is similar to that of the operational component. This similarity raises an issue for the characterisation of these systems as a distinct metafunctional component. They are indeed paradigmatically independent, suggesting a different component, but they have a structural realisation part-way between the prototypically univariate structure of the logical component and the multivariate of the operational.

It was said above that the nesting scale develops through the logical component, while the rank scale derives from the operational component. The paradigmatic choice of which symbols are thematised and which are placed in the Articulation occurs at the level of symbol. These choices, however, realise the Theme and Articulation structures deriving from the level of statement. As well as this, all variation at the rank of element, and all choices at the rank of symbol that preselect types of elements are accounted for by the operational component. Thus the bundle systems organising the Theme-Articulation choices and those of ellipsis sit firmly within the nesting scale.[41]

[40] Excepting those indicated in note 18 above.
[41] There is another set of choices not described here that could also form part of this component at the level of symbol. This involves the distinction between $\frac{mv^2}{r}$ and $m\frac{v^2}{r}$. Ideationally these symbol complexes are the same.

As mentioned above, these systems are independent of all choices in the logical and operational components. As well as this, they sit in the nesting scale but have a distinct multivariate configuration to the logical univariate configuration that produces the nesting scale. For these reasons, it seems appropriate to consider these systems to be part of a separate component altogether.

This component is concerned with the information flow of mathematics. Thus it can be termed the textual metafunction for mathematics. The systems comprising this metafunction are given in Table 3.15.

Metafunction / **Level**	**Textual**
symbol	THEME THEME ELLIPSIS

Table 3.15 Textual metafunction in the grammar of mathematics

3.4.7.4 The Interpersonal Component

From an axial perspective, there is no evidence to propose an interpersonal component in mathematics. The three components outlined so far, the logical, operational and textual, account for all of the systems within this grammar. There are no apparent systems realised through a prosodic structure, nor are there any other systems paradigmatically independent of those already accounted for. Looking notionally, there are no systems that appear to give similar meanings to those of NEGOTIATION or SPEECH FUNCTION, nor those that give the evaluative meanings of APPRAISAL or the power and solidarity dimensions of VOCATION. Nor are there any systems that give choices comparable to those of MOOD. Without paradigmatic independence or distinct syntagmatic structures, there is no reason to suggest a distinct interpersonal component in mathematics.

However, they are organised marginally differently in a way that appears to give some sort of textual meaning. Further description is needed to incorporate this variation into the grammar.

In light of this description, however, it is pertinent to consider a number of important insights made by O'Halloran (2005) regarding the possibility of an interpersonal component. First, O'Halloran points out that there are some similarities in the meanings between certain Relators in mathematics and interpersonal constructions in English. In particular, she suggests a system of POLARITY to distinguish between the positive polarity of = and the negative polarity of $\neq$ (often glossed as *not equal to*) (2005:100, 115). POLARITY in English is considered an interpersonal system (Martin 1984, though not without some contention, see Halliday 1978a:132 where it is treated as experiential and Fawcett 2008 where it is its own functional component), thus O'Halloran considers the distinction between = and $\neq$ to also be interpersonal in mathematics. Along similar lines, we could consider Relators ≈ and ~, both glossed as *approximately equal to*, to notionally give some sort of meaning of GRADUATION, another interpersonal system (Martin and White 2005). These could also therefore form part of an interpersonal component. Developing an interpersonal component based on similarities in meanings such as these, however, goes against the principled axial description built in this chapter. Arguments along these lines rely on notional reasoning that analogises from English. In contrast, if looking at the above axially, we see that each of these Relators, $\neq$, =, ≈ and ~ are entirely dependent on choices within the STATEMENT TYPE network, which was classified as part of the logical component. Thus, the choices giving rise to these Relators are firmly within the logical component. They do not form a distinct paradigmatic system simultaneous with those of other components, nor are they realised by a distinct type of structure. Therefore they do not constitute a distinct functional component. They do, however, suggest that the meanings of polarity and graduation have in some sense been 'ideationalised' when translated into mathematics. These meanings that would be made through the interpersonal metafunction in English are made through the ideational metafunction in mathematics.

An interpretation along these lines allows an understanding of the quantification of certainty through probability, statistics and measurement errors, in relation to linguistic modality. As O'Halloran (2005: 115) states, "In mathematics, choices for MODALITY in the form of probability may be realised through symbolic statements or measures of probability; for example, levels of significance: $p < 0.5$ (where the notion of uncertainty is quantified) and different forms of approximations." Viewed from the description developed in this chapter, the meanings of modality have been ideationalised in a similar way to the polarity and graduation meanings discussed above. What would be expressed interpersonally in language through graded modalisation of probability (e.g. *The hypothesis is <u>probably</u> true*), is expressed through an ideationally organised mathematical statement (such as through p-values: e.g. $p < 0.05$ used to determine the likelihood of a hypothesis being true or false). What is interpersonal in language can be seen as quantified and ideationalised in mathematics. Although statistical mathematics is not studied in detail in this description, it is possible that this system has developed largely to ideationalise what would otherwise in language be fuzzy interpersonal measures of modalisation.

A second important observation by O'Halloran regards interactions between mathematics and language in instances such as $Let\ x = 2$. The use of *Let* before the mathematical statement indicates the construction is a discourse semantic command (demanding goods and services). Without the *Let*, however, the mathematical statement is arguably more similar to a discourse semantic statement (giving information). Thus, the *Let* affects the speech function, giving it variability in interpersonal meaning.

In regards to whether this constitutes evidence for an interpersonal component in mathematics, the grammar developed in this description only considers mathematics in isolation; it does not look at the interaction of mathematics and language. From this perspective, the introduction of language into the statement is immaterial to a discussion of the functionality *internal* to the system of mathematics. It does, however, raise an important challenge that has yet to be fully solved. The introduction of language appears to contextualise the mathematics, transposing the speech-functional meanings from language across to the mathematics. This raises the question of how then we are to model metafunctionality across inter-semiotic systems, or indeed across semiosis in general. Arguing that mathematics does not have an interpersonal component internal to the system does not preclude the possibility that mathematics occurs in texts with resources that do engender interpersonal meaning. More broadly speaking, with multimodal texts, the various functionalities of each semiotic resource are likely to contextualise one another. In the case of $Let\ x = 2$, as mathematics does not have the ability to distinguish between speech-functions, it appears that language is being 'imported' as necessary to make these meanings. Studying the internal functionality of different semiotic resources could provide insights into why some are used in conjunction with others.

One final point regarding the lack of an interpersonal component in mathematics concerns its relation with the register-variable of tenor. In Systemic Functional studies, it is generally accepted that there is a metafunctional "hook-up" with different register variables: shifts in field tend to impact ideational meanings, shifts in tenor tend to impact interpersonal meanings and shifts in mode tend to impact textual meanings. Without an interpersonal component, this register-metafunction correlation could potentially be put at risk. This, however, brings us back to the point made above that mathematics is rarely used in isolation. Other semiotic resources such as language, images and gesture, are regularly used alongside mathematics in various contexts. Multimodal texts that include mathematics are likely to shift interpersonal meanings across multiple resources. Thus tenor would be realised multimodally.

In short, the lack of an internally motivated interpersonal component in mathematics does not preclude mathematics from being involved in interpersonal meaning in a multimodal text. What it does suggest is that mathematics cannot produce variations in interpersonal meaning of its own accord; it must do so in interaction with other semiotic resources. Interpersonal meanings from one resource are likely to be ideationalised when translated into mathematics. It is possible this ideationalising feature of mathematics is a large reason for its powerful role in academic disciplines. The role of mathematics in building academic knowledge will be discussed in relation to genre and field in Chapters 4 and 5.

3.4.7.5 The function-level matrix for the grammar of mathematics

With the discussion of metafunctionality in mathematics, the description has been completed. We can now bring together the metafunctions, level hierarchy and systems of mathematics into a single function-level matrix shown in Table 3.16.

<table>
<tr><th>Nesting</th><th>Rank</th><th>Logical</th><th>Textual</th><th>Operational</th></tr>
<tr><td>statement</td><td></td><td>STATEMENT TYPE
REARTICULATION
COVARIATION
COVARIATE MULTIPLICITY</td><td></td><td></td></tr>
<tr><td colspan="2">symbol</td><td>EXPRESSION TYPE</td><td>THEME
THEME ELLIPSIS</td><td>UNARY OPERATION
SPECIFICATION</td></tr>
<tr><td></td><td>element</td><td></td><td></td><td>ELEMENT TYPE</td></tr>
</table>

Table 3.16 Function-level matrix for the grammar of mathematics

This table shows the most salient features of the architecture of the grammar of mathematics:

- Three functional components: textual, logical and operational (but no interpersonal);
- Two level hierarchies: a nesting hierarchy involving statements and symbols, and a rank-scale involving symbols and elements. The level of symbol operates on both hierarchies;
- Logical and textual systems are confined to levels on the nesting scale;
- Operational systems are confined to levels on the rank scale.

3.5 An axial description of mathematics

This chapter set itself the goal of developing a model of mathematics based on axial principles. It has taken the paradigmatic and syntagmatic axes as primitive and sought to justify larger features of the descriptive architecture from these. To this end, it did not assume macrotheoretical categories such as metafunction or rank, but rather looked to derive these from the systems and structures apparent in the grammar. In doing this, it has produced a description that is in a number of ways "exotic" when compared to language. Rather than a single rank-hierarchy based on constituency, it has shown there are two hierarchies in play, catering to different types of variation. The nesting hierarchy is derived from the logical component and affords the possibility of textual variation. The rank-scale, on the other hand, organises the systems of the operational component. These three components, the logical, operational and textual, were shown to be the only metafunctional variables needed to account for the entirety of the grammar. No interpersonal systems were found. Indeed it was suggested that part of the reason for using language in interaction with mathematics is to make use of language's interpersonal meanings. Mathematics, for its part, was argued to 'digitally' ideationalise meanings that may otherwise have been expressed interpersonally in graded systems in language. The specific meanings made by mathematics, and their role in the knowledge building in physics will be explored in relation to the register variable field in Chapter 5.

The approach taken in this chapter has attempted to utilise a descriptive methodology that can show the functionality of mathematics on its own terms. Axis was chosen as the primitive in relation to i) its potential to be used as the basis for deriving other characteristics of the grammar and ii) its generalisability across semiotics systems. If we wish descriptive and theoretical categories such as metafunction and rank to continue to have utility in the future of Systemic Functional Semiotics, they must be justifiable. The description offered here has shown that metafunction and rank are indeed productive notions, allowing large segments of mathematics and other semiotic resources to be characterised and generalised. But without a principled axial foundation for determining and distinguishing ranks, metafunctions and other categories, descriptive semiotics runs the risk of emptying these terms of meaning, and making everything look like English.

This chapter has looked at mathematics in isolation and on a small, grammatical scale. The following chapter comes at mathematics from a different angle, genre. As part of this, language will be brought into the picture, providing an avenue for understanding intersemiotic relations. With views from both grammar and genre, Chapter 5 will consider mathematics' role in knowledge building in relation to images and language from the perspective of the register variable field.

CHAPTER 4

Genres of Mathematics and Language

Mathematics is just one of many ways of meaning in the discourse of physics. In physics classrooms and textbooks it is regularly used in conjunction with images, gestures, film and demonstration apparatuses. Its most common counterpart, however, is language. The interplay between mathematics and language is inescapable and must be seriously considered if we are to understand texts in use. When mathematics is present, there is a constant exchange of meaning back and forth with language that works toward a variety of purposes. This exchange works not only to move from the meanings made by one resource to the meanings made by the other, but to build new meaning; in Lemke's (1998) terms, it works to multiply the meanings made by each resource.

The previous chapter considered mathematics as a system in its own right. In doing so, it provided a platform for understanding the nature of mathematics, and for discussing its possible variation in texts. However that discussion explicitly avoided consideration of how mathematics interacts with other semiotic resources. This chapter takes the next step by considering physics bimodally, as an academic discipline construed through both mathematics and language. In doing so, it will focus in more detail on the regular co-patternings of mathematics and language in physics texts. This is in pursuit of understanding how the knowledge of physics is construed and the question of why mathematics is used in physics.

The interaction of semiotic resources such as mathematics and language can be considered from many different angles. The most common approach is to propose systems that describe the intersemiotic relations between various semiotic resources (especially image-text relations, see Bateman 2014a). O'Halloran (2005: chapter 6), for example, presents an account of intersemiotic relations among mathematics, language and image at the levels of discourse semantics, grammar and display (a general term used for the expression planes of image, mathematics and written language). These systems consider both the micro- (grammatical) and macro- (discourse) transitions at play in mathematical discourse in order to understand, among other things, how multisemiotic texts mix and adopt features of different resources while maintaining coherence. Approaches such as O'Halloran's consider the movements between semiotic resources within texts, rather than characterising texts as a whole. In contrast, Bateman (2008) in his study of highly multimodal documents focuses on the overall organisation of texts as genres. In his analysis, the semiotic resources in play are considered together from a number of levels, including rhetorical structure, content structure and layout. These in turn are brought together to characterise the text as a whole. Bateman stops short, however, of proposing distinct types of multimodal text with distinct structures (i.e. an inventory of multimodal genres), citing the need for more empirical investigation.

Following Bateman's lead, this chapter will consider bimodal mathematical and linguistic texts from the perspective of genre. It will, however, take one step further by developing an explicit description of genres that involve language and mathematics in relation to their text structures. By doing this the model will show how these distinct genres coordinate the grammatical patternings of language and mathematics, in particular the choice of Articulations (see Section 3.4.2.3 in the previous chapter). As the genres described in this chapter necessarily involve mathematics, with language being optional (though common), they will be termed mathematical genres. This is not, however, to suggest that language is not involved; the genres that will be described are regularly realised bimodally. Indeed when language is used, it often plays a vital role in the construction of the text. The term mathematical genre is simply a shorthand to distinguish these genres from those that are primarily realised monomodally in language (such as those in Martin and Rose 2008).

By coming at bimodal relations in terms of genre, mathematics and language can be seen not simply as isolated systems that each present their own meanings, but as coordinated resources that work together to achieve higher order meaning. The grammatical and discourse textures they assume and the intersemiotic relations they show can be understood as being driven largely by the genre-based functions they jointly realise. This perspective provides a complementary angle on language-mathematics relations to the discourse-semantic and grammatical model developed by O'Halloran. Moreover, it allows for a characterisation of whole texts and the purposes to which they are put to use. Indeed once the model of genre has been built, it will be deployed as a means to map the changes in the use of mathematics in physics through schooling. This map will allow for a broad interpretation of the role of mathematics in building knowledge in physics, and of its utility as a semiotic resource in general.

The chapter is organised around two main themes. Sections 4.1 - 4.3 will build a model of genre based on the axial principles laid out in the previous chapters. This will involve considering the basic elemental genres and genre structures that are jointly realised by mathematics and language, as well as the possibility these genres have for complexing in larger texts (section 4.2). It will also discuss how the description fits into an overall Systemic Functional framework for genre (section 4.3). The following section (4.4) will change focus and consider the second theme for the chapter: the use of mathematics for building knowledge in physics. The grammar and genre models developed in this and the previous chapter will be used to map the changes in the types of mathematics in use in physics through schooling and the text patterns that emerge from them. To explore how these patterns organise the knowledge of physics, they will be interpreted using the dimension of Semantics from Legitimation Code Theory (LCT) (Maton 2014. See chapter 2 Section 2.3). LCT Semantics provides a nuanced understanding of how the various resources of mathematics allow physics to build integrated and generalised knowledge, while at the same time remaining in contact with its empirical object of study. Bringing the two approaches together provides a method for understanding the kinds of mathematics used in physics, why they are used and what the payoff is for physics as a discipline. It will reveal the powerful utility of mathematics, and move toward answering the simple question: why is mathematics used in physics?

By focusing on each of these themes, the chapter will develop a large scale model of mathematics in use and its disciplinary affordances (Fredlund et al. 2012) for physics. Before moving onto the description proper, however, we must review the place of genre in the general theory of Systemic Functional Semiotics.

4.1 Genre in Systemic Functional Semiotics

The previous chapter laid out the principles that guide description in this thesis. It argued that descriptions must in some way bring out the specific functionality of the resource under study, that they must be able to be compared with descriptions of other resources, and finally that they must be based upon explicit methods of argumentation so that they can be compared and judged in relation to competing descriptions of the same resource. This required that each resource be described on its own terms and that any category proposed be justifiable in terms of the system being studied. These principles were not intended just for the grammatical description in this thesis, but for the semiotic description in general. The implication of this is that just as we needed to be hesitant about carrying over the macrotheoretical categories of metafunction and rank from English to mathematics, so should we be hesitant about unquestioningly carrying over a stratum such as genre and the stratification hierarchy this implies. To justify the ranks, nestings and metafunctions proposed in Chapter 3 we used an axial perspective whereby each category was emergent from the interplay of paradigmatic and syntagmatic axes. This kind of axial justification can be used for the development of strata as well. Reasoning along these lines, for a new level to be proposed, it must arise from a distinct system-structure cycle (Martin 2013). This means there must be a distinct set of choices that have their own structural realisations. For this system structure cycle to be considered a level, as opposed to a metafunction, it must indicate a distinct unit to those of other levels. For this level to be considered a stratum, as opposed to a rank or nesting, its units must distinguish themselves from other system structure cycles not through constituency, but through abstraction. Strata do not map onto constituent units – phonology is not a part within a lexicogrammatical whole and lexicogrammar is not a part within a discourse semantic whole – rather they provide levels of meaning without any definite relation between the units they map on to (though as Martin 1992a points out, as one moves higher in strata, the units tend to get bigger). For a stratum to be proposed, therefore, it must have its own systems, structures and units related through abstraction to other strata (see Section 2.4.4 for a more detailed discussion).

The description developed below will show that these criteria for a stratum can be satisfied for mathematical texts. The question that arises from this, however, is why we should consider such a description as being at the stratum of genre, as opposed to discourse semantics or any other stratum. The justification for this lies in the conceptualisation of genre as a connotative semiotic (Martin 1992a: 493, developing Hjelmslev 1943 and Barthes 1967, 1973). A connotative semiotic is a semiotic resource that has another semiotic as its expression plane. In contrast, a denotative semiotic system that has its own expression plane (see Chapter 2, Section 2.1.1). For example the denotative semiotic of language has phonology and/or graphology as its expression plane. As well as this, Cléirigh also suggests that body language (kinology) can be conceptualised as another expression of language (Cléirigh in prep., Martin 2011a, Zappavigna et al. 2010). On the other hand, genre (in Martin's terms) does not have a distinct system such as phonology, graphology or kinology as its expression plane, but rather uses language as its expression The result of this is that genre (and register in Martin's conception) is not language; rather it functions as the content plane of a distinct semiotic system in which language is the expression plane.

The question that arises here is whether we should recognise multiple types of expression plane realising a single system. Language, for example, is realised by two: phonology and graphology (and arguably a third, kinology or body language) If language can be realised by multiple expression planes, this leaves open the possibility for the connotative semiotics of register and genre to also be realised by multiple expression planes at the same time. Indeed if we accept body language as an expression form of language, we have in fact already allowed for this possibility. For our description of mathematical genres, then, we need to explore the possibility that they are realised by more than one system of expression form (i.e. the denotative semiotics of mathematics and language). This is the position this chapter will take. It will be justified in more detail in Ssection 4.3 once mathematical genre systems have been presented.

4.2 Genres of mathematics and language

Texts involving mathematics show a regular progression of meaning. The previous chapter showed that when multiple mathematical statements are presented in sequence, they show complementary patterns of Theme (left side of statements) and Articulation (right side of statements). Articulations tend to show change as the text progresses, while the Themes (left side) tend to remain relatively stable. This pattern emerges in spite of the fact that there is in principle free variation in terms of ideational meaning regarding whether expressions are placed on the left or right side of the statement. Indeed this division of labour was one of the primary reasons for the introduction of the functions Theme and Articulation. The Theme, showing relative stability, maintains the text's hold on its field, and shows each statement's relevance to the surrounding co-text. Articulations, on the other hand, show some kind of progression, which was said to coordinate more with genre staging. Text 4.1 from a senior high school text book illustrates this. The text provides a solution to the problem: *A ball weighing 500g rolls down a hill with an acceleration of 3.0 ms^{-2}. What is the net force acting on it?*[42]

[42] Many of the examples in this chapter respond to some sort of question or problem such as this. These questions are not included in the formal genre systems built through this chapter, however they appear quite

Taking the downhill direction as positive and applying Newton's Second Law to the ball:
$m = 500\ \text{g} = 0.5\ \text{kg}$, $a = 3.0\ \text{m s}^{-2}$

$$\begin{aligned}\boldsymbol{F}_{\text{net}} &= m\boldsymbol{a} \\ &= 0.5 \times 3.0 \\ &= 1.5\ \text{N downhill}\end{aligned}$$

Text 4.1 (a) de Jong et al. (1990: 252)

Focusing on the sequence of mathematical statements beginning with $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$**,** we see that the Theme ($\boldsymbol{F}_{\text{net}}$) is kept constant through ellipsis. This Theme (glossed as the *net force*) is introduced in the question this text answers, which asks for *the net force* to be found. The Themes are thus showing the relevance of the statements to the overall text. The Articulations (right side), on the other hand, display change; they begin with pronumerals ($m\boldsymbol{a}$) before moving onto numbers (0.5×3.0) and then to numbers with units and a direction (1.5 N downhill). The opening statement ($\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$) presents a technical equation, one that is being taught in this section of the textbook as the starting point for the following statements. The second statement ($= 0.5 \times 3.0$) shifts the text to a numerical form by substituting two numbers from the initial written paragraph (0.5 and 3.0) in for the m and $\boldsymbol{a}$ in the first statement. These numbers are taken from the preceding paragraph, where m is specified as equalling 0.5 kg, and $\boldsymbol{a}$ is indicated as 3.0 m s^{-2}. The final statement completes the calculation of the net force and indicates the final result (1.5 N) including the direction of the net force *downhill* (bringing language back into the fold).

The sequence of mathematical statements relies on the initial paragraph that primarily involves language. The initial equation $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$ arises from the technical nominal group *Newton's Second Law* in the second clause, the numbers substituted into the second line are developed from the opening two equations in the paragraph, and the direction in the final statement (*downhill*) derives from opening clause. The opening paragraph works with the sequence of mathematical statements to function as a single coherent whole. Within this whole, however, different sections play distinct roles.

---

regular in their own right, and are of course vitally important for student assessment. It is likely they form part of a larger bimodal (or multimodal) macrogenre, whereby they realise a stage we could call 'Problem', with the mathematical genres built in this chapter embedded in some sort of 'Solution' stage.

Mathematical texts such as this are framed by two stages that in effect determine the form of the rest of the text. These stages provide the beginning and end points of the calculation that in turn govern the path through which the calculation moves. In Ttext 4.1 the opening paragraph indicates the situation in which the calculation takes place. This stage also provides the starting point for the calculation by specifying the opening technical equation, $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$. This stage, which we will call the Situation, provides the assumed understandings from which the text proceeds. It characterises the parameters of the calculation, by including the mathematical formula being used (in Text 4.1 $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$, *Newton's Second Law*), the known numbers (e.g. $m = 500$ g $= 0.5$ kg, $a = 3.0$ m s^{-2}), and other information relevant to the calculation (in Text 4.1's case, the downhill direction is specified as being positive). In Text 4.1, the full Situation is realised by:

> Taking the downhill direction as positive and applying Newton's Second Law to the ball:
> $m = 500$ g $= 0.5$ kg, $a = 3.0$ m s^{-2}
>
> $$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$$

Its counterpart in framing the text is the final Result. In text 4.1 this is realised by the final statement, $(\boldsymbol{F}_{\text{net}}) = 1.5$ N downhill. The Result is the culmination of the previous calculation and in some sense provides the overall raison d'être of the text. Indeed, if we take the informal definition of genre in Martin and Rose (2008) as a 'staged, goal-oriented social process', we can treat the Result as the goal to which the text is orienting. In Text 4.1, by quantifying the $\boldsymbol{F}_{\text{net}}$, the text has answered the initiating question that was posed: *What is the net force acting on it* [the ball]*?* As it is the end goal of the text, Results necessarily occur in every completed text. Without one, the purpose of the text is frustrated.

The two stages, Situation and Result, thus give the beginning and end points of Text 4.1. Together they determine the form of what comes between them. In this text, the second last statement, $\boldsymbol{F}_{\text{net}} = 0.5 \times 3.0$, is the only stretch of discourse not included in the Situation or Result. Rather than giving the initial parameters or the final result this statement provides an intermediary stage that shifts the Articulations from pronumerals ($m\boldsymbol{a}$) to numbers (0.5×3.0). The stage realised by this statement we will call Reorganisation. The Reorganisation details the manipulation of the equations and numbers in the Situation, on its way to the final Result. This stage provides the 'working out' students are asked to do in order to show how they completed the calculation. It is relatively standard practice (at least in Australia), for some marks to be awarded to students if their Reorganisation stage is correct, even if the final Result is not. The Reorganisation stage is, however, optional (despite what teachers try to instil as good practice); when the movement between the Situation and Result is relatively straight forward, it is often left out. In Text 4.1 the Reorganisation is realised by only a single statement. However in principle it can be expanded indefinitely. Nonetheless, the use of a Reorganisation indicates a level of explicitness often useful for both teaching and assessment. Using these stages, we can now analyse Text 4.1 as:

Taking the downhill direction as positive and applying Newton's Second Law to the ball: $m = 500\text{ g} = 0.5\text{ kg}$, $a = 3.0\text{ m s}^{-2}$ $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$	**Situation**
$= 0.5 \times 3.0$	**Reorganisation**
$= 1.5\text{ N downhill}$	**Result**

Text 4.1 (b) de Jong et al. (1990: 252)

4.2.1 Optional stages in mathematical genres

Text 4.1 shows a relatively basic mathematical genre with little elaboration involving its Situation, Reorganisation and Result staging. Although it is most common for these three stages to occur in a text, Situations (as well as Reorgnisations) are in fact optional. The absence of either of these stages tends to occur in larger texts involving longer sequences of mathematical genres (genre complexes, see Section 4.2.3), where information typically given in the Situation can be assumed from preceding co-text, or where the level of explicitness provided by the Reorganisation stage is not needed. Focusing first on texts without the

Reorganisation stage, we see that these are most commonly realised by a single statement involving three expressions. As the distinction between genre and grammar is one of abstraction rather than constituency (Martin 2013), the realisation of a genre in a single statement presents no theoretical roadblocks (compare simple language genres such as stop signs or no smoking signs). As well as tending to be realised in a single statement, genres with only a Situation and Result often do not involve language. Text 4.2 provides an example of this from a student response in a university exam:

$f = \frac{v}{\lambda}$	$= 2.909 \times 10^{15}$ Hz
Situation	**Result**

Text 4.2 University student response

In this text, the Situation is given by the first two expressions that specify the equation being used ($f = \frac{v}{\lambda}$). The following expression (2.909×10^{15} Hz) skips straight to the Result, without an intervening expression that substitutes numbers for $\frac{v}{\lambda}$. This kind of construction is not uncommon, as the Reorganisation stage is in a sense superfluous to the overall goal of the text. It is an intermediary stage that adds an extra level of explicitness to the path between the Situation and the Result; but is not the important take-home message. Nonetheless, the Reorganisation stage is common both in pedagogical texts and in student work, as each context can benefit from added explicitness. For students, they can show their 'working out' (i.e. how they achieved their Result); indeed marks are often given for the working itself. In pedagogical texts, it provides extra scaffolding to help students complete the calculation.

Texts without a Reorganisation stage, on the other hand, are not concerned with the full explicitness of how the Result was achieved. The calculations involved are usually straight forward and involve only a small number of variables. The lesser need for explicitness is related to their position within larger genre complexes. In these texts, the entire genre performs the intermediary function of quickly producing a Result which can be used in following text. In this case, the way the Result is calculated is not particularly important; it is the Result that is needed.

In mathematical texts, there appears to be an unmarked preference for statements to include only two expressions (a Theme and a single Articulation). When there are more than two expressions (multiple Articulations), the final Articulation has informational prominence, while the preceding Articulations are backgrounded. Following this interpretation of the statement, texts where the Situation and Result are realised in a single statement have their Result realised by the final Articulation, while part of the Situation is realised by the preceding Articulation. This foregrounds the Result, while backgrounding the relations that are used to achieve the Result ($\frac{v}{\lambda}$ in Text 4.2). There are thus two main points of prominence: the Theme that holds onto the field (f) and the final Articulation that realises the Result (2.909×10^{15} Hz). With only a Situation and a Result, the text takes a further step in backgrounding the Reorganisation by omitting it altogether. By conflating the Situation and Result with the Theme and Articulations in the grammar, the genre utilises the structural organisation of the statement to augment its own meaning making potential. It is organising its staging to coincide with textual prominence inherent in the statement.

Less common than texts without a Reorganisation are those without a Situation. These texts occur when the information normally given in the Situation (the starting equation, known numerical values etc.) can be assumed, often because they have been given in the previous co-text (the preceding genre). Text 4.3 from a senior high school student exam response shows an example of this:

$Total\ energy = 2 \times 10^{6} \times 1.602 \times 10^{-19}$	**Reorganisation**
$= 3.204 \times 10^{-13}$ J	**Result**

Text 4.3 Senior high school student response

In this text, the starting equation is not given in pronumerical form, nor are the known numerical values specified initially. Rather, the text begins with a Reorganisation stage that involves a numerical Articulation, and concludes with the Result. As in the texts without a

Reorganisation stage, those without a Situation stage tend to occur as part of larger sequences of mathematical texts and offer a relatively brief path to the final Result. The unmarked organisation of mathematical genres thus appears to involve a (Situation) ^ (Reorganisation) ^ Result structure. Where meanings can be assumed or the level of explicitness a full structure provides is not needed, the Situation and Reorganisation can be omitted.

There is one final stage that often occurs in mathematical texts that acts as a kind of coda to the text as a whole. This stage, that we will call the Interpretation, reformulates the mathematical Result as language; in doing so, the Interpretation highlights its significance for the progression of meaning in a larger text, thereby foregrounding the important information or 'take-home message' of the text. As we will see in section 4.4.3 this stage is particularly important for the development of knowledge in physics, as it allows mathematical Results to be named, described and reconciled with the broader field-specific meanings made through language. Text 4.4, from a junior high school textbook, shows an example of a text involving each of the four stages, Situation ^ Reorganisation ^ Result ^ Interpretation:

The acceleration of a car which comes to rest in 5.4 seconds from a speed of 506 km/h is: $$\text{average acceleration} = \frac{\text{change in speed}}{\text{time taken}}$$	**Situation**
$$= \frac{-506\,\text{km/h}}{5.4\ \text{s}} = -\,93.7\,\text{km/h/s}.$$	**Reorganisation^Result**
This negative acceleration can be expressed as a deceleration of 93.7 km/h/s.	**Interpretation**

Text 4.4 (a). Haire et al. (2000: 114)

These four stages provide the basic modular structure of mathematical texts. They involve a multivariate structure whereby each stage performs a distinct function. From these, we can build the preliminary network of mathematical genres shown in Figure 4.1.

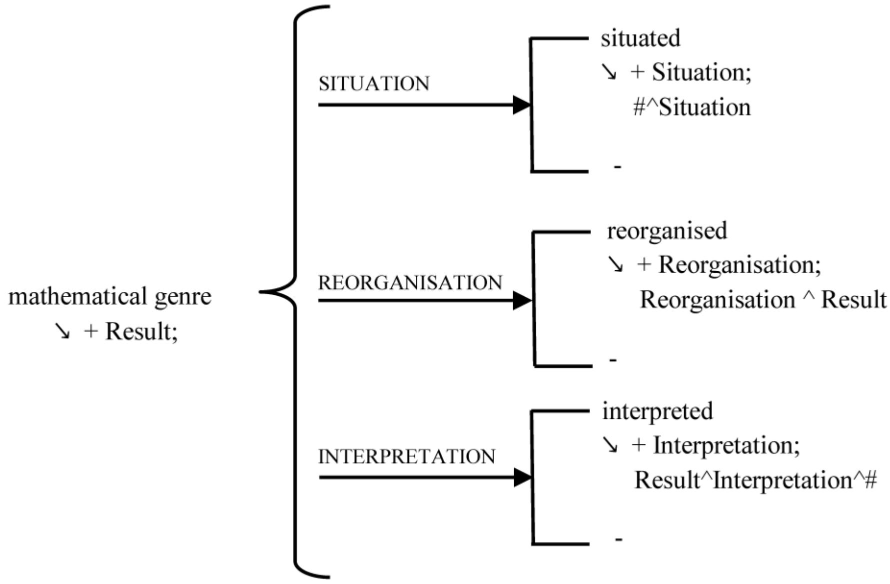


Figure 4.1. Partial system of mathematical genres

In the following sections, we will extend this basic system to show the range of possible variation in mathematical texts. The first step is to consider two types of mathematical genre that cross-classify the options shown in Figure 4.1.

4.2.2 Types of mathematical genre

The texts we have seen so far have all worked toward numerical Results. That is, each of their Results have been realised by statements with numbers in their Articulations. These texts belong to a specific type of mathematical genre that we will call *quantification*. Quantifications are common through schooling from junior high school onwards and allow for the measurement of a specific instance of the object of study. They are not, however, the only mathematical texts that occur. The alternative to quantifications is texts that remain in pronumerical form; we will call these *derivations*. These texts are not concerned with numerically measuring an instance of their object of study, but rather with developing new relations among technical symbols. Derivations begin to appear later on in schooling than quantifications (see Section 4.4), and in general require knowledge of a larger set of technical equations than quantifications. Text 4.5 shows a derivation from a senior high school textbook.

> Consider two colliding objects, as shown in Figure 3.54 [not shown YJD]. Let the velocities before collision be $\vec{u}_A$ and $\vec{u}_B$ and those after collision be $\vec{v}_A$ and $\vec{v}_B$.
>
> From Newton's Third Law, the force of B on A ($\vec{F}_{BA}$) and the force of A on B ($\vec{F}_{AB}$) are related by:
>
> $$\vec{F}_{BA} = -\vec{F}_{AB}$$
> $$m_A\vec{a}_A = -m_B\vec{a}_B$$
> $$\frac{m_A(\vec{v}_A - \vec{u}_A)}{t} = -\frac{m_B(\vec{v}_B - \vec{u}_B)}{t}$$
>
> Since the time of interaction is the same for both objects, we have:
>
> $$m_A(\vec{v}_A - \vec{u}_A) = -m_B(\vec{v}_B - \vec{u}_B)$$
>
> Rearranging, we find:
>
> $$\boxed{m_A\vec{u}_A + m_B\vec{u}_B = m_A\vec{v}_A + m_B\vec{v}_B}$$
>
> This equation shows that:
>
> *The vector sum of the momenta of the objects before collision equals the vector sum of the momenta after collision.*
>
> Momentum has been *conserved* in the collision!

Text 4.5 (a) Warren (2000: 125)

In general terms, derivations have the same structure as quantifications. Each of the four stages, Situation^Reorganisation^Result^Intepretation, occur in the same sequence and with the same possibilities for optionality. The Situation sets up the starting point for the derivation, including the relevant variables and equations, the Reorganisation rearranges the equation given in the Situation (though without substituting numbers in as in quantifications), the Result is the end goal to which the text is orienting, and the Interpretation highlights the significance of the Result in language. The structural analysis for Text 4.5 is as follows:

<table>
<tr><td>

Consider two colliding objects, as shown in Figure 3.54 [not shown YJD]. Let the velocities before collision be $\vec{u}_A$ and $\vec{u}_B$ and those after collision be $\vec{v}_A$ and $\vec{v}_B$.

From Newton's Third Law, the force of *B* on *A* ($\vec{F}_{BA}$) and the force of *A* on *B* ($\vec{F}_{AB}$) are related by:

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

</td><td>Situation</td></tr>
<tr><td>

$$m_A\vec{a}_A = -m_B\vec{a}_B$$

$$\frac{m_A(\vec{v}_A - \vec{u}_A)}{t} = -\frac{m_B(\vec{v}_B - \vec{u}_B)}{t}$$

Since the time of interaction is the same for both objects, we have:

$$m_A(\vec{v}_A - \vec{u}_A) = -m_B(\vec{v}_B - \vec{u}_B)$$

</td><td>Reorganisation</td></tr>
<tr><td>

Rearranging, we find:

$$\boxed{m_A\vec{u}_A + m_B\vec{u}_B = m_A\vec{v}_A + m_B\vec{v}_B}$$

</td><td>Result</td></tr>
<tr><td>

This equation shows that:

The vector sum of the momenta of the objects before collision equals the vector sum of the momenta after collision.

Momentum has been *conserved* in the collision!

</td><td>Interpretation</td></tr>
</table>

Text 4.5 (b) Warren (2000: 125)

Although the general structure of both quantifications and derivations is the same, the realisations of certain stages are markedly different. These differences in realisation reflect the different social purposes of the two genres and provide the main reasoning for their separation as distinct genres. To explore this, we will consider each in turn.

4.2.2.1 Quantifications

Quantifications occur earlier than derivations in physics teaching. They begin to appear at the beginning of junior high school physics (~ 12-16 years old) and by senior high school are very common in both student work and textbooks (~ 16-18 years old). As mentioned above, they aim to achieve a numerical Result in order to measure an instance of the object of the study. This involves a pronumerical Theme (the variable being measured) and a numerical Articulation, as shown by the Result of the quantification in Text 4.1: $(\boldsymbol{F}_{\text{net}}) = 1.5\ \text{N}$ downhill. As this Result illustrates, the Articulation also usually involves units of measurement (here N, glossed as *Newtons*, the units of force). If the Theme involves a vector (a number involving a direction shown by bold $\boldsymbol{F}_{\text{net}}$, see Section 3.4.5), the Result will also include a direction, *downhill*.

As this suggests, the choice of quantification strongly constrains the possibilities for the realisation of the Result in ways that are distinct from the derivation. To show this distinction, we can subclassify the Result of a quantification as a Numerical Result (for subclassification of functions see Huddleston 1981, Halliday 1961 and Appendix A). The insertion of a Numerical Result, specified in these terms, is thus the key criterion for distinguishing quantifications from other genres. It is not just the Result, however, that is affected by the choice of quantification. Since it provides a link between the Situation and Numerical Result, the Reorganisation is also constrained in its possible realisations. In order to move from the initial pronumerical statement in the Situation (e.g.$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$) to the Numerical Result, the Reorganisation of a quantification must substitute numbers into the initial equation. These numbers are often indicated in the Situation or in the previous co-text. We will see in the following section that this type of Reorganisation is in marked contrast to that of a derivation. Accordingly we will also subclassify the Reorganisation of quantifications as the Substitution. The other two stages, Situation and Interpretation, do not have distinctive realisations for quantifications as opposed to derivations and so will not be subclassified. We can now present this more delicate structure with an analysis of Text 4.6, taken from the same page of a junior high school textbook as Text 4.4.

A car travelling at 60 km/h which increases its speed to 100 km/h in 5.0 seconds has an average acceleration of: $\text{average acceleration} = \frac{\text{change in speed}}{\text{time taken}}$	**Situation**
$= \frac{40\,\text{km/h}}{5.0\text{ s}}$	**Substitution**
$= 8.0\,\text{km/h per second}$	**Numerical Result**
That is, on average, the car increases its speed by 8.0 km/h each second.	**Interpretation**

Text 4.6. Haire et al. (2000: 114)

The overall organisation of quantifications and their typical realisations are summarised in Table 4.1 with an example from an undergraduate university textbook.

quantification Aims to provide a numerical result measuring an instance of the object of study. Regularly realised by both language and mathematical symbolism.		
Stages	**Features**	**Example**
Situation (optional)	Situates the quantification in its co-text and context. As part of this it often: - Provides numerical values for variables. - Gives the symbolic equation to be used.	Suppose mercury atoms have an excited energy level 4.9eV above the ground level. An atom can be raised to this level by collision with an electron; it later decays back to the ground level by emitting a photon. From the photon formula $E = hc/\lambda$, the wavelength of the photon should be: $\lambda = \frac{hc}{E}$
Substitution (optional)	Replaces symbols in the Articulation with numbers from the Situation, co-text or field specific knowledge. Often will include multiple lines rearticulating the statement.	$= \frac{(4.136 \times 10^{-15}\mathrm{eV.s})\left(3.00 \times 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}\right)}{4.9\mathrm{eV}}$ $= 2.5 \times 10^{-7}\mathrm{m}$
Numerical Result	Culmination of the quantification. Gives the result of the calculation. Usually has a pronumerical Theme and a numerical Articulation. The Articulation may also include units and a direction.	$= 250\ \mathrm{nm}$
Interpretation (optional)	Reinterprets the mathematical result in language. Indicates the significance of the Numerical Result.	This is equal to the wavelength that Franck and Hertz measured, which demonstrates that this energy level actually exists in the mercury atom. Similar experiments with other atoms yield the same kind of evidence for atomic energy levels. Franck and Hertz shared the 1925 Nobel Prize in physics for their research.

Table 4.1 Stages and typical features of quantifications. Example from Young and Freedman (2012: 1300).

4.2.2.2 Derivations

In contrast to quantifications, derivations do not develop a Numerical Result. Rather, they remain in pronumerical form. The goal of a derivation is not to measure a specific instance of the physical world, but to derive a new set of relationships between technical symbols in the field. For this reason, derivations play a significantly different role in knowledge building to quantifications (see section 4.4). Derivations begin to appear toward the end of junior high school physics, but become a regular feature by senior high school. Like quantifications, the degree to which derivations rely on language can vary. For example, Text 4.5 above utilises language in each stage. The language in 4.5 makes the steps taken through the derivation more explicit.

Consider two colliding objects, as shown in Figure 3.54 [not shown YJD]. Let the velocities before collision be $\vec{u}_A$ and $\vec{u}_B$ and those after collision be $\vec{v}_A$ and $\vec{v}_B$.

From Newton's Third Law, the force of B on A ($\vec{F}_{BA}$) and the force of A on B ($\vec{F}_{AB}$) are related by:

$$\vec{F}_{BA} = -\vec{F}_{AB}$$
$$m_A\vec{a}_A = -m_B\vec{a}_B$$
$$\frac{m_A(\vec{v}_A - \vec{u}_A)}{t} = -\frac{m_B(\vec{v}_B - \vec{u}_B)}{t}$$

Since the time of interaction is the same for both objects, we have:

$$m_A(\vec{v}_A - \vec{u}_A) = -m_B(\vec{v}_B - \vec{u}_B)$$

Rearranging, we find:

$$\boxed{m_A\vec{u}_A + m_B\vec{u}_B = m_A\vec{v}_A + m_B\vec{v}_B}$$

This equation shows that:

The vector sum of the momenta of the objects before collision equals the vector sum of the momenta after collision.

Momentum has been *conserved* in the collision!

Text 4.5 (a) Warren (2000: 125)

On the other hand, Text 4.7 from the same textbook uses relatively little language, limiting it to the Situation and final Interpretation, as well as some conjunctive relations:

> To show the equality of these two different statements of Newton's Second Law [referring to previous co-text YJD], consider the following:
>
> $$\vec{F} = m\vec{a}$$
>
> $$\text{But } \vec{a} = \frac{\vec{v} - \vec{u}}{\Delta t}\text{, therefore}$$
>
> $$\vec{F} = \frac{m(\vec{v} - \vec{u})}{\Delta t}$$
>
> $$= \frac{m\vec{v} - m\vec{u}}{\Delta t}$$
>
> $$= \frac{\Delta m\vec{v}}{\Delta t}$$
>
> Force is the time rate of change of momentum as stated by Newton!

Text 4.7(a) Warren (2000: 123)

Like quantifications, derivations begin with a Situation in which the starting equations are specified. In Text 4.7, this involves a sentence of language situating the derivation within its co-text (*To show the equality of these two different statements of Newton's Second Law, consider the following)*, as well as two equations: $\vec{F} = m\vec{a}$ and $\vec{a} = \frac{\vec{v}-\vec{u}}{\Delta t}$. These two equations are the *two different statements of Newton's Second Law* referred to in the linguistic phase of the Situation. As these equations have not been derived or developed internal to the derivation, but rather have been presented as starting points, they are considered part of the Situation; i.e. they are the basis upon which the rest of the text develops. Following the Situation there are two equations that realise the Reorganisation: $\vec{F} = \frac{m(\vec{v}-\vec{u})}{\Delta t}$ and $= \frac{m\vec{v}-m\vec{u}}{\Delta t}$. The first of these equations, $\vec{F} = \frac{m(\vec{v}-\vec{u})}{\Delta t}$, combines the two initial equations in the Situation by replacing the $\vec{a}$ in the opening equation ($\vec{F} = m\vec{a}$) with the Articulation of the second equation ($\vec{a} = \frac{\vec{v}-\vec{u}}{\Delta t}$) to produce $\vec{F} = \frac{m(\vec{v}-\vec{u})}{\Delta t}$. This is followed by a rearrangement of the Articulation to produce $= \frac{m\vec{v}-m\vec{u}}{\Delta t}$. A final manipulation of the Articulation produces the final Result, $= \frac{\Delta m\vec{v}}{\Delta t}$, which is reinterpreted in language in the Interpretation stage: *Force is the time rate of change of momentum as stated by Newton!*

As with quantifications, the choice of derivation strongly controls the possible realisations of both the Result and the Reorganisation. In this genre, the Result remains in pronumerical form and rarely includes either units or a direction. To distinguish this from the quantification's Numerical Result, we will call the Result of a derivation a Symbolic Result. Similarly, the Reorganisation of a derivation holds a distinct realisation. As the statements in both the Situation and the Symbolic Result are pronumerical, the Reorganisation also remains pronumerical. This stage is therefore not concerned with substituting in numbers like it is in quantifications. Rather it involves rearranging the equations that have so far been presented in order to show new relations. For this reason, we will subclassify the Reorganisation of a derivation as the Rearrangement. With this more delicate structure, we can reanalyse Texts 4.5 and 4.7 as follows.

Consider two colliding objects, as shown in Figure 3.54 [not shown YJD]. Let the velocities before collision be $\vec{u}_A$ and $\vec{u}_B$ and those after collision be $\vec{v}_A$ and $\vec{v}_B$. From Newton's Third Law, the force of *B* on *A* ($\vec{F}_{BA}$) and the force of *A* on *B* ($\vec{F}_{AB}$) are related by: $\vec{F}_{BA} = -\vec{F}_{AB}$	**Situation**
$m_A\vec{a}_A = -m_B\vec{a}_B$ $\frac{m_A(\vec{v}_A - \vec{u}_A)}{t} = -\frac{m_B(\vec{v}_B - \vec{u}_B)}{t}$ Since the time of interaction is the same for both objects, we have: $m_A(\vec{v}_A - \vec{u}_A) = -m_B(\vec{v}_B - \vec{u}_B)$	**Rearrangement**
Rearranging, we find: $\boxed{m_A\vec{u}_A + m_B\vec{u}_B = m_A\vec{v}_A + m_B\vec{v}_B}$	**Symbolic Result**
This equation shows that: *The vector sum of the momenta of the objects before collision equals the vector sum of the momenta after collision.* Momentum has been *conserved* in the collision!	**Interpretation**

Text 4.5 (c) Warren (2000: 125)

To show the equality of these two different statements of Newton's Second Law, consider the following: $\vec{F} = m\vec{a}$ But $\vec{a} = \frac{\vec{v} - \vec{u}}{\Delta t}$, therefore	**Situation**
$\vec{F} = \frac{m(\vec{v} - \vec{u})}{\Delta t} = \frac{m\vec{v} - m\vec{u}}{\Delta t}$	**Rearrangement**
$= \frac{\Delta m\vec{v}}{\Delta t}$	**Symbolic Result**
Force is the time rate of change of momentum as stated by Newton!	**Interpretation**

Text 4.7(b) Warren (2000: 123)

Derivations are highly valued in physics. They are regularly used for student assessment in later years and are a mainstay of the knowledge practices in classrooms and textbooks. A large component of their value arises from their specific role in knowledge building through their development of new relations. These relations not only build new knowledge through mathematics, but also regularly lay the basis for the development of linguistic technicality. We can see a glimpse of this from Text 4.8, a derivation that immediately follows Text 4.7 in a senior high school textbook.

IMPULSE

As we have just seen, we can write Newton's Second Law as:

$$\vec{F} = \frac{(\Delta m\vec{v})}{\Delta t}$$

Situation

Rearranging this equation we can write:

$$\boxed{\vec{F}\Delta t = \Delta(m\vec{v})}$$

Symbolic Result

> **Impulse** ($I = \vec{F}\Delta t$) *is equal to the change in momentum of the object upon which the force is applied*.

The SI unit of impulse is newton second (N.s). This unit is hence an alternative unit for momentum (see earlier).

It follows that the same impulse can result from either a small force applied for a long time or a large force applied for a short time; the changes in momentum will be the same in both cases.

Interpretation

Text 4.8 (a) Warren (2000: 123)

The opening equation of this derivation, $\vec{F} = \frac{(\Delta m\vec{v})}{\Delta t}$, is taken from the Symbolic Result of the previous derivation (Text 4.7). This is then reorganised to produce the final Result shown in the box: $\vec{F}\Delta t = \Delta(m\vec{v})$. Crucially for our discussion, the final Interpretation reinterprets this Symbolic Result in language, and in doing so, introduces a new piece of linguistic technicality, *Impulse*. Impulse is used to name one of the relations developed in the derivation ($\vec{F}\Delta t$), and is immediately elaborated linguistically in relation to other technical terms (i.e. *change in momentum* and *force*) and given units (*newton second* N.s.). The derivations used in Texts 4.7 and 4.8 have not only developed new relations within mathematics but have also engendered new linguistic technicality. We will consider this text again in Section 4.4, but importantly for the discussion here, the derivation genre has provided a framework through which meanings built in one resource can be built upon and expanded by those of another.

The bimodal nature of derivations (and quantifications) is thus crucial for their knowledge building potential.

We can now provide an overview of the derivation genre and its stages in Table 4.2. The example is a university student exam response solving the problem:

Using Bernoulli's principle, explain why a wind blowing horizontally across the top surface of an open, upright umbrella can cause it to invert.

derivation Aims to develop a new symbolic relationship by deriving a new mathematical statement. Regularly realised by both language and mathematical symbolism.		
Stages	**Features**	**Example**
Situation (optional)	Situates the derivation in its co-text and context. As part of this it often: - Specifies the initial equations to be used - Introduces symbols or techniques for deriving the result.	Let p_1 be the pressure above the umbrella, and p_2 be the pressure below it. $p_1 + \rho g y_1 + \frac{1}{2}\rho {v_1}^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho {v_2}^2$ $y_1 = y_2$
Rearrangement (optional)	Rearranges the initial equations, through substituting one set of relationships in for another, or manipulating the equation. Often will include multiple lines rearranging the statement.	$\therefore p_1 + \frac{1}{2}\rho {v_1}^2 = p_2 + \frac{1}{2}\rho {v_2}^2$ $p_1 - p_2 = \frac{1}{2}\rho({v_2}^2 - {v_1}^2)$
Symbolic Result	Culmination of the derivation. Gives the result of the manipulation of the statements. Will have pronumerical symbols in both the Theme and Articulation.	Since $v_1 > v_2, \quad p_1 - p_2 < 0$
Interpretation (optional)	Reinterprets the mathematical result in language. Indicates the significance of the Symbolic Result.	Since the pressure above the umbrella is less than the pressure below it. Air is "pulled" up towards the lower pressure area, because the umbrella is blocking the path of the air, it is pulled up as well, inverting the umbrella in the process.

Table 4.2 Stages and typical features of derivations. Example from university student exam response.

The full system of elemental mathematical genres can now be presented in Figure 4.2. Following Halliday (1961), subclassification of structural functions will be shown through a subscript, e.g. $\text{Result}_{\text{Numerical}}$.

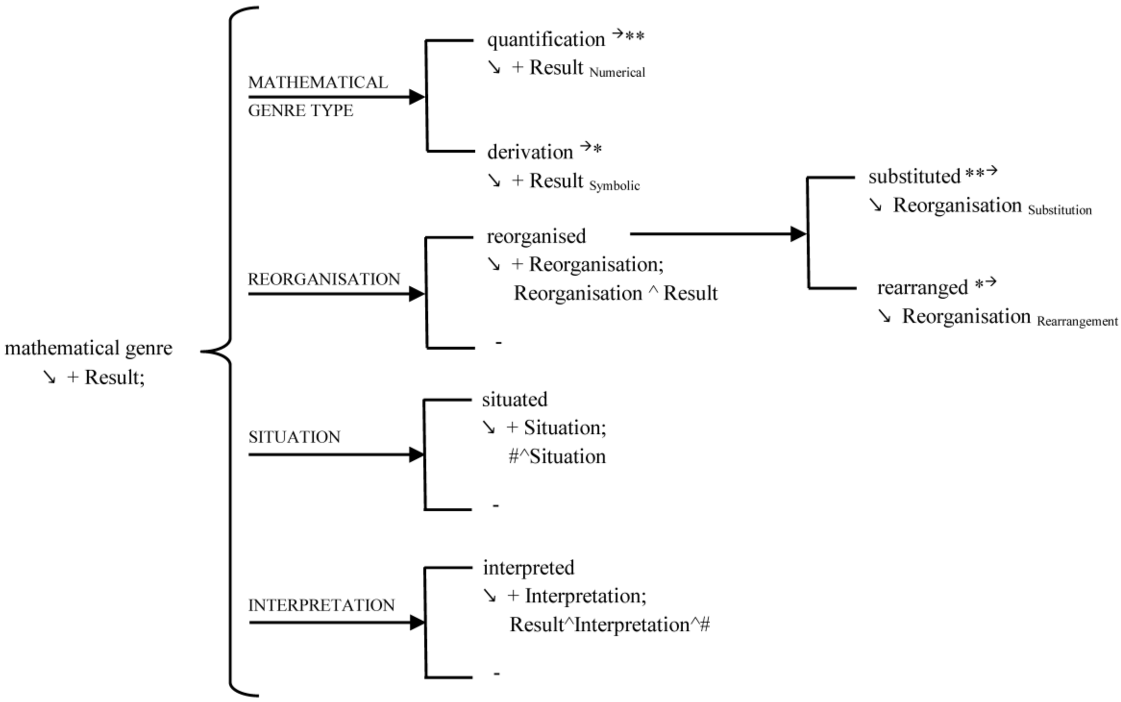


Figure 4.2. Network of elemental mathematical genres

Now that we have developed this model, we can reflect on the relations between the grammar and genres of mathematics. In particular, we can return to the progression of Articulations in relation to the Theme. As we have discussed above, the Theme provides stability, anchoring each statement to its co-text and field. Articulations, on the other hand, show flux. They progress in a relatively consistent manner. This section has shown that this progression patterns with genre staging. The seemingly predictable logogenetic ordering of statements is a consequence of these statements occurring within different stages of mathematical genres. Moreover, the number of statements in sequence is largely determined by the number of optional stages in these genres. Finally, the choice of numerals or pronumerals within the Articulation is almost entirely determined by the choice of quantification and derivation. By developing a model of both genre and grammar, we are able to predict and explain highly consistent patternings of mathematics and language across texts. So far, however, we have only considered relatively short texts instantiating a single elemental genre. As emphasised in Chapter 3, much of the power in mathematics comes from its regular ability to complex indefinitely. This complexing ability occurs not just in grammar but also within genre. The next section will thus develop a model of genre complexing that will allow an understanding of larger mathematical texts.

4.2.3 Mathematical genre complexing

The texts we have seen so far have been relatively short. They have ranged from single statements to about a page and have all moved toward a single Result. In stark contrast to the grammar of mathematics, the multivariate structure of mathematical genres constrains the possibility for indefinite iteration. But mathematical genres regularly interact with other genres to build more complex texts. To see this, we can consider Text 4.9 that we saw in the previous chapter, from a senior high school textbook. This text provides a solution to the prompt:

Calculate how much weight a 50 kg girl would lose if she migrated from the earth to a colony on the surface of Mars.

On the earth:

$$\begin{aligned}\boldsymbol{W}_{\text{earth}} &= m\boldsymbol{g}_{\text{earth}}\\ &= 50 \times 9.8\\ &= 490\ \text{N downwards}\end{aligned}$$

On Mars:

$$\begin{aligned}\boldsymbol{W}_{\text{Mars}} &= m\boldsymbol{g}_{\text{Mars}}\\ &= 50 \times 3.6\\ &= 180\ \text{N downwards}\end{aligned}$$

Loss of weight = 490 – 180 = 310 N. But there is no loss of mass!

Text 4.9 (a) de Jong et al. (1990: 249)

At first glance, this text looks like a relatively straightforward quantification; it begins with symbols and ends with numbers. However the text does not unfold through a single sequence of Situation ^ Substitution ^ Numerical Result ^ Interpretation. Rather, it comprises a string of three quantifications, framed with boxes below.

On the earth:	On Mars:
$\boldsymbol{W}_{\text{earth}} = m\boldsymbol{g}_{\text{earth}}$ $= 50 \times 9.8$ $= 490 \text{ N downwards}$	$\boldsymbol{W}_{\text{Mars}} = m\boldsymbol{g}_{\text{Mars}}$ $= 50 \times 3.6$ $= 180 \text{ N downwards}$

Loss of weight = 490 – 180 = 310 N. But there is no loss of mass!

Text 4.9 (b) de Jong et al. (1990: 249)

Looking at the structure of each of these genres confirms them as quantifications in their own right.

	quantification$_1$
On the earth: $\boldsymbol{W}_{\text{earth}} = m\boldsymbol{g}_{\text{earth}}$	Situation
$= 50 \times 9.8$	Substitution
$= 490 \text{ N downwards}$	Numerical Result
	quantification$_2$
On Mars: $\boldsymbol{W}_{\text{Mars}} = m\boldsymbol{g}_{\text{Mars}}$	Situation
$= 50 \times 3.6$	Substitution
$= 180 \text{ N downwards}$	Numerical Result
	quantification$_3$
Loss of weight = 490 – 180 = 310 N.	Substitution^ Numerical Result
But there is no loss of mass!	Interpretation

Text 4.9 (c) de Jong et al. (1990: 249)

This larger text can be accounted for as a sequence of quantifications. These three quantifications however work together to form a larger whole. None of the genres on their own functions to answer the problem on the basis of the information provided; but together they do. The question thus arises what the relationship is among the quantifications that allow them to function as a coherent whole?

We can first consider the relation between the two opening quantifications, prefaced with *On the earth* and *On Mars*. Neither of these genres are dependent on the other. Although they appear to take a very similar form, the numbers and equations they use are gathered not from each other, but from the initial problem (50 in each Substitution) and from field-specific knowledge (the equations in each Situation and the 9.8 and 3.6 in the Substitutions). Because they are not dependent on each other, neither requires the other in order to be completed Indeed their sequence could be swapped around with no consequences for (ideational) meaning.

Their independence from one another contrasts with their relation to the final quantification. This quantification necessarily depends on the first two quantifications having been completed. The two numbers used in the Substitution stage of the final quantification (490 and 180) are taken from the Numerical Results of the first two quantifications. If these first two quantifications had not been completed, the final quantification would be stranded at the Situation stage. We can thus make a distinction between the dependency relation involving the final quantification, and the coordination relation between the first two quantifications.

Because the final quantification is dependent on both previous quantifications having been completed, its relation is not with each of the other quantifications individually, but with both of them. In other words, there are two layers of complexing in this text. The highest layer involves the final quantification and the first two as a pair, with the second layer relating the first two quantifications individually. If we mark the dependency relation using Greek letters α, β etc. and the coordination relation using Arabic numerals 1, 2 etc. the analysis of Text 4.9 becomes:

	α	1	**quantification$_1$**
On the earth: $\boldsymbol{W}_{\text{earth}} = m\boldsymbol{g}_{\text{earth}}$			Situation
$= 50 \times 9.8$			Substitution
$= 490$ N downwards			Numerical Result
		2	**quantification$_2$**
On Mars: $\boldsymbol{W}_{\text{Mars}} = m\boldsymbol{g}_{\text{Mars}}$			Situation
$= 50 \times 3.6$			Substitution
$= 180$ N downwards			Numerical Result
	β		**quantification$_3$**
Loss of weight = 490 – 180 = 310 N.			Substitution^ Numerical Result
But there is no loss of mass!			Interpretation

Text 4.9 (d) de Jong et al. (1990: 249)

Using Greek letters and Arabic numerals to mark the complexing relations follows Halliday's analysis of the taxis distinction for complexing relations in English grammar (Halliday and Matthiessen 2014). The division between hypotaxis and parataxis is one based on whether the complexing relation shows dependency and differentiation in status, or whether it does not. Hypotaxis indicates relations where one unit is dependent upon another and holds a lower status. Hypotaxis is thus more closely aligned with the traditional category of subordination (though it is by no means the same; subordination generally also includes what SFL would term embedding, Martin 1988). Parataxis, on the other hand, shows an equal status relation whereby neither unit is reliant on the other; they each may occur independently of the other. Parataxis thus more closely resembles the traditional category of coordination. The dependency and status distinction indicated by the taxis in Halliday's description of English grammar shows strong affinities with the relations we have seen between mathematical genres, and so is useful for the discussion of these relations.

At first glance, this appears to be a subtly different interpretation of genre complexing to Martin's (1994) model. In this model, genre complexing is interpreted not in terms of taxis, but in terms of Halliday's logico-semantic relations (Halliday and Matthiessen 2014). In doing so, Martin is able to distinguish between projection, where one genre reports or quotes what another says, and expansion, where a genre in some way develops the meaning of another. Projection relations do not appear to occur between mathematical genres (though see section 4.3 where they do occur between exclusively linguistic genres and mathematical genres) and so will be set aside here. Within expansion, Martin is able to distinguish between three types of expansion: elaboration, where a genre in some sense restates or exemplifies the meanings of another; extension, where new a new genre adds meaning to another; and enhancement, where a genre embellishes or qualifies the meaning of another.

Martin suggests that at the level of genre there does not appear to be a distinct taxis system independent of the logico-semantic types. That is, each logico-semantic relation also indicates a particular dependency (taxis) relation. For Martin, enhancement relations (marked by ^{X}) show a higher degree of dependency and differential status (that is, hypotaxis), whereas extension relations (marked by +) show equal status and lower degrees of dependency between genres (parataxis) (c.f. Nesbitt and Plum 1988 who show a strong probabilistic association between each of these variables within English clause complexing). This model allows an alternative interpretation of the relations between mathematical genres above, whereby the first two quantifications simply add meaning to the other with their sequence determined textually (that is, they show an extension $^{+}$ relation), while the final quantification enhances the meanings of the others with its ordering necessarily controlled by the ideational meanings being developed (it shows an enhancement relation ^{X}).

It remains to consider whether mathematical genres can elaborate one another (=). Martin suggests that at the level of genre elaboration primarily involves one genre exemplifying another. For mathematical genre complexing we can see that this does indeed occur, primarily as a relation between derivations and quantifications. Text 4.10 shows an example of elaboration from a university physics lecture.

		derivation
$V_{AC} = hf_{\max} = \frac{hc}{\lambda_{\min}}$		Situation ^ Rearrangement
$\lambda_{\min} = \frac{hc}{eV_{AC}}$		Symbolic Result
	=	**quantification**
$\text{e.g. If} \quad V_{AC} = 30\ \text{kV}$		Situation
$\lambda_{\min} = 41 \times 10^{-12}\text{m}$		Numerical Result

Text 4.10 University Lecture genre complex

In this text, the quantification provides an example of a calculation utilising the Symbolic Result of the previous derivation. The exemplification relation is in fact marked by the elaborating abbreviation *e.g.* Although the second genre utilises the Result from the first to develop its own Result, elaboration relations such as these show relatively equal status with minimal dependency, in comparison to enhancement relations. The second text is not relying on the first in order to be completed, as is the case for enhancement; rather it is exemplifying the first. Put another way, the entire genre complex is not geared toward the Result shown in the final quantification, as is the case in Text 4.9. Rather the derivation and quantification are offering two angles on the same scenario. For this reason, the elaboration relation is treated as closer to parataxis than hypotaxis.

The analysis of complexing relations between mathematical genres is thus similar to that of expansion relations between linguistic genres. Elaboration and extension show relatively equal status and so can be interpreted as having a paratactic structure, while enhancement shows unequal status and so can be interpreted as having a hypotactic structure. In our analysis, to emphasise both the taxis distinction and the logico-semantic relations, elaboration and extension will be indicated by Arabic numerals 1, 2 etc. as well as being marked by = (elaboration, e.g. $^{=}2$) or + (extension, $^{+}2$), while enhancement will be marked by both Greek letters α, β, and × (e.g. $^{\times}\beta$). Text 4.11 shows an analysis of a larger mathematical text from a senior high school student exam response to the problem:

Calculate the number of photons, $\lambda = 4.5 \times 10^{-4}\ mm$*, which are required to transfer 2.0 MeV of energy.*

$E = hf \qquad c = f\lambda$	α	1	α	**derivation** Situation
$\therefore E = \frac{hc}{\lambda}$				Symbolic Result
$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{4.5 \times 10^{-4} \times 10^{-3}}$			$^{x}\beta$	**quantification$_1$** Substitution
$= 4.4173333.. \times 10^{-19}$ J (per photons)				Numerical Result
$Total\ Energy = 2 \times 10^{6} \times 1.602 \times 10^{-19}$		$^{+}2$		**quantification$_2$** Substitution
$= 3.204 \times 10^{-13}$ J .				Numerical Result
$\therefore photons\ need = \frac{3.204 \times 10^{-13}}{4.4173 \ldots \times 10^{-19}}$	$^{x}\beta$			**quantification$_3$** Substitution
$= 725534.4793$ ≈ 730000 photons (2 sig fig)				Numerical Result

Text 4.11 High school student exam response

Analysis reveals that hypotactic relations between mathematical genres are progressive (Halliday 1965). That is, the head of the complex (the α) comes first, with each dependent genre following afterwards (in contrast to, say, the English nominal group that is regressive, with α in final position and the dependents preceding). This progressive structure makes sense as it is the opening genre (in particular its opening Situation) that determines the following sequence of Results. Any particular final Numerical Result necessarily arises from the starting Situation of the opening genre. At the highest layer in Text 4.11 above for example (the first three genres as α and the final as $^{x}\beta$), the starting equations and substituted numbers in α determine what the final Numerical Result will be: $= 725534.4793 \approx 730000$ photons (2 sig fig). Any change in the numerical values in α will almost definitely change the final Numerical Result. Any change in the starting equations will likely change the sequence of genres needed to produce the final Numerical Result. The α thus plays a large determining role in the development of the text.

Once the α has laid down its starting point, the genre complex does not simply meander in any direction, however. It moves toward a definite finishing point, which is shown by the final genre in the complex (in particular the Result of the final genre). If the final genre is a quantification, then at some stage the intervening genres must shift to numerical terms. If the final genre is a derivation, they will remain in pronumerical form. The final genre thus determines in broad terms what the form the genre complex will take, while the opening genre determines its starting point. Between the two, they strongly characterise how the genre complex will progress. Any intermediate genres work simply to specify the path between the two.

Incorporating the complexing relations and the generalisations of taxis into the system of mathematical genres leads to the network in Figure 4.3.[43]

[43] In order to formalise the different type of complexing relations, it is necessary to insert a generalised function X at the primary delicacy feature [mathematical genre]. This function is then subclassified as 1 (if paratactic) or α if hypotactic.

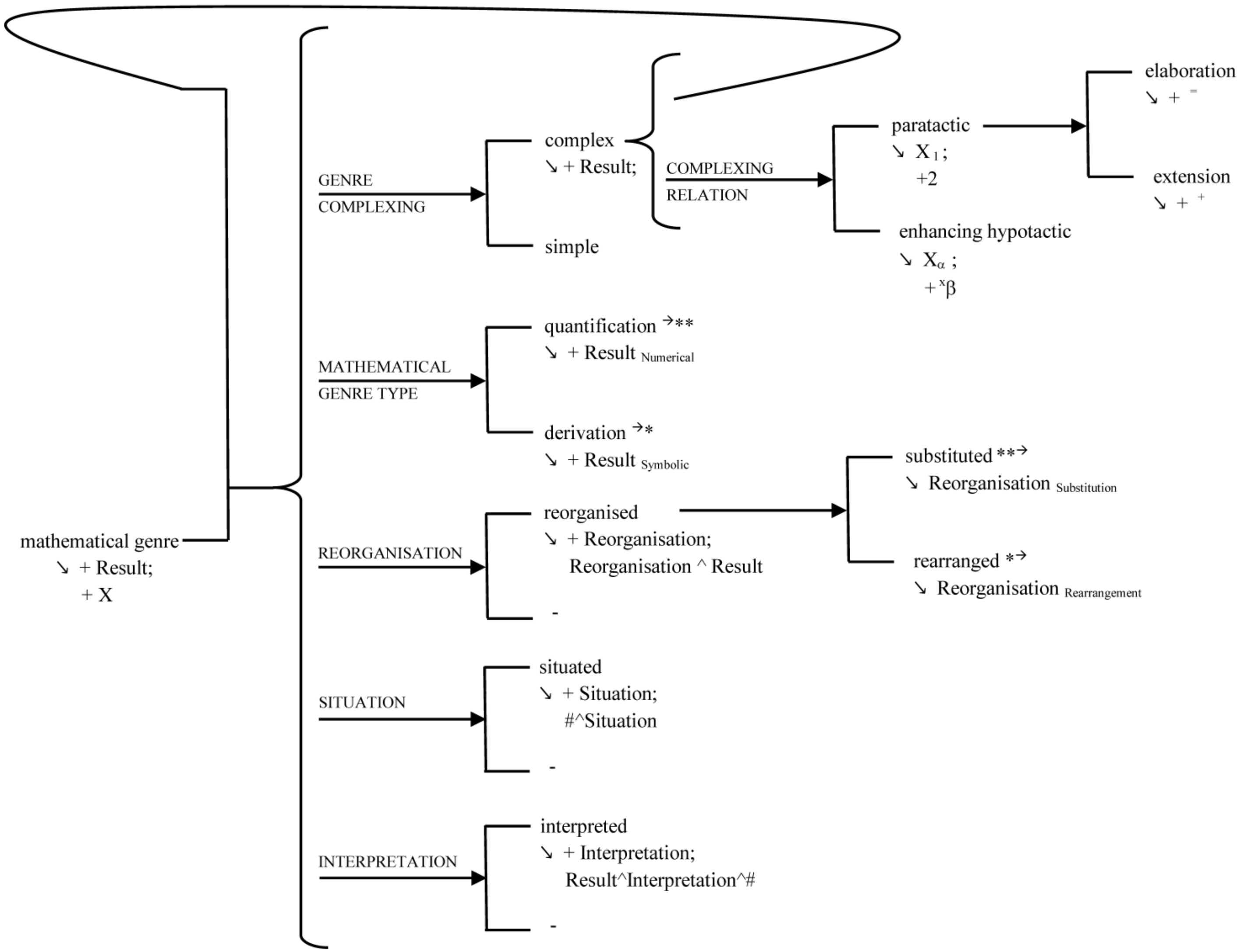


Figure 4.3 Full network of mathematical genres

4.3 Genre as a distinct stratum

The genre description presented above has been developed with an eye to the large scale variation in mathematical texts. It proposes bimodal realisation across mathematics and language as an avenue for understanding their interaction. It built a model of the types of internal structuring of mathematical genres and the influence these have on the mathematical grammatical patterns, and thus it offered a relatively predictable link between the grammatical and genre-based text patterns. Finally it considered the relations between genres as complexing relations in order to open the possibility for indefinitely long texts built through recursion. This model presents a detailed understanding of mathematical texts which as we will see in Section 4.4 can provide a basis for understanding more broadly the role of mathematics in the discourse of physics. But it has not yet fully resolved the question raised in Section 4.1 as to why we can consider this model as being at the stratum of genre, on a deeper level of abstraction from the grammatical stratum built in the previous chapter.

This takes us to the core of the descriptive apparatus used to justify each of the macrotheoretical categories in this thesis. Throughout the description axis has been used to develop the large-scale descriptive architecture of mathematics, including its metafunctional organisation and its levels of rank and nesting within the grammar. The thesis has avoided assuming macrotheoretical categories, without firm justification on axial grounds. This same method can be used to justify stratal organisation involving mathematics. As discussed in Section 4.1 and Chapter 2, grounds for proposing a distinct level (rank, nesting or strata) are that it must have its own system-structure cycle (Martin 2013); there must be a distinct set of paradigmatic choices with their own syntagmatic realisations. The networks built in the chapter to this point satisfy this criterion. They present a series of choices with their own structural realisations that are in principle largely independent of the grammar (although they may skew the probabilities of grammatical systems as part of their realisation). Once a system structure cycle has been established as a new level, the question remains what type of level it is. It is possible that the system developed be interpreted as another level within the grammar (some sort of rank of genre) rather than a distinct stratum in itself.

To determine whether the network is another level within the grammar (i.e. implicating another rank or nesting) or whether it is a separate stratum, we need to consider its relation to extant grammatical networks. If it is a higher rank or nesting within the grammar, it should be related in some way through constituency (whether developed through a multivariate structure or through univariate layering). In constituency hierarchies, structural functions at the higher level (i.e. Situation, Reorganisation, Result and Interpretation) must be realised by a one or more of the units at the level below (Halliday 1961). If we focus just on the mathematical symbolism, this means each genre stages needs to be realised by either a single statement or a whole-multiple of statements. The examples we have already seen show this is not the case. For example, Text 4.4 (reproduced below) shows multiple stages can be realised by single statements. In this example, both the Reorganisation and Result stages are realised in a single stage (shown boxed).

The acceleration of a car which comes to rest in 5.4 seconds from a speed of 506 km/h is: $$\text{average acceleration} = \frac{\text{change in speed}}{\text{time taken}}$$	**Situation**
$$\boxed{= \frac{-506\text{km/h}}{5.4\text{ s}} = -93.7\,\text{km/h/s.}}$$	**Reorganisation^Result**
This negative acceleration can be expressed as a deceleration of 93.7 km/h/s.	**Interpretation**

Text 4.4 Reorganisation and Result stages realised in one grammatical statement

This example makes it clear there is not a strict constituency relation between genre stages and grammatical statements. Equally importantly, the genre network is realised by both mathematics and language (in this case English), with both resources potentially working together to realise a single stage (such as the Situation in Text 4.4). In contrast, both the SFL descriptions of English (Halliday and Matthiessen 2014, Martin 1992a) and of mathematical grammar (in this thesis) are exclusively monomodal. The genre system cannot be included within either semiotic resource's grammatical stratum if we want to keep these grammatical systems in some way distinct. The conclusion is thus that the system developed in this chapter is not related through constituency to the grammatical systems, but through an interstratal realisation of a more abstract stratum. That is, the relation between the genre system and the grammatical systems is not that of distinct ranks or nestings internal to the grammatical stratum, but one of distinct strata.

As discussed in Section 4.1, the fact that the genres are realised by both language and mathematics indicates that this stratum is a connotative semiotic. It is a semiotic resource in its own right, realised by two other semiotic resources, mathematics and language. This stratum thus constitutes part of a communicative plane above language and mathematics (Martin 1992:501). In Martin’s model, the communicative plane above language involves both register and genre (the third stratum in Martin 1992a, ideology, no longer forms part of this model, Martin 2006b). So far, however, we have only justified that it is a stratum within the communicative plane. We have yet to justify axially that it is a stratum of genre, as opposed to register or another stratum all together. To do this, we will consider the role of mathematical genres in larger texts that also include linguistic genres.

In physics and many other disciplines, it is standard that linguistic and mathematical genres co-occur in the same text. For example, Text 4.12 shows an excerpt from a two page spread in a junior high school text book constituted by a genre complex involving five elemental genres: two linguistic genres (both reports) and three mathematical genres (all quantifications). For ease of reference, each elemental genre is numbered with Roman numerals I-V.

Speed is a measure of the **rate** at which an object moves over a distance. In other words, it tells you how quickly distance is covered. The **average speed** can be calculated by dividing the distance travelled by the time taken. That is: $\text{average speed} = \dfrac{\text{distance travelled}}{\text{time taken}}$ In symbols, this formula is usually expressed as: $v = \dfrac{d}{t}$	1	1	report **I**
Which *unit*? The speed of vehicles is usually expressed in kilometres per hour (km/h). However, sometimes it is more convenient to express speed in units of metres per second (m/s). The speed at which grass grows could be sensibly expressed in units of millimetres per week. Speed must, however, always be express as a unit of distance divided by a unit of time.		$^{+}2$	report **II**
Some examples (a) The average speed of an aeroplane that travels from Perth to Sydney, a distance of 4190 km by air, in 5 hours is: $v = \dfrac{d}{t}$ $= \dfrac{4190\ \text{km}}{5\ \text{h}}$ $= 838\ \text{km/h}$	$^{=}2$	1	q'cation **III**
The formula can also be used to express the speed in m/s. $v = \dfrac{d}{t}$ $= \dfrac{4\,190\,000\ \text{m}}{5 \times 3600\ \text{s}}$ (converting kilometres to metres and hours to seconds) $= 233\ \text{m/s}$		$^{+}2$	q'cation **IV**
(b) the average speed of a snail that takes 10 minutes to cross an 80 cm wide concrete paving stone in a straight line is: $v = \dfrac{d}{t}$ $= \dfrac{80\ \text{cm}}{10\ \text{cm}}$ $= 8.0\ \text{cm/min}$		$^{+}3$	q'cation **V**

Text 4.12. Haire et al. (2000: 110-111)

Just as we considered the relations between mathematical genres in larger texts, so can we consider their relations with linguistic genres. Text 4.12 shows a relatively straightforward and coherent text that moves from linguistic genres (I-II) to mathematical genres (III-V). To understand the relation between the first two reports and the final three quantifications, we can heed the fact that the same complexing relations used for linguistic genres were used for mathematical genres. Reports I and II are related through extension (+) with one simply being added to the other. The following quantifications, however, gives examples of the application of the equation presented in the first report and the units presented in the second report. We can thus consider the relation between the reports and quantifications as one of elaboration (=). To account for this text, then, we have used the same relations for complexing between linguistic and mathematical genres as we used within each type. This suggests that we can generalise these relations across both types of genre. In addition, as far as projection is concerned, math genres can be projected by linguistic genres (although as discussed above they cannot themselves project linguistic genres). This is shown by the simple fact that throughout this chapter full mathematical genres from other sources have been used as examples. Each time a quantification or derivation has been exemplified it has been projected by the rest of the text.

Looking systemically, we can thus generalise the complexing relations across both mathematical and linguistic genres as simultaneous with both these systems. The implication of this is that the systems of linguistic and mathematical genres can be interpreted as being on the same stratum. Viewed from this angle, the stratum of genre is thus a connotative semiotic realised through both mathematics and language. The two semiotic resources do not have their own individual communicative planes involving genre, but can be brought together through their joint role in realising a single stratum of genre.

This conclusion can also be arrived at from another angle. As emphasised throughout the description, the realisation of mathematical genres is not just mathematics but also language. By the same token, report I in Text 4.12 shows that realisations of linguistic genres are also not simply limited to language. This report involves the presentation of two mathematical statements: $\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$ and $v = \frac{d}{t}$. This genre is not a mathematical genre of quantification or derivation; it does not move through a series of steps manipulating equations or substituting numbers in from a starting situation to conclude with a final Result. Rather it presents a description of the technical term *speed*, with one of its features being the equation used to calculate it; it is a descriptive report (Martin and Rose 2008). Both linguistic genres and mathematical genres, therefore, can be realised by both mathematics and language. If mathematical and linguistic genres were separated into different unrelated strata, it is difficult to see how we could theoretically motivate this diversity of realisation. Thus through their realisation, the simplest solution is to allow a single stratum of genre coordinating both mathematics and language.

From this conclusion, we can return to the issue raised earlier of how we can argue that the stratum involving mathematical genres is at the level of genre and not register. We can do this by showing that it is systemically simultaneous with linguistic genres and enters into complexing relations with them. They thus share the same stratum, i.e. the stratum of genre. Through axial argumentation, we have therefore been able to set up a stratal model for mathematics to complement the rank, nesting and metafunction models developed in the previous chapter.

Mathematical and linguistic genres can therefore be brought into a single network with their shared complexing relations, shown in Figure 4.4. The network for linguistic genres is derived from Martin and Rose (2008) and Veel (1997). The division of complexing relations between parataxis and hypotaxis developed in Section 4.2.3 has been generalised here. Following Martin (1994), projection relations link genres of equal status, and thus have been grouped under parataxis.

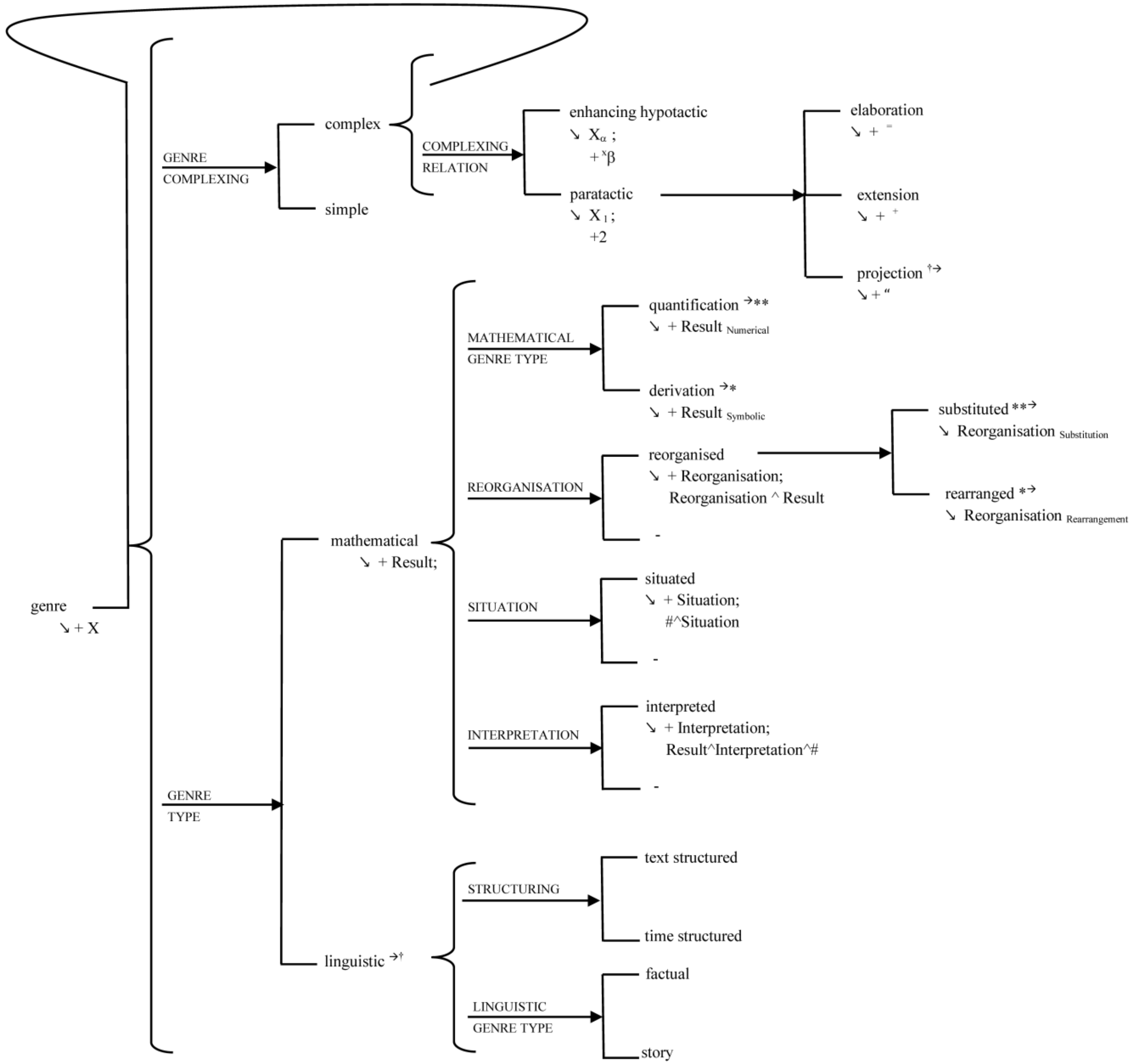


Figure 4.4 Network of genre including linguistic genres, mathematical genres and complexing

This network concludes the descriptive sections of this chapter. It presents the options for mathematical genres, and shows them in relation to linguistic genres. In doing so, it formalises a description based on axial foundations for both a stratal organisation of mathematics, and a single generalised stratum of genre that is realised by both mathematics and language. This stratal organisation supplements the metafunctional, rank and nesting organisation developed axially in the previous chapter to show the variability and functionality possible for mathematics in use. The full set of system networks across genre and grammar is presented in Appendix B. With the grammatical and genre descriptions completed, we can now turn to the other broad focus of this thesis, asking what role mathematics plays in the overall organisation of knowledge in physics.

4.4 Mathematics for building knowledge in physics

As we have seen, mathematics is pervasive in physics. It is used in both schooling and research, and it forms part of the high stakes texts students read to learn physics and those they write for assessment. Chapters 3 and 4 have shown mathematics involves its own distinct grammar and realises a unique set of genres. But why is it used in physics? As discussed in Chapter 2, this question has come to prominence from a recent concern in educational linguistics and social realist sociology with the structure of knowledge in academic disciplines (Christie & Martin 2007, Christie & Maton 2011). Various studies of science within the Systemic Functional tradition have shown that the natural sciences such as physics, along with academic discourse in general, are far removed from our everyday discourse (e.g. Martin & Veel 1998, Lemke 1990). The sciences tend to involve distinctive sets of factual genres (see Figure 4.4) and use language to construe both multi-tiered sequences of causality and deep taxonomies of composition and classification (Halliday & Martin 1993, Martin and Rose 2008, Veel 1997, see Section 2.2 chapter 2). From the viewpoint of Bernstein's code theory, they can be characterised as 'hierarchical knowledge structures' that attempt to create very general propositions and theories, integrating knowledge to account for an expanding range of different phenomena (Bernstein 1999: 162). According to Legitimation Code Theory (Maton 2014), the principles underlying these knowledge structures emphasise epistemic relations between knowledge and its object of study, and downplay social relations between knowledge and its author or subject. At the same time, one of the most salient features of scientific discourse is its heavy use of non-linguistic semiotic resources, in particular mathematics (Parodi 2012). As this thesis has shown (see also O'Halloran 2005), mathematics organises its meanings in considerably different ways to language and thus offers a complementary system for construing the knowledge of science.

The prevalence of mathematics and the distinctive structuring of scientific knowledge begs the question as to whether these two attributes are related. Does mathematics contribute to science's ability to develop integrated and abstracted models of the natural world, and does it aid in linking these models to empirical studies of their object of study? If so, how does mathematics do this? Framed in Fredlund et al.'s terms (2012), what are the disciplinary affordances of mathematics for science? Since physics appears the scientific discipline in which mathematics is most widely used (Parodi 2012), we can probe these questions by tracing mathematics as it develops through physics schooling. This section will follow mathematics as it shifts through primary (elementary) school, junior high school, senior high school and undergraduate university through the data collected in New South Wales, Australia (see Appendix C for details of the corpus), in order to understand the changing forms of mathematics and what this means for knowledge building in physics.

Through schooling there is a distinct development in how mathematics is used. The changes across the years correlate with different roles mathematics plays in organising the knowledge of physics. To understand the impact these changes have, we will first map the development in terms of the Systemic Functional model developed in this thesis. These changes will be illustrated in terms of the distinct mathematical genres deployed, the primary grammatical organisation used, as well as mathematics' intersemiotic interaction with language. To explore how these patterns organise the knowledge of physics, they will be interpreted using the Semantics dimension of Legitimation Code Theory (Maton 2014). LCT provides a complementary angle through which we can understand the structuring of knowledge, and has been used productively with SFL across a number of studies (see Maton and Doran in press 2017, Martin 2011b, Maton et al. 2015 for surveys of the dialogue between SFL and code theory). As explained in Chapter 2, Semantics is concerned with two main variables: *semantic gravity*, which explores the degree to which meanings are dependent on their context, and *semantic density*, which explores the degree of condensation of meaning in a practice (Maton 2014, Maton and Doran in press 2016a, b). Each of these will be reviewed in further detail as they become relevant in the chapter. Utilising these concepts from Semantics enables a nuanced understanding of how the various resources of mathematics allow physics to build integrated and generalised knowledge, while at the same time remaining in contact with its empirical object of study. Bringing the two approaches together provides a method for understanding the kinds of mathematics used in physics, why they are used, and what the payoff is for physics as a discipline.

To organise this section of the chapter, a subsection will be devoted to each of the primary, junior high, senior high and undergraduate university sectors of schooling (as these are organised in the state of New South Wales, Australia). For primary and junior high school, the data under study arises purely from textbooks, while for senior high school and university physics, the textbook data is complemented by classroom discourse and student exam responses (see Appendix C for details of the corpus). The progression from primary school through to university physics will develop an expanding understanding of the utility of mathematics. Throughout the section, presentations of what type of mathematics occurs and its frequency will be discussed in terms of tendencies and typicalities. It is not meant as a detailed or quantitative report of the mathematics used in physics at each stage; it is used simply as a vehicle to understand mathematics' role in knowledge building. For this reason, the examples chosen are illustrative; they are selected to demonstrate the explanation of the functionality of mathematics, rather than as representative examples for any particular level of schooling. The final section will pull together the strands raised in each section to characterise physics as a whole when viewed from mathematics.

4.4.1 Mathematics in primary school physics

The late primary school years (ages ~10-12) are the first to introduce mathematics in the service of physics.[44] At this stage, mathematics is not a prominent feature of the discourse; the physics covered depends more heavily on language and images to construe its knowledge. Nonetheless, the mathematics that is used gives a glimpse of how it will organise the knowledge of physics in later stages. In order to contribute to the knowledge of physics, however, it first must be invested with technical meaning from physics. Text 4.13 shows an example of how this can take place, via mathematics' interaction with language. In this text

[44] In primary school in New South Wales, Australia, physics is not a stand-alone subject. Rather, it forms part of a core science syllabus that also includes other natural sciences such as chemistry, biology (Board of Studies NSW 2012). From early primary school, mathematics is taught separately as a topic area independent of scientific concerns.

mathematics is being used to introduce the relationship between force, mass and acceleration known as Newton's Second Law.

FORCE EQUATION

The relationship between force (F), mass (m) and acceleration (a) is summed up in the equation:

$$F = ma$$

This shows the force of an object depends on the combination of its mass and acceleration. This is why the impact of a slow-moving truck and a fast-moving bullet are equally devastating. Both have tremendous force – the truck because of its large mass, the bullet because of its huge acceleration. The equation can also be swapped:

$$a = F/m$$

This shows the acceleration goes up with the force but down with the mass.

Text 4.13 Farndon (2003: 19)

The opening sentence of this text introduces three mathematical symbols, F, m and a. Each of these symbols are named using linguistic technicality: F is force, m is mass and a is acceleration. By naming these symbols, the text is investing them with technical meaning from the field. By encoding the symbols with instances of technicality in language, the symbols and linguistic technicality in effect become synonymous. The result is that changes in meaning in one semiotic resource, whether language or mathematics, necessarily changes the meaning of the other. For example, when Text 1 specifies that *the force of an object depends on the combination of its mass and acceleration*, language is indicating an unspecified dependency between *force, mass* and *acceleration*. As F, m and a have been made synonymous with these terms, this dependency necessarily transfers to the mathematical symbols. Similarly, any meanings built around these symbols in mathematics automatically implicates language. In $F = ma$ the relationship between the three symbols is given more precisely, specifying the dependency mentioned in language. Utilising the covariate relations developed in Chapter 3 (Section 3.4.2.5) to describe the relations, F is directly proportional to both m and a. This means crudely that as either m or a increases, F does too at the same rate. Similarly, the other equation introduced, $a = F/m$, indicates that a is inversely proportional to m, meaning that if a decreases, m increases and vice-versa. As the sentence following this equation explains: *This shows the acceleration goes up with the force but down with the mass*. In this text, the meanings of each of *force*, *mass* and *acceleration* are now linked to the meanings of the others. This interaction between language and mathematics is a vital first step for mathematics to contribute to the knowledge of physics. Before it can perform the functions it does in later years, it must be invested with meaning from the field of physics.

Even at this early stage, Text 4.13 shows that full mathematical equations are used, albeit rarely. By linking symbols in equations, precise relations are being set up that may hold across the entire field (in the next chapter, we will term these relation *implication complexes*). Indeed, when the relations between these symbols change, a change in the field is also signified. For example, $F = ma$ is applicable for classical 'Newtonian' mechanics, which, to put it crudely, involves the study of motion on a scale of size and speed comparable to that we experience in our everyday life. However, when moving to other fields of physics such as special relativity (concerning situations where speeds are close to the speed of light) and quantum mechanics (concerning the workings of very small things) the relations among these symbols are different. For each subfield of physics, relations specified in mathematics constitute one part of the knowledge of the field.

The iterative univariate structure of mathematics is such that an indefinite number of symbols can be related in any one equation. Although in this early stage equations do not expand much beyond the three symbols shown in $F = ma$, at later stages physics involves more complex equations such as: $K + U = \frac{1}{2}mv^2 + \frac{-GMm}{r}$. The possibility for indefinite iteration allows mathematics to play a powerful role in physics as it allows large sets of technical relations to be distilled into small snapshots.

As we have seen, even in this early stage, there is a give and take between language and mathematics in physics. Mathematics gains meaning by being encoded with technical meaning from the field. At the same time it develops meaning by setting up novel and precise relations among symbols that have been given technical meaning.

To conceptualise this burgeoning of meaning, we can enact the concept of *semantic density* from Legitimation Code Theory (Maton, 2014). Semantic density is concerned with the degree of condensation of meaning in an item. If an item has more meaning, it is said to have stronger semantic density (SD+); if it has less meaning it has weaker semantic density (SD–). A key metric for determining whether something has stronger or weaker semantic density is its degree of relationality (Maton and Doran in press 2016a): i.e. how many relations the item has with other items in a field. For example when introducing a term such as *energy,* it can be specified that it has subtypes of *potential energy* and *kinetic energy*. This sets up relations of classification between each of the terms, thus increasing their relationality and strengthening their semantic density.

Viewed from this perspective, mathematics as used in primary school physics primarily works to increase the semantic density of physics, i.e. to build technical meaning. First, the individual symbols are invested with meaning from technicality in language, e.g. F is given the meanings of *force*. This strengthens the semantic density of the mathematical symbols. Beyond this, the symbols are developed in equations, such as in $F = ma$. These equations specify sets of relations between symbols, further strengthening their semantic density. Since these symbols are associated with linguistic technicality, this semantic density is transferred over to the linguistic realm as well. That is, the relations between F, m and a specified in the mathematics transfers back to the relations between their linguistic correlates, *force, mass* and *acceleration*. As these meanings constitute part of the field, this interplay between mathematics and language strengthens the semantic density of the field itself.

At primary school, then, mathematics works to extend the semantic density of the field of physics. However as mentioned above, mathematics is only rarely used at this level. Physics at this stage relies more heavily on language and image to construe its knowledge. It is when moving into junior high school that mathematics comes into its own as a crucial component of physics. Not only is it used to a much larger degree, but the mathematics begins to be developed in quantifications (Section 4.2.2.1). This allows mathematics to function considerably differently to the way it does in primary school.

4.4.2 Mathematics in junior high school physics

Physics in junior high school (years 7-10, ages ~12-16) significantly increases its use of mathematics. While still relatively marginal in comparison to the use of language and images, the fledgling use of quantifications present opportunities for physics to reach toward the empirical world. Quantifications are the key mathematical innovation for this stage and build upon the basis for knowledge development introduced in primary school to further enhance the possibilities for knowledge building of physics. As in primary school, mathematical equations and symbols are introduced and named, and thereby invested with technical meaning, which increases the semantic density of the field. An example of this, again involving the equation $F = ma$, is shown in Text 4.14.

> Newton's Second Law of Motion describes how the mass on an object affects the way that it moves when acted upon by one or more forces. In symbols, Newton's second law can be expressed as:
>
> $$a = \frac{F}{m}$$
>
> where a = acceleration
> F = the total force on the object
> m = the mass of the object.
>
> If the total force is measured in newtons (N) and the mass is measured in kilograms (kg), the acceleration can be determined in metres per second squared (m/s^2). This formula describes the observation that larger masses accelerate less rapidly than smaller masses acted on by the same total force. It also describes how a particular object accelerates more rapidly when a larger total force is applied. When all of the forces on an object are balanced, the total force is zero. Newton's second law is often expressed as $F = ma$.

Text 4.14 Haire et al. (2000: 118).

Text 4.14 again encodes technical meaning in individual symbols: *a* is equated with *acceleration*, *F* with *the total force on the object* and *m* with *the mass of the object*. As well as this, the full equation is named as *Newton's second law*. The text continues to build meaning into the symbols and equation in the final paragraph, strengthening their semantic density.

Moving further into the page in which Text 4.14 is situated, mathematics is again used. This time, however, the text is not concerned with condensing meaning into the symbolism, but with using the mathematics to calculate the acceleration of a space shuttle taking off. As Text 4.15 shows, this involves using a quantification.

> Newton's second law can be used to estimate the acceleration of the space shuttle at blast off:
>
> $$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{7\,000\,000 \text{ N upwards}}{2\,200\,000 \text{ kg}} \\ &= 3.2 \text{ m/s}^2 \end{aligned}$$
>
> In other words the space shuttle is gaining speed at the rate of only 3.2 m/s (or 11.5 km/h) each second. No wonder the blast off seems to take forever!

Text 4.15 Haire et al. (2000: 119)

As discussed previously, quantifications aim to produce numerical results that measure a specific instance of the object of study (which for physics is the physical world). The opening Situation orients the text to the problem it is calculating (in this case the acceleration of the space shuttle) as well as specifying the equation to be used. Moving through the Substitution that gathers numerical values form the previous co-text (not shown), the final Numerical Result ($= 3.2 \text{ m/s}^2$) completes the calculation. The Numerical Result allows the physics to be grounded in a specific instance of the empirical world; it allows it to move outside the realm of abstracted theory. The final Interpretation reconstrues the Result linguistically and relates it back to the acceleration of the space shuttle specified in the opening stage, as well as linking it to a more everyday, common-sense understanding of the motion of the space shuttle (*No wonder the blast off seems to take forever!*).

The introduction of quantifications is the key marker that boosts the role of mathematics in junior high school in comparison to primary school. To understand the role of quantifications in physics, we can utilise the second concept from the Semantics dimension of LCT: *semantic gravity*. Semantic gravity is concerned with the degree to which meanings are dependent on their context. Stronger semantic gravity (SG+) indicates meanings are less dependent on their context, whereas weaker semantic gravity (SG–) indicates greater context-dependence. Semantic gravity and semantic density are independent variables that allow an understanding of how meanings vary in their relations to other meanings and to their context.

Mathematical equations that do not involve numbers are not tied to any particular physical context. $F = ma$, for example, describes an abstract set of relations that hold for a very large set of situations – essentially all physical situations that can occur in our everyday life. The equation does not, however, address any particular situation. It does not, for example, say how much force, acceleration or mass will occur at any particular instance, rather it simply shows their generalised relations. The equation, then, is characterised by relatively weak semantic gravity.

On the other hand, the final equation in numerical form,$(a) = 3.2\ \mathrm{m/s^2}$, precisely describes a specific situation. The numerical form of the equation does not mention anything about the generalised relationships between force, acceleration and mass, but measures a specific instance of acceleration. It is thereby characterised by relatively strong semantic gravity. The quantification genre thus involves a shift from weaker to stronger semantic gravity; it is a tool for gravitation (for strengthening semantic gravity, Maton 2014:129). This allows physics to keep in touch with its object of study.

In junior high school physics, mathematics continues to strengthen semantic density by specifying equations and condensing them with technical meaning from language. At the same time, mathematics' role in quantifications allows physics to strengthen its semantic gravity by reaching out to specific empirical situations. Both the use of quantifications and the encoding of technical meaning from language continues into senior high school. This stage sees a further innovation, the introduction of derivations, that highlights the increasing role of mathematics in building knowledge in physics.

4.4.3 Mathematics in senior high school physics

Senior high school (years 11-12, ages ~16-18) physics continues the trend of increasing reliance on mathematics. By this stage, mathematics is a crucial component of the high-stakes assessment in physics. Students must not only read mathematics as it is used in classrooms, textbooks and assessment, and in doing so, gain technical physical meaning, they must also write mathematics as a means to solving physical problems. In senior high school, the forms of mathematics used in junior high school are consolidated, expanded and built upon. In terms of sheer quantity, there is an enormous increase in the number of equations introduced. Text 4.16, for example, shows a snapshot of slightly over a quarter of the forty-eight equations specified in the formula sheet of the final state-wide exam of high-school physics.

$$v_{\mathrm{av}} = \frac{\Delta r}{\Delta t}$$

$$a_{\mathrm{av}} = \frac{\Delta v}{\Delta t} \quad \text{therefore} \quad a_{\mathrm{av}} = \frac{v-u}{t}$$

$$\Sigma F = ma$$

$$F = \frac{mv^2}{r}$$

$$E_k = \frac{1}{2}mv^2$$

$$W = Fs$$

$$p = mv$$

$$\text{Impulse} = Ft$$

$$F = \frac{Gm_1 m_2}{d^2}$$

$$E = mc^2$$

$$l_v = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$t_v = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m_v = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Text 4.16 Board of Studies, Teaching & Educational Standards NSW (2014: 42)

Each of the equations in this formula sheet must be understood by students to be successful in assessment. Although the equations themselves are given, there is no explication of what they mean, to which situations they apply or how to use them. They are technical equations that students need to understand in relation to the broader field. The equations build a very large complex of relations among technical meanings. The increase in equations arguably accelerates in future years, with a larger part of the technical meaning of physics organised through the mathematics deployed in the field.

Complementing the increase in the use of mathematics, the complexity of quantifications also increases. In junior high school, it is typical for single quantifications to occur in isolation, but in senior high school it is common for larger quantification complexes to occur that aim to produce a single result (see Section 4.2.3). Text 4.9, reproduced below, shows an example of this with each quantification boxed.

On the earth:

$$\begin{aligned} \boldsymbol{W}_{\text{earth}} &= m\boldsymbol{g}_{\text{earth}} \\ &= 50 \times 9.8 \\ &= 490 \text{ N downwards} \end{aligned}$$

On Mars:

$$\begin{aligned} \boldsymbol{W}_{\text{Mars}} &= m\boldsymbol{g}_{\text{Mars}} \\ &= 50 \times 3.6 \\ &= 180 \text{ N downwards} \end{aligned}$$

Loss of weight = 490 – 180 = 310 N. But there is no loss of mass!

Text 4.9 (c) de Jong et al. (1990: 249)

The quantifications in this text all work toward achieving a single final result. The importance of quantification complexes such as the one in Text 4.9 is that they allow physics to calculate numerical results from relatively distant starting points. In single quantifications numerical values must be available for every symbol other than the one being calculated. For example, in Text 4.15, discussed in relation to junior high school, the aim was to calculate *a* (acceleration) from the equation $a = \frac{F}{m}$. The numerical values of both *F* and *m* were known from the previous co-text, leaving only *a* to be determined. This allowed *a* to be calculated with a single quantification. In Text 4.9 from senior high school, on the other hand, calculation of the loss of weight requires that both the weight on Earth and the weight on Mars be known. These are not specified in the text and so require calculation through other quantifications. This can be seen from the fact that the Substitution in the final quantification (= 490 – 180) gathers its numerical values from the Numerical Results of the previous quantifications. Each of these quantifications use the formula: $\boldsymbol{W} = m\boldsymbol{g}$. The symbols *m* (mass) and *g* (gravity on earth or Mars) are both known from the previous co-text allowing the weight on both Mars and Earth to be calculated. So based on the previously differentiated knowledge of the mass and gravity on both Mars and Earth, a sequence of quantifications can be used to calculate the loss of weight.

With regard to the knowledge of physics, the emergence of quantification complexes in senior high school builds upon the role of single quantifications used in junior high school. Single quantifications allow physics to reach from generalised theory to specific empirical situations. Based on single quantifications, however, empirical situations can be explored only if a relatively specific set of numerical knowledge was available. For example, if we wished to quantify the symbol F through the equation $F = ma$, with only a single quantification, we would need values for both m and a to complete the calculation. With quantification complexes, however, if these values are not known, we can do further calculations to find them. In principle, we could find F without initially knowing m or a, as quantification complexing allows these to be calculated first. Indeed this is what occurred in Text 4.9 above. To calculate the Loss of weight, both the weight on Mars and on Earth need to be specified. However initially, they weren't known. Through quantification complexing, these values could be calculated first, and then used in the final quantification to complete the final Numerical Result. Quantification complexes thus allow a larger range of possible starting points for calculations than single quantifications. This allows knowledge that is further removed from the empirical object of study to be put to use. As the number of quantifications that can occur in a complex is in principle indefinite, this complexing provides a powerful tool for physics to reach toward its object of study from very distant starting points.

As well as the introduction of quantification complexes and a greater reliance on mathematics, senior high school sees derivations come to prominence. As we have seen, derivations are concerned not with measuring specific empirical instances, but rather with developing new mathematical relations. Text 4.7, reproduced below, showed an example of this.

To show the equality of these two different statements of Newton's Second Law [referring to previous co-text YJD], consider the following:

$$\vec{F} = m\vec{a}$$

$$\text{But } \vec{a} = \frac{\vec{v} - \vec{u}}{\Delta t}, \text{therefore}$$

$$\vec{F} = \frac{m(\vec{v} - \vec{u})}{\Delta t}$$

$$= \frac{m\vec{v} - m\vec{u}}{\Delta t}$$

$$= \frac{\Delta m\vec{v}}{\Delta t}$$

Force is the time rate of change of momentum as stated by Newton!

Text 4.7(c) Warren (2000: 123)

The new equation developed in the derivation makes explicit relations between symbols implied but not yet specified in the field. Derivations are thus deployed to deepen the technical knowledge of physics. They develop and specify new sets of relations that become part of the field. These new relations can in turn be used in quantifications to extend the range of empirical situations accounted for by physics. Derivations are thus used not just to build new mathematical relations, but also to contribute to the development of new linguistic technicality. We have already seen an example of this in Text 4.8, reproduced below, a text that immediately follows Text 4.7.

IMPULSE

As we have just seen, we can write Newton's Second Law as:

$$\vec{F} = \frac{(\Delta m\vec{v})}{\Delta t}$$

Rearranging this equation we can write:

$$\boxed{\vec{F}\Delta t = \Delta(m\vec{v})}$$

Impulse ($I = \vec{F}\Delta t$) *is equal to the change in momentum of the object upon which the force is applied.*

The SI unit of impulse is newton second (N.s). This unit is hence an alternative unit for momentum (see earlier [referring to previous co-text not shown YJD]).

It follows that the same impulse can result from either a small force applied for a long time or a large force applied for a short time; the changes in momentum will be the same in both cases.

Text 4.8 (b) Warren (2000: 123) (boxes in original).

Text 4.8 begins with another short derivation. The opening equation $\vec{F} = \frac{(\Delta m\vec{v})}{\Delta t}$ is taken from the Symbolic Result of the previous equation, and is reorganised to produce the final Symbolic Result, $\vec{F}\Delta t = \Delta(m\vec{v})$. As discussed in section 4.2.2.2, this Result is then reinterpreted in language in the Interpretation stage. In doing so, a new piece of linguistic technicality is introduced, that in some sense encapsulates the meanings made by the mathematical Result. The sequence of derivations has worked not simply to develop new relations in the mathematics, but to also develop new technical meaning in language.

Derivations develop new equations by making relations which are otherwise implied in the field explicit. With the growth of technical symbols and equations, a large combinatorial potential arises. Each symbol carries around a large set of implied relations that can be brought to bear in any particular situation (considered in more detail in the following chapter).

For example, as discussed throughout, the equation $F = ma$ specifies relations between F, m and a. These relations remain even when one of the symbols is mentioned without the others, such as for F in $F = \frac{E_k}{s}$ (E_k is glossed as kinetic energy, s as displacement). These two equations, $F = ma$ and $F = \frac{E_k}{s}$, set up relations between F and ma, and F and $\frac{E_k}{s}$ respectively. As both sets of relations hold at the same time, relations between ma and $\frac{E_k}{s}$ can also be specified as $ma = \frac{E_k}{s}$, or rearranged, $E_k = mas$. Derivations bring implicit relations between symbols into actuality.

We can again interpret this in terms of the LCT dimension of Semantics. Derivations are tools that make new relations explicit, and lay a platform for the introduction of new linguistic technicality. In this way, they work to build meaning in the field. Whereas in earlier years, mathematics tends to encode technicality developed in language, in senior high school derivations build relations which have not yet been specified. Derivations thus strengthen the semantic density of the field; they are a tool for epistemological condensation (Maton 2014, Maton and Doran in press 2016b). This condensation role is particularly powerful. It allows mathematics to make explicit relations not previously known and to push into the new areas, thereby expanding the horizons of knowledge. When used in conjunction with quantifications, this new knowledge can be tested to see how usefully it construes the empirical world.

Based on this understanding of derivations, we can now review them in relation to the increasing use of mathematics, and its relation to language. As we saw in primary school (and continued through junior and senior high school), language initially works to invest mathematics with technical meaning of physics. Drawing on this investment, derivations can then produce new relations which have not previously been made explicit. The relations in mathematics, and the symbols involved in them, can then be named in language. This transfers the meanings developed in mathematics back to language, which can in turn utilise its own ways of meaning making. By handing meaning back and forth in this way, mathematics and language work in tandem to considerably strengthen the semantic density of the field.

4.4.4 Mathematics in undergraduate university physics

The jump to undergraduate university physics sees the reliance on mathematics to organise the knowledge of physics increase once more. In all subfields of physics at this level mathematics is in regular use. Students are expected to both read and write large quantification and derivation complexes to a degree far surpassing that of senior high school. However as well as simply using more mathematics, university physics changes the form of its mathematics to consolidate and extend its knowledge-building role. In particular, university physics begins to more readily deploy the operational component of the grammar realised through multivariate structures (see Section 3.4.7.2 of Chapter 3), in particular the unary operations of calculus. Although not be explicitly included in the grammar developed in the previous chapter, the general multivariate structuring of calculus follows the same principles as other operations in the operational component of the grammar. To see how this calculus is put to use, we will consider the derivation in Text 4.17 from a first year undergraduate university textbook. This derivation is excerpted from a much larger sequence and provides an intermediate step to a final Result further into the text (but not shown here). The equation numbers on the right of the text are those of the original.

Consider the following wave function for a wave of wavelength λ and frequency *f* moving in a positive *x*-direction along a string:

$$y(x,t) = A\cos(kx - \omega t) + B\sin(kx - \omega t) \quad \text{(sinusoidal wave on a string)} \quad (40.2)$$

Here $k = 2\pi/\lambda$ is the *wave number* and $\omega = 2\pi f$ is the *angular frequency*. (We used these same quantities for mechanical waves in Chapter 15 and electromagnetic waves in Chapter 32.) The quantities *A* and *B* are constants that determine the amplitude and phase of the wave. The expression in Eq. (40.2) is a valid wave function if and only if it satisfies the wave equation, Eq. (40.1) [from previous co-text not shown YJD]. To check this, take the first and second derivatives of *y*(*x*, *t*) with respect to *x* and take the first and second derivatives with respect to *t*:

$$\frac{\partial y(x,t)}{\partial x} = -kA\sin(kx - \omega t) + kB\cos(kx - \omega t) \quad (40.3a)$$

$$\frac{\partial^2 y(x,t)}{\partial x^2} = -k^2 A\cos(kx - \omega t) - k^2 B\sin(kx - \omega t) \quad (40.3b)$$

$$\frac{\partial y(x,t)}{\partial t} = \omega A\sin(kx - \omega t) - \omega B\cos(kx - \omega t) \quad (40.3c)$$

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\cos(kx - \omega t) - \omega^2 B\sin(kx - \omega t) \quad (40.3d)$$

If we substitute Eqs. (40.3b) and (40.3d) into the wave equation, Eq. (40.1) [not shown YJD], we get:

$$-k^2 A\cos(kx - \omega t) - k^2 B\sin(kx - \omega t)$$

$$= \frac{1}{v^2}[-\omega^2 A\cos(kx - \omega t) - \omega^2 B\sin(kx - \omega t)] \quad (40.4)$$

Text 4.17 Young and Freedman (2012: 1329-1330)

Each of the equations in this derivation involve at least three unary operations, including the trigonometric operations cos and sin, and a generic operation of $y(x,t)$ (described in Section 3.4.3). Further to this, each uses a number of operators involved in calculus. These are shown by $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial t}$, $\frac{\partial^2}{\partial x^2}$ and $\frac{\partial^2}{\partial t^2}$ in the Themes (left side) of equations (40.3a-d). These equations are excerpted below with the symbols involving calculus operators shown in bold.

$$\frac{\boldsymbol{\partial y(x,t)}}{\boldsymbol{\partial x}} = -kA\sin(kx-\omega t) + kB\cos(kx-\omega t) \qquad (40.3a)$$

$$\frac{\boldsymbol{\partial^2 y(x,t)}}{\boldsymbol{\partial x^2}} = -k^2A\cos(kx-\omega t) - k^2B\sin(kx-\omega t) \qquad (40.3b)$$

$$\frac{\boldsymbol{\partial y(x,t)}}{\boldsymbol{\partial t}} = \omega A\sin(kx-\omega t) - \omega B\cos(kx-\omega t) \qquad (40.3c)$$

$$\frac{\boldsymbol{\partial^2 y(x,t)}}{\boldsymbol{\partial t^2}} = -\omega^2A\cos(kx-\omega t) - \omega^2B\sin(kx-\omega t) \qquad (40.3d)$$

Within calculus, these unary operators signal operations known as first derivatives, $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial t}$ in 40.3a,c, and second derivatives, $\frac{\partial^2}{\partial x^2}$ and $\frac{\partial^2}{\partial t^2}$ in 40.3b,d. [45] These operators each modify the same Argument, $y(x,t)$, given in the Theme of the earlier equation (40.2). We will focus on the first of the operator $\frac{\partial}{\partial x}$ in the Theme of (40.3a):

$$\frac{\partial y(x,t)}{\partial x} = -kA\sin(kx-\omega t) + kB\cos(kx-\omega t) \qquad (40.3a)$$

Like the other unary operators, $\frac{\partial}{\partial x}$ modifies the $y(x,t)$ given as the Theme of equation (40.2). By modifying this symbol, it necessarily changes what it equals. That is, in the first equation, $y(x,t)$ without modification, equals $A\cos(kx-\omega t) + B\sin(kx-\omega t)$. With modification, however, $\frac{\partial y(x,t)}{\partial x}$ equals $-kA\sin(kx-\omega t) + kB\cos(kx-\omega t)$. By modifying $y(x,t)$ with the calculus operator $\frac{\partial}{\partial x}$, the relations on the right hand side are also changing.

For a student within a university physics course, the movement from the first equation to the second should be relatively straight forward. Students should by this stage should know how to manipulate equations when this operator $\frac{\partial}{\partial x}$ is applied (in New South Wales, Australia where the data was collected, this is taught in the penultimate year of high school). As such, the text shows no intermediate steps between equation (40.2) and equation (40.3a). The way

[45] For a full grammar that includes the operators of calculus, it would need to include both the univariate and multivariate structures shown by these operators. At the highest level, $\frac{\partial}{\partial x}$ functions as a multivariate unary operator on $y(x,t)$ to produce $\frac{\partial y(x,t)}{\partial x}$. Within this however, each ∂ (related to the change operator Δ) also functions as an operator, producing ∂y and ∂x. These in turn are related to each other by the univariate binary operation of division, shown by the vinculum —. The tension between the multivariate and univariate allows a great deal more variability than can be discussed here.

the mathematics is manipulated when one takes the first derivative (when one applies $\frac{\partial}{\partial x}$) is assumed. Understanding the movement between these two equations goes to the crux of the role of multivariate structures in mathematics and their importance for the knowledge of physics.

As discussed in Section 3.4.3 of the previous chapter, multivariate operators such as this can be equated to a set of binary relations. The example we used in chapter three was of the change operator Δ. This operator shows the differences between two specific points of the same variable. For example, if Δ modifies *x* to become Δ*x*, it can be equated with $x_2 - x_1$. In this expressions x_1 and x_2 are two instances of the same variable modified by Δ. Similarly, if Δ modifies *y*, this results in $\Delta y = y_2 - y_1$, and if it modifies *z* it results in $\Delta z = z_2 - z_1$ and so on. The Δ thus encodes a set of 'dummy' relations that can be applied to the symbol it is modifying. This same principle applies for the calculus operators above. However whereas the change operator Δ encodes a single relation between two symbols, $y_2 - y_1$, the calculus operator $\frac{\partial}{\partial x}$ encodes significantly more. Formally, the relations encoded by $\frac{\partial}{\partial x}$, when modifying *y*, are given by:

$$\frac{\partial y}{\partial x} = \lim_{h \to 0} \frac{f(y+h) - f(a)}{h}$$

This immediately signifies a more complex set of relations to be applied. It includes larger set of binary relations (addition +, subtraction – and division shown by the vinculum —), and compounding this, the relations also involve other multivariate unary operators, shown by each of $\lim_{h \to 0}$, $f(y+h)$ and $f(a)$. Being multivariate operators, these also encode their own set of relations. Thus the multivariate operators of calculus are built upon other multivariate operators, which are in turn built upon other relations. There is thus a very large set of relations encoded into the single modifier $\frac{\partial}{\partial x}$.

In practice, a series of shorthand procedures for manipulating equations are taught so the above equation is rarely used (at least in undergraduate level physics). Nonetheless, by modifying a symbol by $\frac{\partial}{\partial x}$ or any other calculus operation, a vast swathe of procedures and relations are immediately brought to the fore. These relations and procedures are distilled into the modifier in a similar fashion to technicality in language. However there is a significant difference between linguistic technicality and the multivariate operators we see here. This revolves around the fact that in mathematics these operators are grammaticalised. They are both regular and relatively indelicate choices in the grammar that retain essentially the same meaning across most fields. This allows them to be applied across a large range of fields and variables. For university level physics, what this means is that large sets of grammaticalised relations are brought to bear on the technicality of the field, allowing for new possibilities for manipulation of the mathematics and a significant expansion of the combinatory potential of technical meanings.

As with previous stages of schooling, we can interpret this shift in terms of the LCT variable of semantic density. As discussed previously, the key feature of semantic density is its degree of relationality; the number of relations any particular item invokes. Using this interpretation, the choice of a multivariate unary operator such as $\frac{\partial}{\partial x}$ dramatically increases the number of relations that an equation resonates out to. It thus provides a marker of significantly stronger semantic density. Notably, however, without modifying any particular symbol with technical meaning in physics, the relations invoked are entirely internal to the system of mathematics. They do not have any impact on physics' relations to its object of study.[46] Once they modify a technical symbol within physics, however, these relations become available to the field of physics. They can thus be utilised to link large constellations of meaning within the field, which can then in turn be manipulated by derivations or quantifications. The fact that these operators are grammaticalised as regular and indelicate choices applicable across a large range of fields pushes the power of mathematics for strengthening semantic density to a new level. The mathematics in university physics thus enhances the scope of knowledge building in physics, by both building upon the grammar and genres available in previous years, and more readily utilising a new grammatical form.

4.4.5 Mathematics in the knowledge structure of physics

The survey of mathematics in physics schooling undertaken here reveals its powerful utility. Through its interaction with language, each symbol can garner technical meaning, which can

[46] In terms of the 4K model of LCT (Maton 2014), these operators (and seemingly the entire operational metafunction of mathematics) are primarily concerned with *discursive* relations – relations between items internal to the theory. The multivariate unary operators thus indicate relatively strong discursive semantic density.

then be related to an indefinite number of symbols in a single snapshot. Its univariate structure provides the potential for an indefinite array of technical meanings to be related, while its multivariate structure allows sets of procedures and relations to be employed across a vast array of fields. When used in derivations, physics can employ the large combinatory potential of mathematics to bring together relations in the field that have not yet been specified, and in doing so develop new knowledge in the field. When used in quantifications, mathematics can use these relations to account for specific instances in the real world. This opens the possibility for theory to be tested against empirical data and the physical world to be predicted.

In LCT terms, the mathematics used in physics is a tool for both condensation (strengthening semantic density) and gravitation (strengthening semantic gravity). This allows meanings to be related and proliferated in a large number of combinations, while maintaining the capacity to connect with the empirical world. Without the strong potential for condensation, technical meanings would have a limited possibility for combination. This means that they would be tied to their contexts and become segmentalised. Physics would thus lose the potential to develop generalisable theories that account for a broad range of phenomena. In contrast, without the possibility for gravitation, physics would have no capacity to reconnect with its object of study; there would be no counter-balance to ensure the proliferation of theory maintains relevance in the study of the physical world. The condensation and gravitation shown by mathematics arise from the two genres introduced in this chapter. On the one hand, derivations strengthen the semantic density of both individual symbols and the field. The semantic density developed in mathematics can be condensed into language and vice versa¸ allowing each semiotic resource to utilise its own meaning-making resources. Mathematics thus provides a platform for physics' hierarchical knowledge structure by incorporating a tool for creating general propositions and theories, and integrating knowledge across a range of phenomena (Bernstein 1999). On the other hand, quantifications strengthen the semantic gravity of a text and give physics the ability to link abstracted theory to specific instances. This expanded semantic range gives an avenue for physics to strengthen its epistemic relations between its knowledge and its object of study (Maton 2014). Through mathematics, theory can be tested by data, and data can be predicted by theory.

We can now return to the question posed at the beginning of this chapter: why is mathematics used in physics? Mathematics is used because it provides tools for both theoretical development and bringing theory to bear on data. It is thus an instrument for expanding the frontiers of knowledge and for keeping that knowledge in touch with the empirical world.

4.5 Genre coordinating mathematics and language

This chapter set itself the goal of understanding the interaction of mathematics with language. It approached this problem from the perspective of genre to show the large scale patterns of mathematics and language in producing larger texts. As with the previous chapter, however, it did not simply assume a stratum of grammar, but rather sought to derive it axially. In doing so, it showed that a productive model of genre can incorporate both primarily mathematical and linguistic genres in a single stratum. That is, it showed that a generalised genre network realised by both mathematics and language can coordinate the large scale text patterns at work in physics.

These genres and the broad grammatical patterns that realise them were then put to work mapping the changes in mathematics through physics schooling. This map allowed a broad understanding of the development of mathematics, and its utility for organising the knowledge of physics. Mathematics was shown to aid both building new technical meaning and linking this to the empirical world. It lays the basis for both the internal and external languages of description for physics.

Chapter 3 and 4 have thus put forward a comprehensive model of mathematics in Systemic Functional terms. It has shown predictable relations between large text patterns and the grammatical organisation of mathematics, and derived in a methodologically consistent manner a stratal, metafunctional, rank and nesting architecture of mathematics. This architecture allows mathematics to develop its own meaning making patterns to relate technical meanings in physics. The question remains, however, what precisely these meaning making patterns are. From the model of grammar and genre, what can we say about the specific affordances of mathematics in contrast to language? This will be the focus of the following chapter. It will firstly look at mathematics and language from the perspective of the register variable field, before considering in detail the final semiotic resource considered in this thesis, image. The following chapter will thus round off the three perspectives on the discourse of physics in this thesis: grammar, genre and field, and move from a bimodal discussion to a trimodal one.

CHAPTER 5

Images and the Knowledge Structure of Physics

Physics is often spoken of as the archetypical natural science. As discussed in Chapter 2, within the tradition of code theory it is regularly positioned as the prototypical hierarchical knowledge structure (e.g. Christie et al. 2007, Maton and Muller 2007, O'Halloran 2007a), with both strong verticality (the ability to develop ever more integrative and general propositions encompassing larger sets of phenomena) and strong grammaticality (the ability to specify relatively unambiguous empirical referents, Muller 2007). Physics is thus characterised as being able to link theory to the empirical world and to generate new knowledge that subsumes current understandings. This characterisation offers a useful insight into the overarching knowledge structure of physics, however it does not allow us to see the organising principles that underpin this structure. In other words, it does not specify the mechanisms that produce this structure, nor how we 'see' this in data. As Maton (2014: 109) argues, categorising a discipline such as physics in terms of knowledge structure is good to think with, but it does not provide analytical tools to understand how this comes about.

In order to access the organising principles underpinning the knowledge structure of physics, this chapter will pick up the thread introduced in the previous chapters and consider the discourse of physics through the Legitimation Code Theory dimension of Semantics; in particular, the variables of semantic gravity (SG) and semantic density (SD).[47] Recapping the explanations in Chapters 2 and 4, semantic gravity is concerned with the degree to which meanings are dependent on their context (with stronger semantic gravity, SG+, being more dependent on context and weaker semantic gravity, SG –, being less dependent) while semantic density is concerned with the degree of condensation of meaning in a term or practice (with stronger semantic density, SD+, indicating more condensation of meaning and weaker semantic density, SD –, indicating less condensation of meaning) (Maton 2014, Maton and Doran in press 2016a). The chapter proposes that a hierarchical knowledge structure's ability to establish integrative and general propositions encompassing large sets of empirical phenomena depends in large part on being able to generate strong semantic density.

[47] More specifically, it will focus on *epistemic* semantic gravity and semantic density (Maton 2014, Maton and Doran in press 2016a,b). This chapter will not consider the values, morals or aesthetic meanings prevalent in physics that are more associated with axiological semantic density.

Similarly, the ability to produce unambiguous empirical referents arises largely from its broad range of semantic gravity. If we consider physics as a hierarchical knowledge structure, we should thus be able to see in its discourse the potential for strong semantic density, and for movement between a large range of semantic gravity (such as those shown in Conana 2015).[48]

The previous chapter gave an insight into the role mathematics plays in organising the knowledge of physics by tracing the development of mathematical genres and grammar through physics schooling. This chapter will build on this base by characterising the discourse of physics in terms of the images it uses in relation to mathematics and language. By doing so, it continues the progression traced through the previous chapters. Chapter 3 considered mathematical symbolism in isolation and focused on its grammatical organisation. Chapter 4 brought in language and considered its interaction with mathematics from the perspective of genre. This chapter brings in images and considers their role alongside mathematics and language in construing the knowledge of physics. Whereas the previous chapters have focused on grammar and genre, this chapter will view the role of each resource in the discourse of physics through the register variable field (see Section 2.2 Chapter 2). By developing a field-based description, this chapter builds upon the rich descriptions of scientific language in SFL (e.g. Halliday and Martin 1993, Martin and Veel 1998) and offers a common perspective for comparing mathematics, language and image.[49] By combining this with an analysis using LCT's Semantics, this chapter allows us to understand the specific knowledge-building affordances of each resource and the organisation of knowledge in physics as a whole.

Like any academic discipline, physics has its own distinctive ways of meaning. It puts its language, image and mathematics to work in specific ways to establish its own disciplinary discourse. This discourse manifests itself in texts that are 'semiotic hybrids' (Lemke 1998) constituted by a critical constellation of modes (Airey and Linder 2009), where each resource construes disciplinary knowledge in complementary ways. At the same time, different semiotic resources can organise similar disciplinary meanings. Although meanings are often resemiotised (Iedema 2003) across a text, a curriculum or a discipline through an array of

[48] Semantics is not the only way in to seeing the structuring of knowledge in physics. Dimensions of LCT such as Specialisation (Maton 2014) and Autonomy (Maton 2005) also provide useful perspectives on how physics is organised. However for reasons of space, they will not be dealt with in this thesis.

[49] The chapter will not, however, offer detailed networks for field as it did in the previous chapters for genre and grammar. In keeping with the axial orientation of this thesis, the suggestions in this chapter are thus offered as steps toward full formalisation and a full justification for register as a stratum in multimodal semiotics.

different semiotic resources, the disciplinary meanings realised by each resource are still related. For example, an instance of technicality in physics such as *Wien's displacement law* can be illustrated through language, mathematics or images. In language it can be described by stating that for blackbodies 'as the temperature increases, the peak of the spectral emittance curve becomes higher and shifts to shorter wavelengths' (Young and Freedman 2012: 1311). Alternatively, in mathematics this law can be presented as:

$$\lambda_m T = 2.90 \times 10^{-3} \mathrm{m} \cdot \mathrm{K}$$

Or through images it can be displayed as:

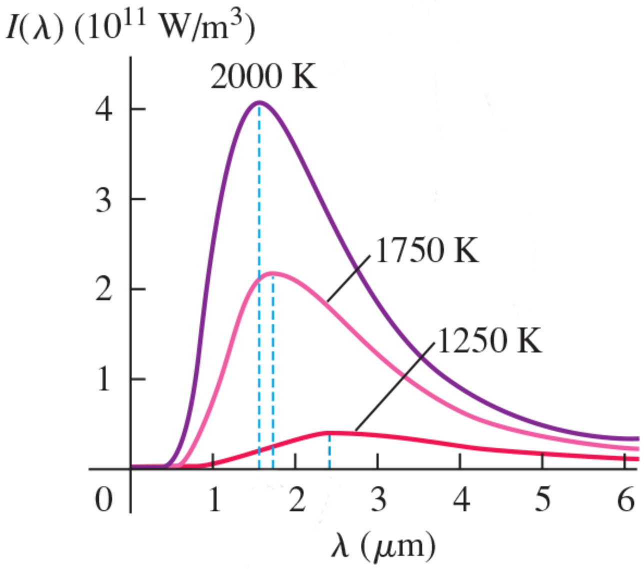


Figure 5.1 Graph of Wien's displacement law (Young and Freedman 2012: 1311)

Or, as in one undergraduate university textbook, it can be elaborated by all three on a single page (Figure 5.2):

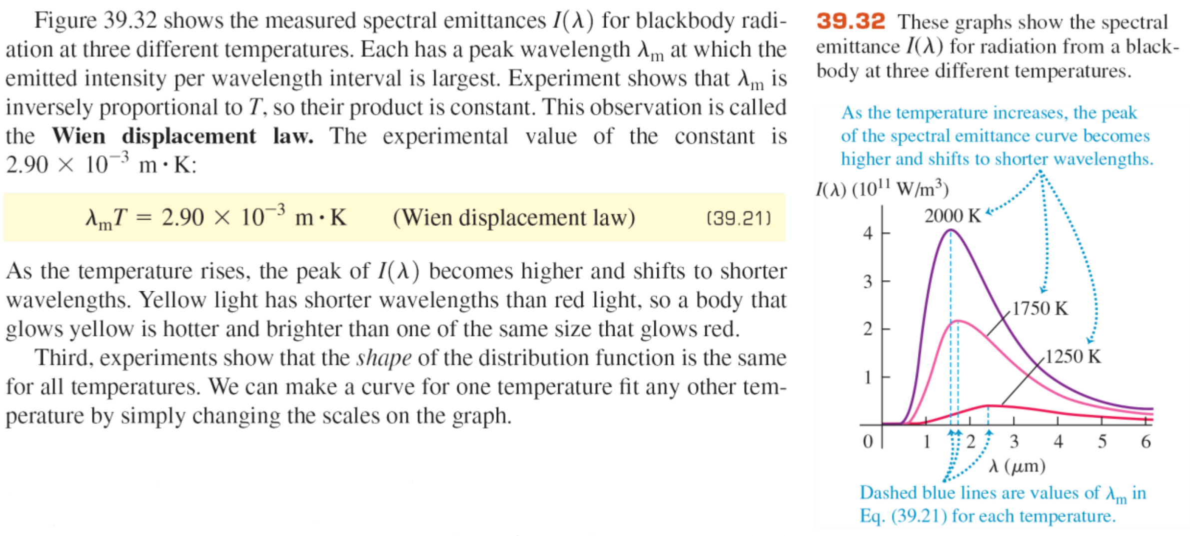


Figure 39.32 shows the measured spectral emittances $I(\lambda)$ for blackbody radiation at three different temperatures. Each has a peak wavelength λ_m at which the emitted intensity per wavelength interval is largest. Experiment shows that λ_m is inversely proportional to T, so their product is constant. This observation is called the **Wien displacement law.** The experimental value of the constant is 2.90×10^{-3} m · K:

$$\lambda_m T = 2.90 \times 10^{-3} \text{ m} \cdot \text{K} \qquad \text{(Wien displacement law)} \qquad (39.21)$$

As the temperature rises, the peak of $I(\lambda)$ becomes higher and shifts to shorter wavelengths. Yellow light has shorter wavelengths than red light, so a body that glows yellow is hotter and brighter than one of the same size that glows red.

Third, experiments show that the *shape* of the distribution function is the same for all temperatures. We can make a curve for one temperature fit any other temperature by simply changing the scales on the graph.

39.32 These graphs show the spectral emittance $I(\lambda)$ for radiation from a blackbody at three different temperatures.

Figure 5.2 Wien's displacement law (Young and Freedman 2012: 1311)

Each of these semiotic resources present meanings associated with *Wien's displacement law* and so they are part of the field of physics and resonate out to other meanings in the field. However each resource also presents these meanings in a way that is not precisely translatable across the resources. Understanding the similarity and difference of resources in the construal of physics knowledge is crucial to understanding the knowledge of physics itself, and so will be a constant theme throughout the chapter. We will see how texts in physics manage to build vast networks of interlocking meanings, and how these meanings contribute to physics' knowledge structure.

The discussion will begin with a brief review of field as viewed from language and an interpretation of mathematics in field-based terms. We will see that language and mathematics organise different dimensions of field, and thus complement each other in construing the knowledge of physics. Second, we will consider images, in particular diagrams and graphs, to show the significant meaning potential available in individual images, and the possibilities for types of meaning not readily available through language or mathematics. Throughout the chapter, we will interpret the field-based perspective of each resource from the perspective of Semantics in Legitimation Code Theory and thus build up a picture step-by-step of the knowledge structure of physics.

5.1 The field of physics viewed from language and mathematics

As discussed in Chapter 2 (Section 2.2) language arranges the knowledge of science into deep taxonomies and long sequences of activity (following Martin 1992a, Halliday and Martin 1993). Scientific taxonomies are either compositional, arranging technical terms into part-whole relations (such as the relation between an atom and its constituent nucleus and electrons), or classificational, arranging technical terms into type-subtype relations (such as the relation between atoms and its subtypes, hydrogen atom, helium atom etc.). Complementing these taxonomies are activity sequences that show progressions of events associated with a field. In science, these are typically sequences of implication where the unfolding of events is based on absolute contingency. That is, the progression of happenings is such that there is no possibility for counterexpectation. In language, these implication sequences are often realised by relations of causality, where each event necessarily causes or implicates the next. In certain situations, however, the sequences can be less deterministic. Other sequences, known as expectancy sequences, simply display expected or typical unfolding and thus open the possibility for unexpected events to occur. These expectancy sequences are often realised in language through temporal, rather than causal relations.

From the perspective of language, therefore, the field of physics is organised through a large set of relatively delicate taxonomies of composition and classification, and a series of activity sequences involving entities that comprise these taxonomies. Every technical term gains a large swathe of meaning from its position in these intersecting dimensions, and thus displays relatively strong semantic density (Maton and Doran in press 2016a,b). Language is, however, only one component of the technical discourse of physics. As the previous chapters have shown, mathematics construes its meanings in considerably different ways to language through a distinct grammar and by realising distinct genres. Accordingly, in physics, the technical meanings arising from mathematics are organised along markedly different dimensions than those arising from language. In order to contrast mathematics with language in these terms (and with images further into the chapter), we must now briefly reinterpret mathematics in terms of field.

The overarching grammatical organisation of mathematics raises the question of what type of field-relation it construes. The relations are not ones of composition: in $p = mv$, m is not a part of p or v; nor are they classification: m is not a type of p or v. Nor do they organise any sort of expectancy sequence. Rather, the grammatical organisation appears to present a large network of interdependency. When the value of one symbol changes, at least one of the others must necessarily also change. In this way, the relations are closer to those of implication sequences in language that indicate that if one event occurs another must necessarily also occur. However unlike the implication relations realised in language those of mathematics do not suggest any type of unidirectional sequence. Each element in the vast network of interdependency is contingent on every other. Moreover, the grammar of mathematics can specify large sets of these relations in one synchronic snapshot. This offers a subtle distinction to implication sequencing in language, and suggests the need for a small reinterpretation of implication for mathematics. Whereas language indicates implication sequences with a definite direction, in mathematics we can view the vast interconnected networks of meanings as implication complexes without any directionality. Seen in these terms, mathematics in physics works to construe large complexes of interdependency whereby each symbol is contingent on every other.

This interpretation allows the realisational relationship between field and the grammar of mathematics to be traced. First, each field specifies its own particular implication complexes. These implication complexes are then realised by particular covariate relations in the grammar of mathematics (see Section 3.4.2.5 of Chapter 3), which remain stable across the field. In conjunction with the choice of Theme and Articulation, these covariate relations in turn coordinate the univariate organisation of symbols within both statements and expressions. The result is that only a small set of mathematical statements become acceptable within any particular field. In effect, these statements and the symbols that comprise them constitute the mathematical technicality of physics.

If we look at this from the perspective genre, we see that the function of derivations is thus to build and make explicit field-specific implication complexes. However this raises the question of what quantifications do in terms of field. Where derivations build new relations between symbols, leading to new implication complexes, quantifications move from generalised statements to specific numerical measurements. In LCT terms, they work to strengthen the semantic gravity of physics by offering a pathway from abstracted theory to specific empirical instances. However this movement is not accounted for in the current conception of field. The relation between generalised theory and specific instance is not one of taxonomy, nor is it sequencing or complexing. Rather, it appears to be a distinct field-specific dimension. Therefore as an exploratory step, I will propose a new scale that describes the movement between generalised relations and specific instances, termed *generality*. Under this scale, pronumerical symbols, such as E_i, m, F etc. are at a higher generality than their numerical values, such as 5, -1.5 etc.

Such a scale allows physics to talk both in general terms about the broader physical world, and in specific terms about a particular instances in the world. Although very preliminary in its formulation, it also offers an interpretation of the field-based meanings made by quantifications. Whereas derivations work to build implication complexes, quantifications work to shift physics from higher generality to lower generality (from a generalised description to a measurement of a specific instance). Notably, this occurs in only one direction. Quantifications only allow a movement from the general to the specific, not the other way around. In the data under study, there is no mathematical genre that offers a shift from number to generalised equations. Although at a higher level of mathematics, such a shift is possible through statistical and mathematical modelling, the algebra and calculus in physics schooling only affords a single direction. In Section 5.2.2, however, we will see that graphs do offer movements in the opposite direction. They complement quantifications by arranging empirical data into patterns that can be incorporated into generalised theory.

Before moving on, we must note that the scale of generality in physics is not the same as the instantiation dimension in SFL. Instantiation is specifically concerned with the movement between generalised description and specific instance of semiosis. The difference between the two scales can be seen by the fact that any point in the scale of generality can occur in a text (the instance pole of instantiation). For example, a physics text can present a relation with higher generality involving only pronumerals, e.g. $\Delta E_{emitted} = E_i - E_f$, or it can present a relation with lower generality involving numbers, $\Delta E_{3\rightarrow 2} = 3.04 \times 10^{-19}$ Joules. If it were the case that generality and instantiation were the same thing, then only the low generality (i.e. numbers) could be instantiated in a text.

Generality and instantiation are related, however. The scale of generality allows a field to describe the relation between system and instance in its own object of study. In physics, this object of study is the physical material world. Generality shows the relations between specific instances of the real world and the more general descriptions of physical systems. As another example from the separate field of meteorology, generality describes the relation between the generalised climate and instances of the weather. Similarly, if we view Systemic Functional Semiotics as a field in itself (along the lines of physics or meteorology), generality describes the relation between systems of semiosis and instances of text. That is, it describes the scale of instantiation. The dimension of instantiation is thus a scale of generality focusing on semiosis (rather than the physical world or the climate).[50]

Closing the discussion of mathematics, we see that for the field of physics it displays two distinctive features. It realises large complexes of implication relations developed through derivations, and shifts generality from highly general theory to specific empirical instances through quantifications. From the perspective of LCT, implication complexing affords condensation (strengthening semantic density) and decreasing generality affords gravitation (strengthening semantic gravity) in physics. As the previous chapter discussed, these two movements offer strong potential for developing the knowledge structure of physics. They allow physics to construe integrative propositions and engage with a large number of phenomena, while also creating a pathway for physics to connect its theory with its empirical object of study.

As discussed earlier, a look at any physics text shows that mathematics works not just with language but also with image. So far we have reflected on field as it is developed by both language and mathematics, the next step is to consider the role of images. In the following section, we will see that images provide a powerful resource for displaying a very large number of technical meanings in a single snapshot. In addition, they bring to the fore their own field specific meanings; meanings that are not readily apparent in mathematics and language. By drawing on image in conjunction with mathematics and language, physics presents a multifaceted field that utilises the affordances of multiple semiotic resources to construe the various dimensions of its hierarchical knowledge structure.

5.2 Images and field

Aside from language, the only resource to rival mathematics for its pervasiveness in physics is images. Images are used earlier than mathematics in primary schooling, are prevalent throughout high school and university, and form a critical resource in research. They are used to explain processes, report descriptive features and present raw data. They display a multifaceted functionality for organising the technical knowledge of physics and they

[50] Generality is also distinct from SFL's variable of presence (Martin and Matruglio 2013, Martin in press 2016). Presence is a cover term for a range of resources across metafunctions that contribute to the contextual dependency of a text. If mathematics is taken into account, it is likely that the scale of generality would be one factor in the iconicity dimension of presence. However as Martin makes clear, there is a very large range of other resources at play in presence at any one time.

complement the meanings made in language and mathematics. Thus an account of the discourse and knowledge of physics that avoids images would be significantly lacking.

Much of the power of images comes through the amount of meaning that can be displayed in a single snapshot. As we will see, physics images can present large taxonomies, long sequences of activity, extensive arrays of data and a broad range of generality all in a single image. Like both mathematics and language, images contribute to the hierarchical knowledge structure of physics as they can display a large degree of meaning and can develop generalised models from empirical data. This means they can scaffold the strong semantic density apparent in physics and at the same time invert the shift in semantic gravity afforded by mathematical quantifications. This will be developed in two main sections below. First we will consider diagrams (broadly interpreted), to highlight the possibility of multiple field-based structures in a single image. This will illustrate images' strong potential for semantic density, and also offer an insight into their utility for presenting overviews of these meanings. Second, we will focus on graphs (again interpreted broadly) to show how information can be organised into multiple arrays in ways not readily instantiated in language or mathematics.[51] These arrays of information allow for the generalisation and abstraction of patterns, and indicate shifts in semantic gravity in the reverse direction to those seen in mathematics. In conjunction with mathematics and language, we will see that images play a vital role in developing meaning and linking the theory of physics to the empirical world.

5.2.1 Diagrams in physics

Diagrams are regularly used in physics to illustrate, exemplify, explain, explore and present a large range of technical meanings. In many textbooks, it is unusual for a page to go by without there being at least one diagram complementing the linguistic and mathematical text. Like language, diagrams present a range of field-specific meanings including activity sequencing and taxonomy. In fact, many of these dimensions appear more easily articulated through image; it is no accident, for example, that the standard presentation of taxonomies in SFL involves images. Kress and van Leeuwen's grammar of images (1990/1996/2006) is

[51] We will not consider whether a stratum of genre can be justified for images in this thesis, however as we will see, there does appear to be a division of labour in terms of field between different 'types' of images. Such a coordination of field and mode suggests that a stratum of genre may be a productive category for images. This is briefly discussed in Chapter 6.

particularly useful when considering field (introduced in Section 2.4.1 of Chapter 2). Many of the types of representational images (2006: 59) within their ideational metafunction resonate strongly with field as it has been conceived in relation to language. Narrative images for example tend to realise activities in field through Vectors of motion, while many conceptual images that show Carrier•Possessive-Attribute or Superordinate•Subordinate structures articulate taxonomies. Indeed much of the grammar of images appears to have been developed with field-based meanings in mind. As we will see, physics regularly uses many such images to convey its technical meanings.

In their grammar, Kress and van Leeuwen are unclear as to whether an image can contain multiple structures of the same status. They do indicate that a single structure defining the image as a whole can recur in parts of that structure, but it is not clear whether an image can display multiple structures at the highest level simultaneously. For the purposes of this chapter, I will argue for treating physics diagrams as potentially containing many structures of the same status. This means that various elements in images can play different structural roles, and therefore realise multiple field-based meanings. Indeed the regular use of multiple structures in a single image is one of the most powerful features of images for construing the technical meanings of physics. However before introducing these complex images we will begin by exploring relatively simple images that realise a single dimension of field.

As mentioned above, physics regularly utilises narrative images and these images construe activity. Narrative images include events and their participants, and minimally involve a Vector. This Vector displays some sort of motion or direction and is often accompanied by a number of participants. Figure 5.3 (a), from a senior high school textbook, exemplifies this pattern.

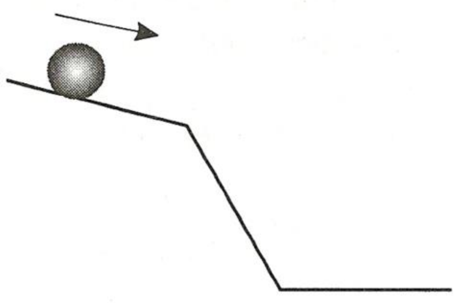


Figure 5.3 (a) Image with a single activity (Warren 2000: 141)

This image illustrates a ball rolling down an inclined plane. The arrow above the ball is the Vector that makes explicit the direction of motion, while the ball is the Actor (the participant that moves). In Kress and van Leeuwen's terms, this image is a non-transactional image as it involves a moving Actor, but without any definite end-point or Goal. To analyse this image, we will highlight different elements according to their structural role. In the analysis below, the Vector is highlighted in yellow and the Actor is in red.[52]

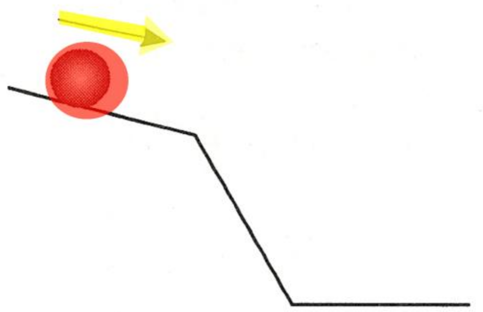


Figure 5.3 (b) Analysed image with a single activity (Vector in yellow, Actor in red) (Warren 2000: 141)

This image represents a relatively simple image with only a single structure,[53] and realises a single activity at the level of field. In contrast, it is common for images to display multiple Vectors in a sequence and therefore realise an activity sequence. Figure 5.4 (a), from a junior high school textbook, for example, illustrates this (with the analysis in 5.4 (b) to its right).

[52] The image analysis for this thesis was originally done using Multimodal Analysis Image (O'Halloran 2012).
[53] We will not consider the possible circumstantial meanings of the inclined plane (the line that the ball is rolling down). Throughout the analyses, many images will display meanings that are somewhat tangential to the field-based meanings that we wish to consider. As such, these will be left out of the analysis.

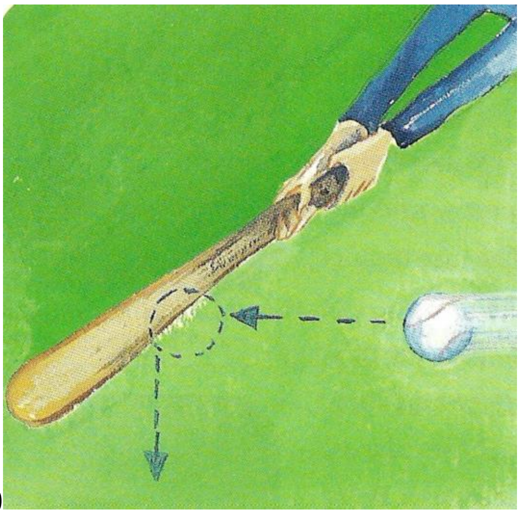
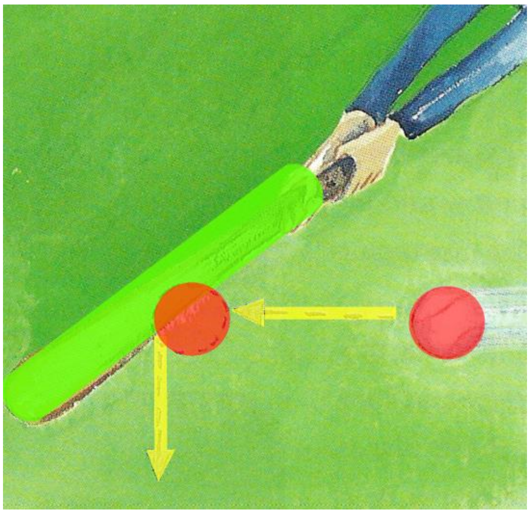
(a)  (b) 

Figure 5.4 (a) Activity sequence in an image (Mau 1999: 8)
(b) Analysis showing Actors (red), Vectors (yellow) and Goal (green)

In interpreting this image, the ball, functioning as an Actor, first moves toward the bat (the Goal, highlighted green). Its movement is indicated by a yellow arrow Vector. Once it has hit the bat, the ball changes direction and moves downwards. The dotted circle indicates the Actor from which the second Vector emanates. Interpreted along these lines, the image shows an activity sequence where one event, the ball moving to the left, is followed by another event, the ball moving down.

Activity sequences such as this can become quite complex, with long strings of Vectors emanating from a single Actor. This is demonstrated by Figure 5.5 (a) that illustrates a nuclear chain reaction (from a senior high school textbook). To analyse this image, it is useful to make an addition to Kress and van Leewuen's grammar. This addition is to distinguish the participant from which the Vector emanates, but which itself does not move, from the participant that is actually moving. In Kress and van Leeuwen's grammar, both of these are considered the Actor; however for the purposes of the discussion in this chapter, the function of Actor will be reserved for the participant that is moving, while the participant from which the Vector emanates (but which does not itself move), will be called the Source (and highlighted with pink).

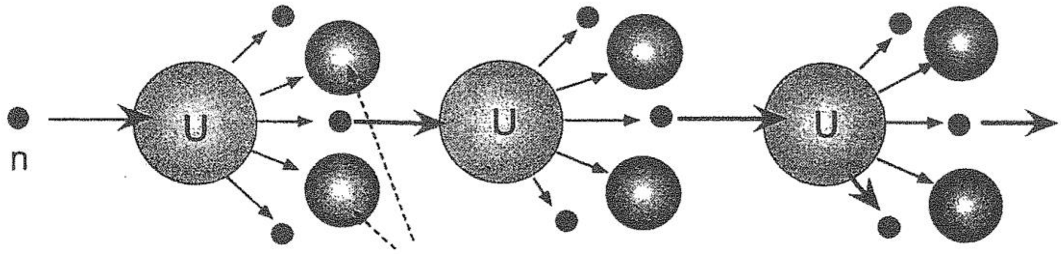


Figure 5.5 (a) Long activity sequence (Marsden 2003: 15)

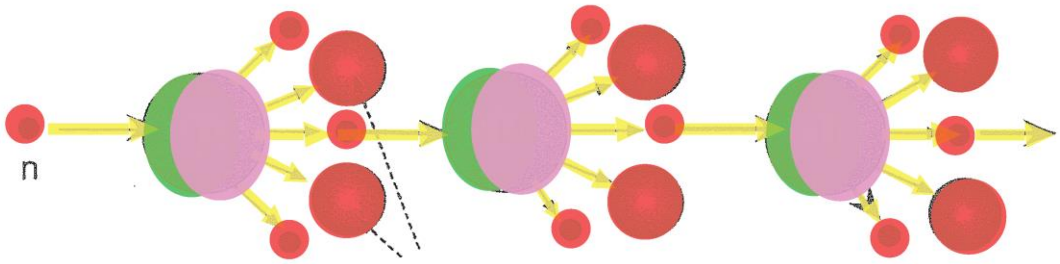


Figure 5.5 (b) Analysis of long activity sequence
(Actor in red, Vector in yellow, Goal in green, Source in pink)
(Marsden 2003: 15)

In this image, a longer activity sequence is illustrated. It begins with the Actor labelled *n* (representing a neutron), moving toward its Goal (in green), a uranium atom (labelled U), with the direction of motion indicated by an arrow Vector (in yellow). In addition to being the Goal of the initial Vector, this element also functions as the Source (shown by overlapping pink on the Goal's green) from which five other Vectors emanate in different directions. Each of these Vectors in turn represent the motions of other particles, shown as Actors in red. One of these particles (also a neutron, but not labelled), moves toward the second uranium atom as its Goal, from which in turn five more Vectors emanate. This process is repeated once more, resulting in nineteen Vectors and nineteen participants being displayed, with three of the participants performing two functions (Source and Goal). In a single snapshot, this image realises a very large sequence of activity. We will see, however, that this is by no means the upper limit in the degree of meaning that can be displayed in an image.

Before moving on to other structures, it is important to note that multiple activities may be represented in an image without any explicit sequencing between them. This is illustrated in

Figure 5.6 from a senior high school textbook. This image shows a ‘free body diagram’ of the forces impacting a car that is coasting without any pressure on the accelerator. For this image, only the Vectors representing forces have been highlighted.

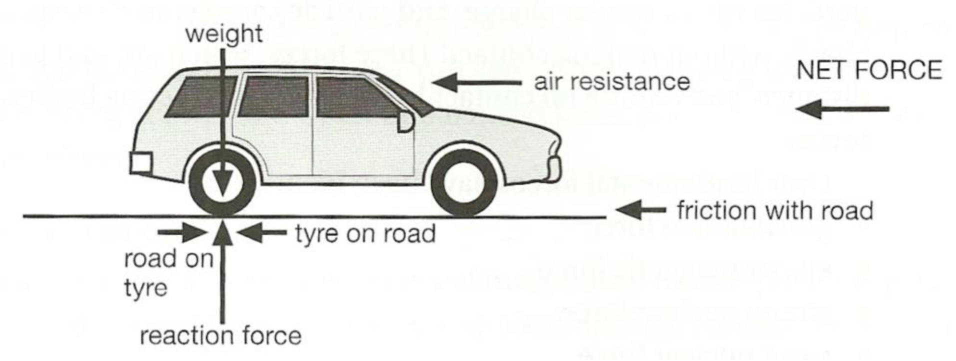


Figure 5.6 (a) Free body diagram (Wiecek et al. 2005: 216)

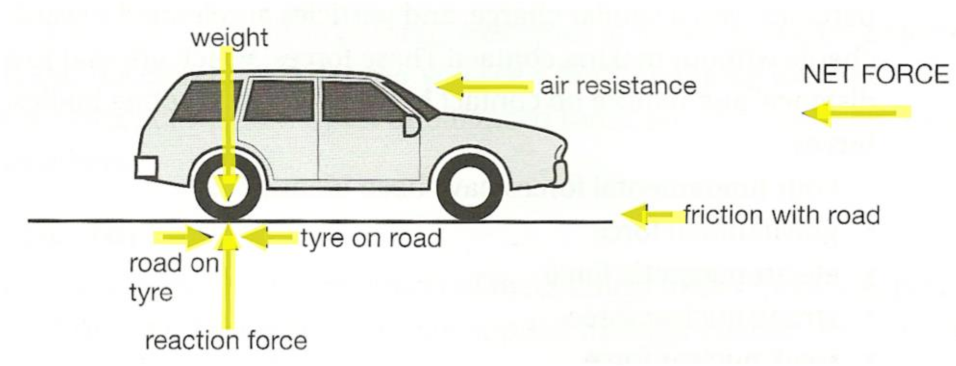


Figure 5.6 (b) Vectors (yellow) in free body diagram (Wiecek et al. 2005: 216)

This image shows seven Vectors, however they are not sequenced in relation one another. Each Vector construes its own relation to the car, without any indication of a sequence. Although in the field of physics the NET FORCE Vector on the right of the image is the sum total of every other force shown, this is not made explicit in the image. In mathematics, the relation between the net force and the other forces would be shown by an equation such as $\boldsymbol{F}_{\mathrm{net}} = \sum \boldsymbol{F}$, which is read as *the net force is the sum of all component forces*. This implication relation is not, however, given in the image. This highlights an important feature of images. Images do not appear to have the same capacity as mathematics to construe nuanced implication complexes. They can display multiple activities that are related implicationally, but they do not grammaticalise the precise nature of these relations. In addition, images do not appear to make a clear distinction between implication and expectancy sequencing. They can show that one Vector follows another; but without labelling, there is no way to distinguish whether sequences of Vectors are related temporally or causally. Nonetheless despite these restrictions, images can display several activity sequences in a single eyeful.

In addition to realising activity, images display a strong capacity for realising delicate taxonomies of both classification and composition. Figure 5.7 presents a relatively simple compositional outline of an atomic model, known as the Rutherford atom. Grammatically, this is an analytical image, constituted by a Carrier (the entire atom, shown in blue) and five Possessive Attributes (shown in purple) (Kress and van Leeuwen 2006: 87). In terms of field, it realises a two-level compositional taxonomy, with the highest level (the whole) being the atom itself, and its constituent being the electrons and the nucleus.

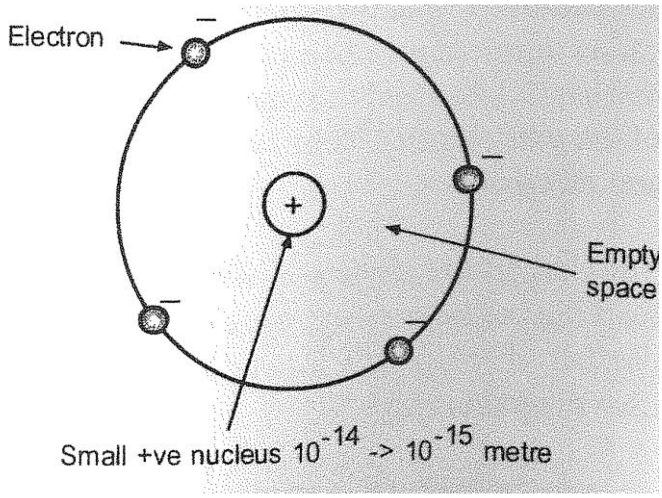


Figure 5.7 (a) The Rutherford atom (Marsden 2003: 2)

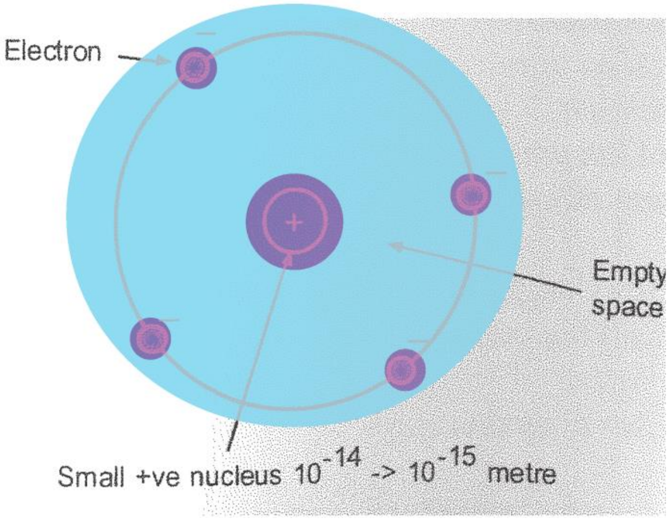


Figure 5.7 (b) Compositional analysis of the Rutherford atom (Carrier in blue, Possessive Attribute in purple) (Marsden 2003: 2)

In contrast, Figure 5.8 that we saw in Chapter 2 presents a simple classification of types of matter, with the Superordinate shown in brown and the Subordinates shown in orange.

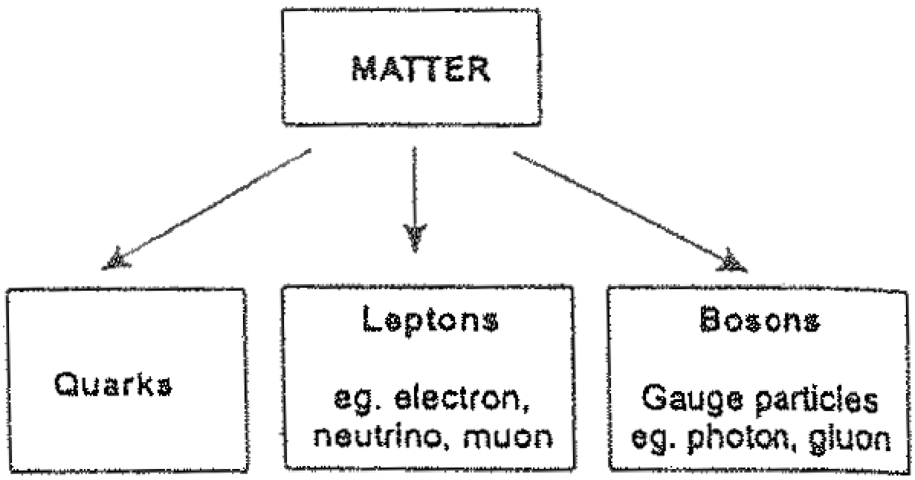


Figure 5.8 (a) Types of matter (Warren 2000: 155)

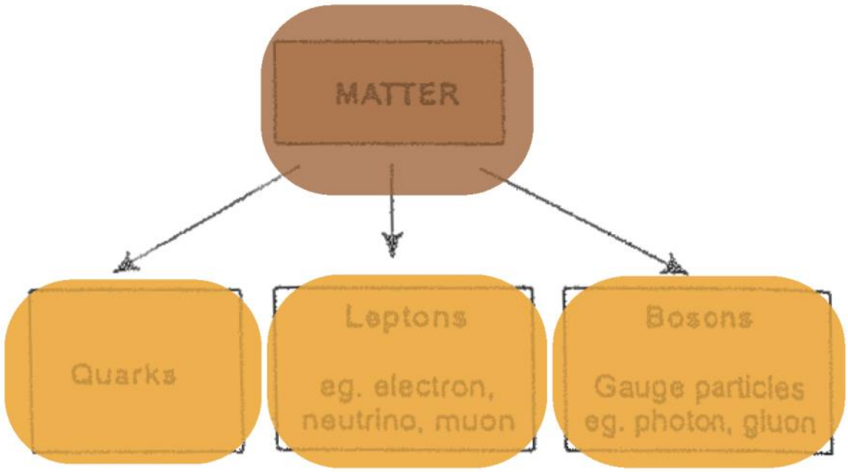


Figure 5.8 (b) Classification analysis of types of matter (Superordinate in brown, Subordinates in orange) (Warren 2000: 155)

Each of these images realises a single dimension of field, either activity or taxonomy. The real power of images, however, comes through their ability to present multiple structures that realise multiple types of field-based meanings. This greatly expands the meaning potential of images and often results in elements performing multiple functions. Through this multifunctionality, different field structures can be related, allowing the interlocking meanings of physics to be displayed in a single snapshot. To illustrate this, we will focus on Figure 5.9, an image from a university textbook. This image outlines two experimental apparatuses designed to view patterns of light emitted from different sources (known as the *emission line spectrum*).

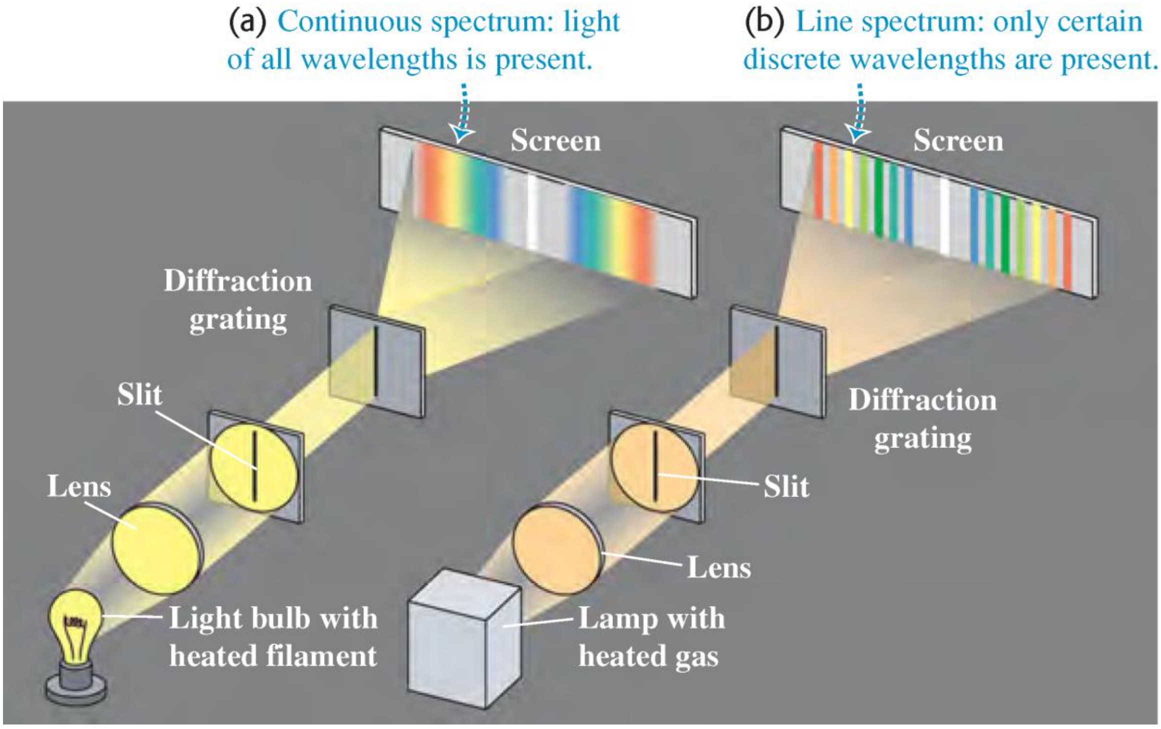


Figure 5.9 Experimental apparatus diagram (Young and Freedman 2012: 1292)

This image illustrates two examples of the same experimental set up – one on the left and one on the right – that differ only in their source light (the *light bulb with heated filament* on the left and the *lamp with heated gas* on the right). Due to their set up, and the similarity in their overall purpose, these two apparatuses are of the same type; they both illustrate sub-types of an experimental set up known as a *single slit experiment*. At first glance, then, the grammatical structure of the image displays a covert classificational taxonomy involving two Subordinates (shown in orange in Figure 5.10), but without explicitly showing the Superordinate (Kress and van Leeuwen 2006: 87).

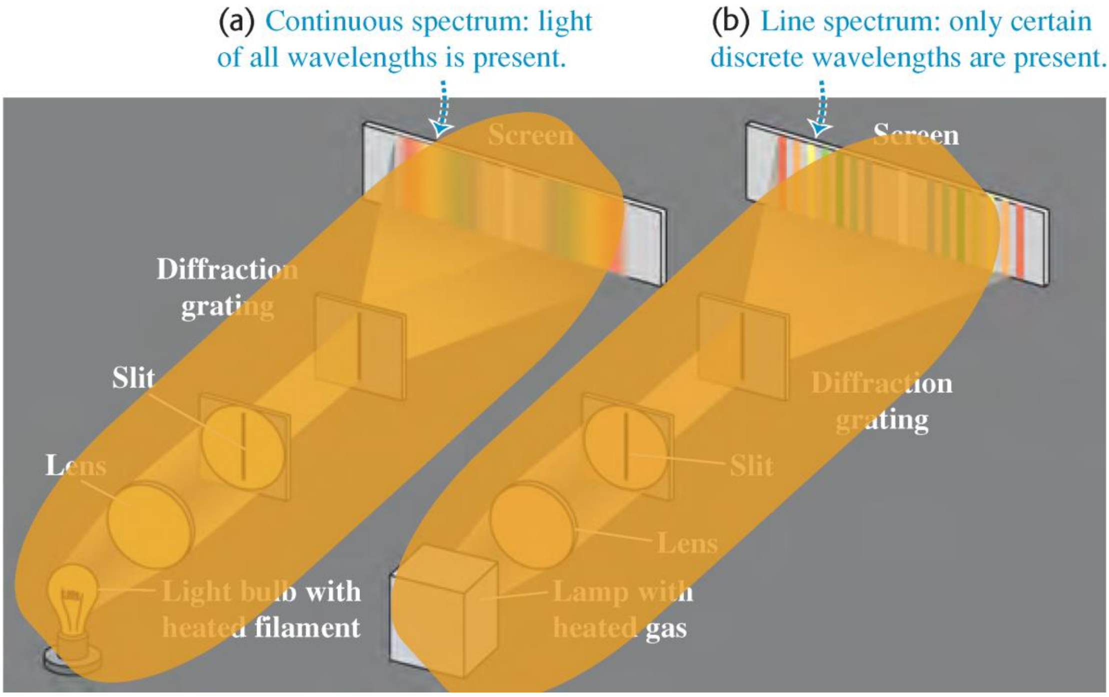


Figure 5.10 Classification analysis of experimental apparatus diagram (Subordinate in orange) (Young and Freedman 2012: 1292)

In terms of field, this image realises a classification taxonomy whereby each apparatus is a subtype of the single slit experimental apparatus. If we look further, however, it is clear these classification relations are by no means the only structures in the image. The image can also be read as an analytical image, displaying part-whole relations between each apparatus and their components. Figure 5.11 highlights these composition relations: light blue indicates the Carriers (the wholes) and purple indicates the Possessive Attributes (the parts).

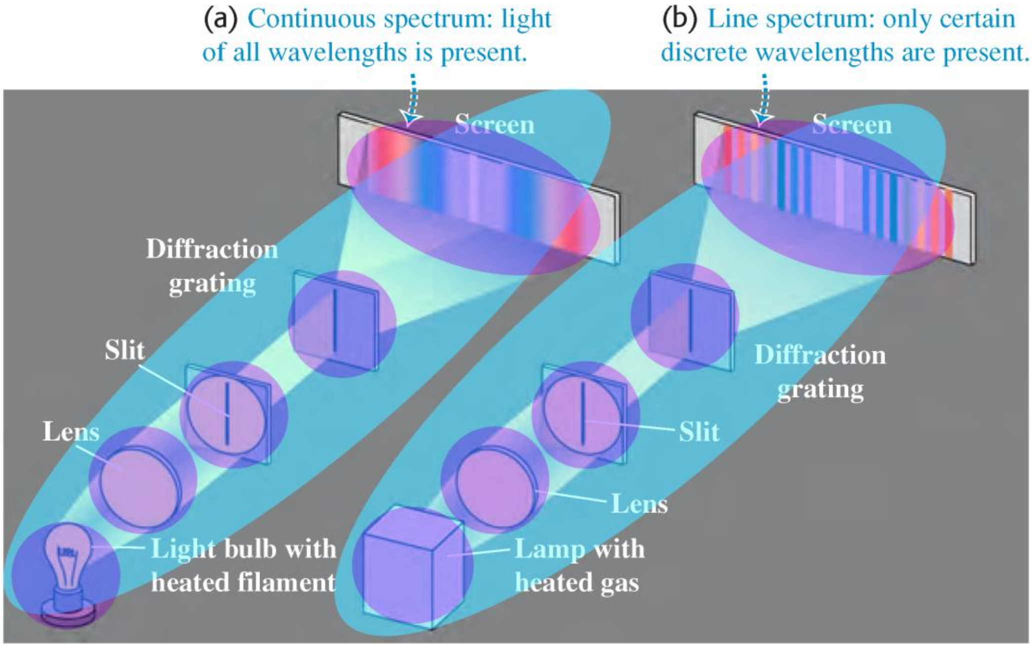


Figure 5.11 Composition analysis of experimental apparatus diagram (Carriers in blue, Possessive Attributes in purple) (Young and Freedman 2012: 1292)

This analysis shows that each apparatus contains five pieces of equipment: a lens, slit, diffraction grating, screen and light source. This similarity in composition justifies our previous analysis of the two apparatuses being of the same general type; indeed four of their components (the lens, slit, diffraction grating and screen) are exactly the same. The difference between the two set-ups comes from their choice of light source. The apparatus on the left utilises a light bulb with a heated filament, while the apparatus on the right uses a lamp with heated gas. This difference in a single component distinguishes the two apparatuses as different subtypes. This image therefore relates two interlocking taxonomies in one go; their composition taxonomies justify the classification taxonomy, and the classification taxonomy anticipates the composition taxonomy. It also shows that the apparatuses themselves function in both of these taxonomies as co-parts in a classification taxonomy and as wholes in two composition taxonomies.

As each apparatus is a different sub-type with a slightly different composition, the results of each experiment are different. This is encoded in the image through a narrative structure. This structure depicts the path of light from the source through each piece of equipment until it hits the screen. Figure 5.12 shows this narrative analysis (Source in pink, Vector in yellow, Goal in green and Resultative Attribute (see below) in dark blue).

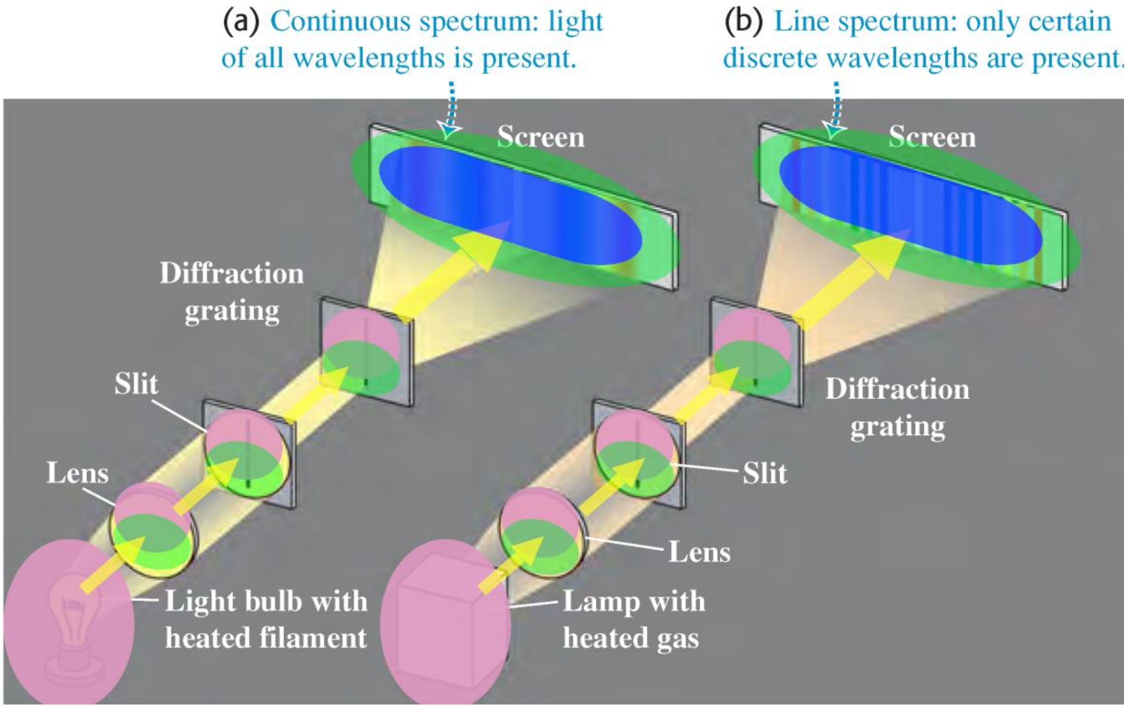


Figure 5.12 Narrative analysis of experimental apparatus diagram (Sources in pink, Vectors in yellow, Goals in green, Resultative Attributes in blue) (Young and Freedman 2012: 1292)

The analysis indicates that the Vector emanating from the light Source moves toward the lense, functioning as a Goal. The lenses also function as Sources from which another Vector of light emanates. This pattern continues for each piece of equipment until the final Vector reaches the screen, i.e. the final Goal. This structure thus involves four Vectors for each apparatus (justified by the fact that after each piece of equipment, the shape and size of the light changes, indicating distinct Vectors, rather than one continuous Vector).

In addition, we can note the difference in the patterns on each screen, labelled as: *(a) Continuous spectrum: light of all wavelengths is present* and *(b) Line spectrum: only certain discrete wavelengths are present*. These patterns are a result of the different narrative structures, in particular the different elements realising the original Source (the *light bulb with heated filament* and the *lamp with heated gas*). The differences in these patterns are important as they effectively present the results of the experiment. In lieu of an appropriate function in Kress and van Leeuwen's grammar that relates this pattern to the narrative analysis, we will consider these patterns to function as Resultative Attributes (analogous to

Resultative Attributes of English, such as *straight* in *he bent that rod straight*; Halliday and Matthiessen 2014: 327, Martin et al. 2010: 116). The Resultative Attributes are shown in dark blue above.

The image thus realises two activity sequences – one for each apparatus – in addition to the classification and composition taxonomies shown previously. These activity sequences involve light moving from each apparatus' light source to the lens, and then from the light source to the slit, and then from the slit to the diffraction grating, and finally from the diffraction grating to the screen, resulting in their particular light patterns (their spectra). The differences in each activity sequence (i.e. the different patterns on the screen) arise from the different apparatus' set ups (i.e. the particular sub-type of the apparatus and its corresponding composition). The particular activity sequences that occur are intertwined with the particular classification and composition taxonomies. Moreover they are all displayed in one image.

To give an idea of the amount of meaning presented in the image (not including labels), each analysis is overlayed in Figure 5.13.

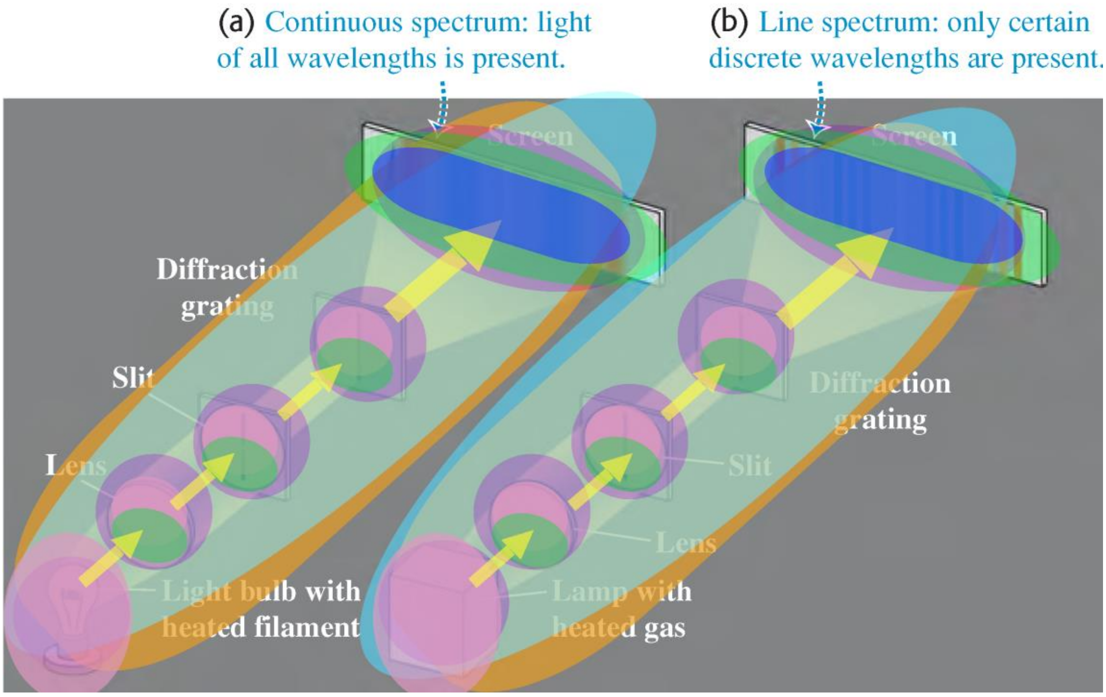


Figure 5.13 Full analysis of experimental apparatus diagram (Sources in pink, Vectors in yellow, Goals in green, Resultative Attributes in dark blue, Subordinates in orange, Carriers in light blue, Possessive Attributes in purple) (Young and Freedman 2012: 1292)

When analysed for each function impacting on field, the amount of meaning given in a single image becomes clear (and the analysis becomes hard to view in a single take). By realising multiple activity sequences, compositional taxonomies and a classification taxonomy all from the same field, each dimension is explicitly related.

If we take a further step to include the dozen labels in the image, the number of meanings displayed increases further. Each label indicates a synonymous relation between the linguistic label and the imagic element being labelled. The effect of this is twofold. First, elements that share the same label are seen to be the same. For example, the two pieces of equipment labelled *slit* are specified as being the same type of element. This further confirms the compositional analysis given previously, and thus also reinforces the unity between the apparatuses in terms of their place in the classification taxonomy. Second, the labels allow language and image to share meanings. The field-specific meanings associated with the linguistic technicality are linked with those associated with the elements in the image. For example, the distinction between the *continuous spectrum* and *line spectrum* as two sub-types in a classification taxonomy of *spectra* are related to the patterns displayed on each apparatuses' screen. As we have discussed, these patterns are determined by the particular activity sequence in each apparatus. This activity sequence is in turn determined by the compositional taxonomy of each apparatus, which is similarly determined by the different types of apparatus in the classificational taxonomy. Therefore from this single image, we can form an unbroken chain of relations between the separate classification taxonomy of spectra and that of types of apparatus, as well as the different activity sequences and composition taxonomies. A single image makes manifest the interlocking lattice of field-specific meaning.

This image is by no means unusual. It is in fact a relatively unremarkable image that would be easily understood by someone sufficiently trained in physics. In the following section we will consider an image that shows all of these meanings plus those realised by graphs. Such is the pervasiveness of images like this – especially at the higher levels of physics – that a presentation of this amount of meaning is a common occurrence.

This potential to realise field-based meaning has significant implications for the semantic density of physics discourse. By explicitly relating multiple field structures of activity and taxonomy, images can indicate tremendously strong semantic density in a single 'eyeful'.

This allows the field of physics to be extended (if these relations had not previously been made explicit), and also offers an efficient method of displaying this meaning. Through images, relatively large components of the field can be illustrated in a small stretch of imagic discourse. By sharing meaning with mathematics and language, each resource can utilise their own affordances to build the expanding network of meaning that constitutes hierarchical knowledge structure of physics.

Activity and taxonomy are not the only dimensions of field images may realise, however. In the following section, we will see that graphs bring forward further meaning potential not readily apparent in language, mathematics nor any of the images we have seen so far.

5.2.2 Graphs in physics

Graphs are regularly employed in physics to record measurements, illustrate patterns and highlight salient interrelations between technical meanings in physics. They allow a broad range of empirical observations to be related along multiple dimensions and establish a means for these relations to be incorporated into theory. Graphs first become prominent in junior high school before becoming regular features in senior high school and undergraduate university, and ubiquitous in research publications. Like diagrams, they display a rich and multifaceted functionality for organising the technical knowledge of physics. However the meanings they organise are of a different order to the taxonomy and activity we saw in the previous section. By virtue of their organisation, graphs expand the meaning potential of physics by realising new and distinct dimensions of field. This section will be concerned with highlighting these dimensions and characterising their specific roles in constructing the knowledge of physics. First, it will show that graphs order technical meanings along axes in order to utilise images' capacity for topological representation (Lemke 1998). This establishes arrays of meaning with the potential for a continuous gradation of empirical observations in terms of degree, quantity or amount. Second, it will highlight that through these arrays graphs enable patterns to be abstracted and generalised from empirical measurements. This reverses the direction of generality shown by mathematical quantifications and thus allows the empirical object of study to speak back to the theory of physics. Finally, it will show that like the diagrams discussed in the previous section, graphs can be added to other images to enrich the relations between field-specific activities, taxonomies, arrays and generality. From this we will see that the meaning potential of graphs complements that of mathematics, language and diagrams to extend the range of resources needed to construe physics' hierarchical knowledge structure.

Graphs exhibit a significant degree of variability. They can show single or multiple dimensions, they can arrange discrete points or continuous lines and they can specify precise measurements or relative degrees. Minimally, a graph is realised by a single axis that allows data points to be ordered along a single dimension. Figure 5.14 from a university textbook exemplifies a one dimensional graph such as this. This graph presents an array of light wavelengths known as the Balmer series (that are emitted from a transitioning electron in a hydrogen atom). It arranges a set of discrete points along the horizontal axis, with each point's relative position indicating its wavelength.

Figure 5.14 One dimensional graph of the Balmer series (Young and Freedman 2012: 1304)

Looking from field, the labels H_α, H_β, H_γ etc. suggest that each point on the graph is related through classification. They are each subtypes of H-lines (standing for hydrogen), with the far right being the H-α line, the next from the right being the H-β line, the next being the H-γ line and so on. In addition, they are all labelled numerically, as 656.3nm (nanometres), 486.1 nm etc. This suggests that these points sit at a relatively low level of generality, i.e. they represent empirical instances rather than generalised patterns. However these field-specific meanings of classification and generality are indicated by the labels, not by the layout of the graph itself. In fact its spatial layout establishes a different type of relation. This relation contrasts and orders the points in terms of their specific wavelength. Those to the right are construed as having a longer wavelength (also indicated by the larger number) than those to the left. Moreover, their relative distance apart specifies their relative difference in wavelength. For example, the larger gap between the H_α line (in red on the far right) and the H_β line (one to the left in blue) indicates a significantly larger difference in wavelength than that shown by the smaller gap between H_β and H_γ to its left. Although each point is a co-hyponym (co-type) in a classification taxonomy of emission lines and also realises a specific level of generality, this spatial arrangement realises a further relation. In terms of field, this relation can be interpreted as a field-specific array. Arrays organise technical meanings in a field along a particular dimension. In this case, the emission lines are being ordered along an array of wavelength. More generally, graphs primarily realise arrays through the spatial ordering of points or lines along an axis. Due to their facility for displaying topological meaning (Lemke 1998) images can in principle construe arrays with infinitely small degrees of gradation. This allows an indefinite number of terms to be related and, in the case of multidimensional graphs, offers the possibility of both continuous and discrete variation.

One dimensional graphs such as Figure 5.14 are relatively infrequent in the discourse of physics. More commonly, graphs are presented with two intersecting dimensions. These graphs are known as Cartesian planes. Figure 5.15 illustrates a two dimensional Cartesian plane used in an undergraduate university lecture (but originally sourced from an art project focusing on global warming, Rohdes 2007). The graph presents the range of wavelengths of light emitted by the Sun and arriving at the Earth. It arranges two sets of points, shown by the red and yellow bars. The yellow bars indicate the spectral irradiance emitted by the sun (crudely, the amount of sunlight) that hits the top of the atmosphere, while the red bars indicate the spectral irradiance that travels through the atmosphere and hits sea level.

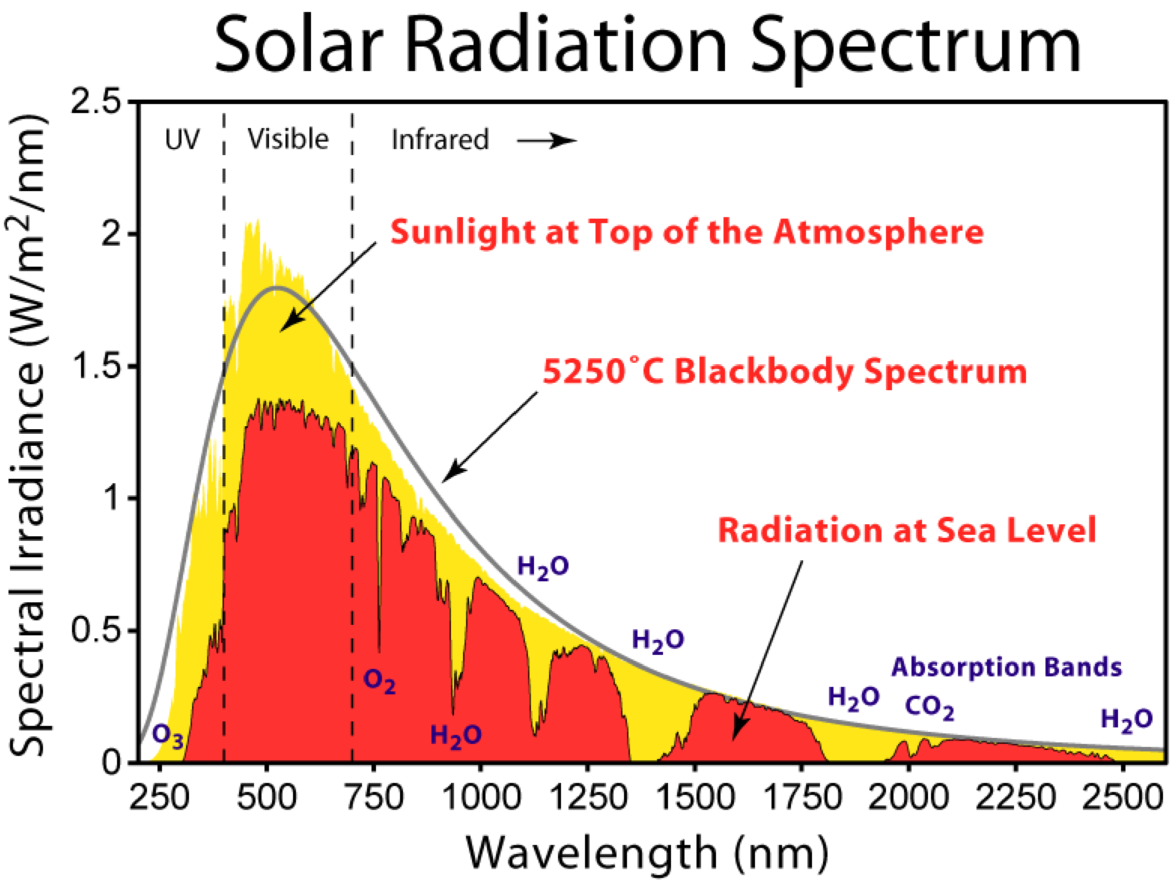


Figure 5.15 Solar radiation spectrum (Rohde 2007)

This graph coordinates two axes, the vertical y-axis, labelled *Spectral Irradiance (W/m^2/nm)* and the horizontal x-axis labelled *Wavelength (nm)*. By presenting two dimensions, each point is characterised by two variables: its spectral irradiance measured in W/m^2/nm (read as Watts per square metre per nanometre) and its wavelength in nm (nanometres, a billionth of a metre). For example, the red bar (labelled *radiation at sea level*) at a 500nm wavelength has a spectral irradiance of ~1.4 W/m^2/nm. As can be seen, each point in the graph shown by the red or yellow bars is miniscule. This means the arrays present very small gradations in relation to each other and allow a great deal of precision to be encapsulated in the field. In this particular field, this graph establishes an interrelation between the two arrays of spectral irradiance and wavelength. Thus one of the realisations of the field of the solar radiation spectrum is that each value of wavelength will have the specific value of spectral irradiance specified by this graph.

The arrangement of points into arrays directs us to the second feature of graphs that is significant for knowledge in physics: its potential for increasing generality. Both the yellow and red bars present empirical observations, i.e. relatively specific measurements of spectral irradiance for each wavelength based on tables published by the American Society for Testing and Materials (2012). In terms of field, they represent relatively low generality. However by arranging these measurements along an array, the graph abstracts a general pattern of change. Both spectral irradiances peak around 500nm wavelength, drop off quickly at lower wavelengths (on the left), but more slowly at higher wavelengths (on the right). The graph presents this general pattern in the form of a line, shown in grey and labelled *5250° C Blackbody Spectrum*. This line represents the spectral irradiance vs wavelength for a theoretical construct known as a black-body. By fitting this line to the empirical measurements, the graph portrays the solar spectrum as approximating that of a black-body (specifically, a black-body at a temperature of 5250° C). It relates the empirical and low generality measurements to the theoretical and higher generality black-body spectrum. The graph thus offers the potential to abstract generalised theory from physical observations, i.e. to move from low to high generality.

In addition, by overlaying the set of red points on the yellow points, the graph highlights a second dimension of generalisation. As the graph states, the yellow bars represent the sunlight that hits the top of the atmosphere. On the other hand, the red bars represent the sunlight that makes it through the atmosphere to sea level. The difference in height (spectral irradiance) between the yellow and red bars signifies the amount of light that is absorbed by the atmosphere and thus does not reach sea level. Whereas the array of light at the top of the atmosphere (yellow) closely resembles the idealised line of the black body, the light at sea level (red) is much less smooth. The red displays gaps and bumps where the yellow doesn't. These gaps indicate wavelengths where the absorption is highest, i.e. where the atmosphere stops the most light. Importantly, these gaps are empirical differences born of observation. By layering the red and yellow measurements on top of each other, the graph compares the two by labelling the gaps *absorption bands*. Each absorption band is then given a specific classification (H_2O, CO_2, O_2 and O_3) that signifies the molecule that does the absorbing. The graph therefore groups empirical measurements and generalises them into a classification taxonomy. In doing so, it again construes new higher generality field specific relations from lower generality observations.

As this figure shows, graphs present opportunities for heightening generality. They allow arrays of specific measurements to be generalised into patterns, which then opens the path for these patterns to be abstracted into other field relations (such as classification). However, this is not to say that graphs only allow a shift from low to high generality. The nature of images is such that this reading path can be reversed; we could have begun at the generalised black-body line and moved to the empirical observations. But it does illustrate that graphs offer opportunities to shift from lower to higher generality. This movement contrasts with that afforded by mathematical quantifications. Mathematics organises movements from generalised theory in the form of implication complexes to specific measurements in the form of numbers (from high generality to low generality). Graphs, on the other hand, offer the possibility of movements from specific measurements to generalised theory (from low generality to high generality). In LCT Semantics terms, physics thus may both strengthen and weaken semantic gravity. Quantifications present movements from weaker to stronger semantic gravity, while graphs present movements from stronger to weaker semantic gravity. Put another way, quantifications offer a tool for gravitation, graphs for levitation (following the terminology used in Maton 2014). The two resources are thus complementary for physics' knowledge structure. Together, they provide the means for the theory of physics to reach toward its empirical object of study, and for the empirical to speak back to the theory.

In terms of semantic density, the arrays in graphs allow an enormous set of measurements with indefinitely small gradations to be related along a single dimension. This supports relatively strong semantic density and bolsters the range of empirical phenomena that can be encompassed in a single image. Moreover, semantic density can be strengthened by the abstraction of field-structures such as taxonomies, as greater constellations of meaning are assembled. The fact that, like diagrams, graphs can be combined with other structures in a single image further expands the strongest potential for semantic density. Series of activities, taxonomies, arrays and levels of generality can be presented in a single image, offering great power for integrating physics' knowledge structure. To illustrate this, we will consider in detail at Figure 5.16, an 'energy level diagram' from a university physics textbook. This image illustrates a set of possible energy transitions available to an electron in a hydrogen atom.

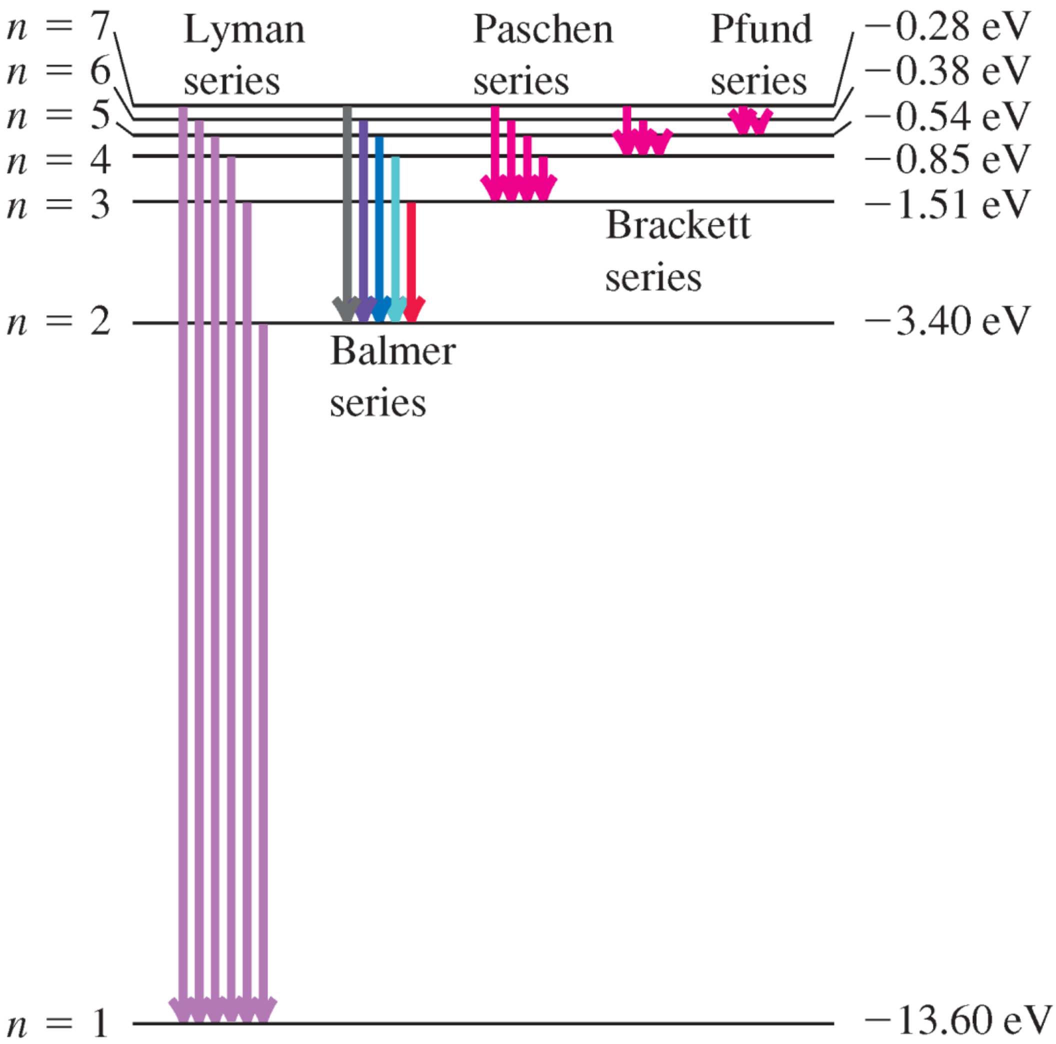


Figure 5.16 Energy level diagram for a hydrogen atom (Young and Freedman 2012: 1303)

First, this figure presents a one dimensional graph. It arranges its points, shown by horizontal lines, along the vertical axis and measures them in terms of their energy (e.g. −3.40 eV, −13.60 eV). It thus construes an array of energy levels in the hydrogen atom. Figure 5.17 below highlights this by displaying the vertical axis with a red line and highlighting each point light blue.

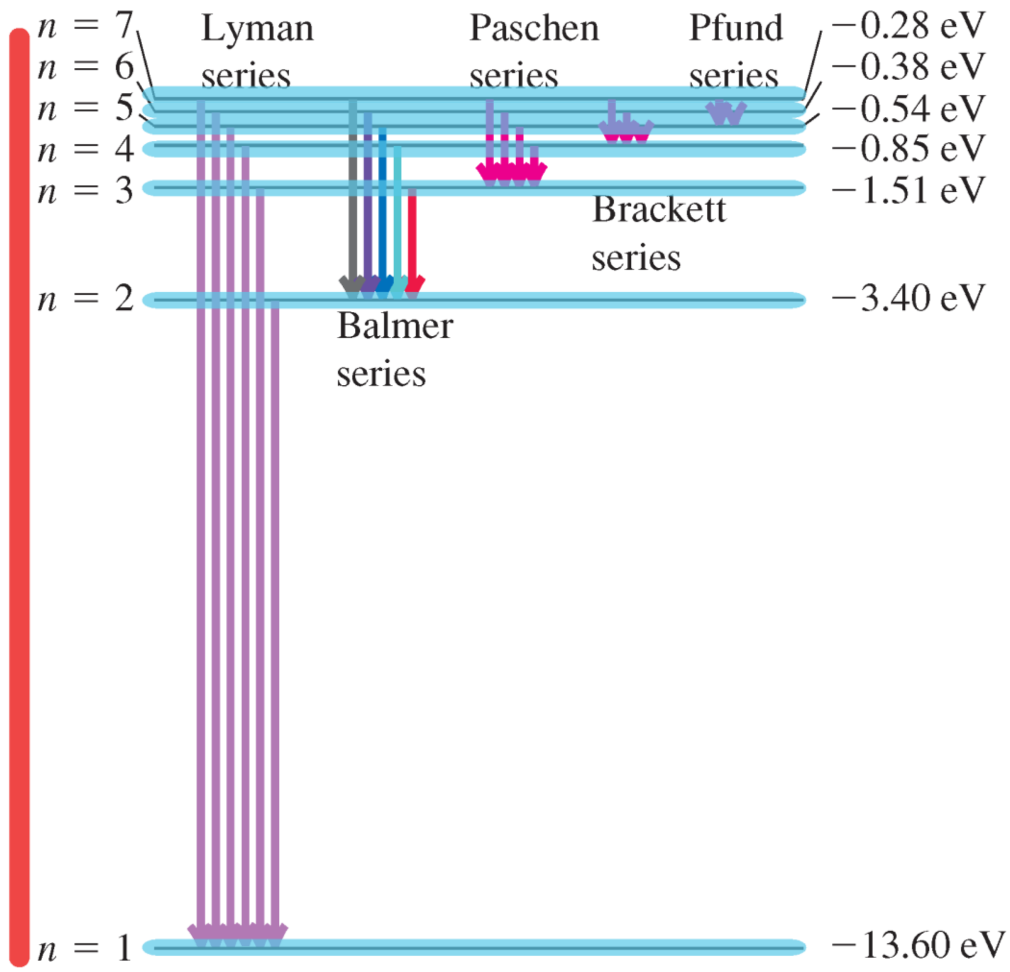
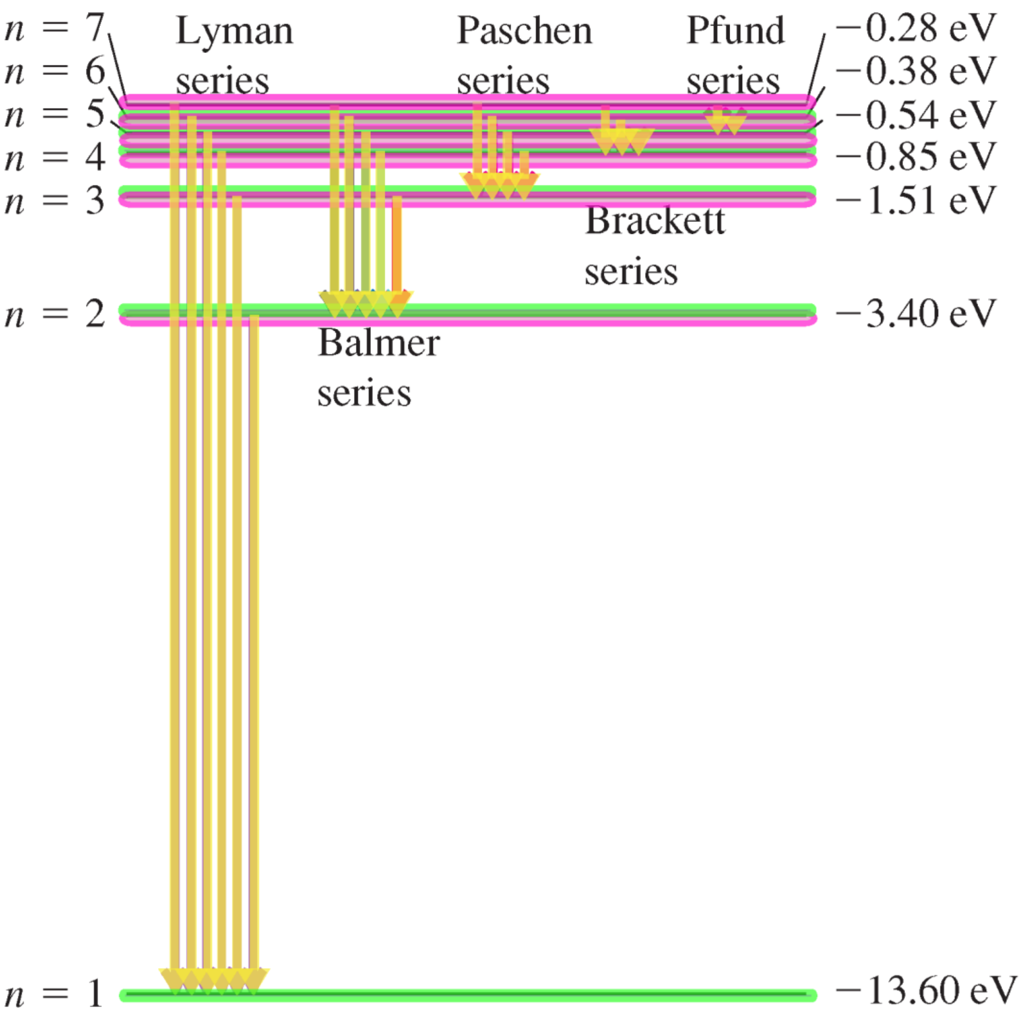


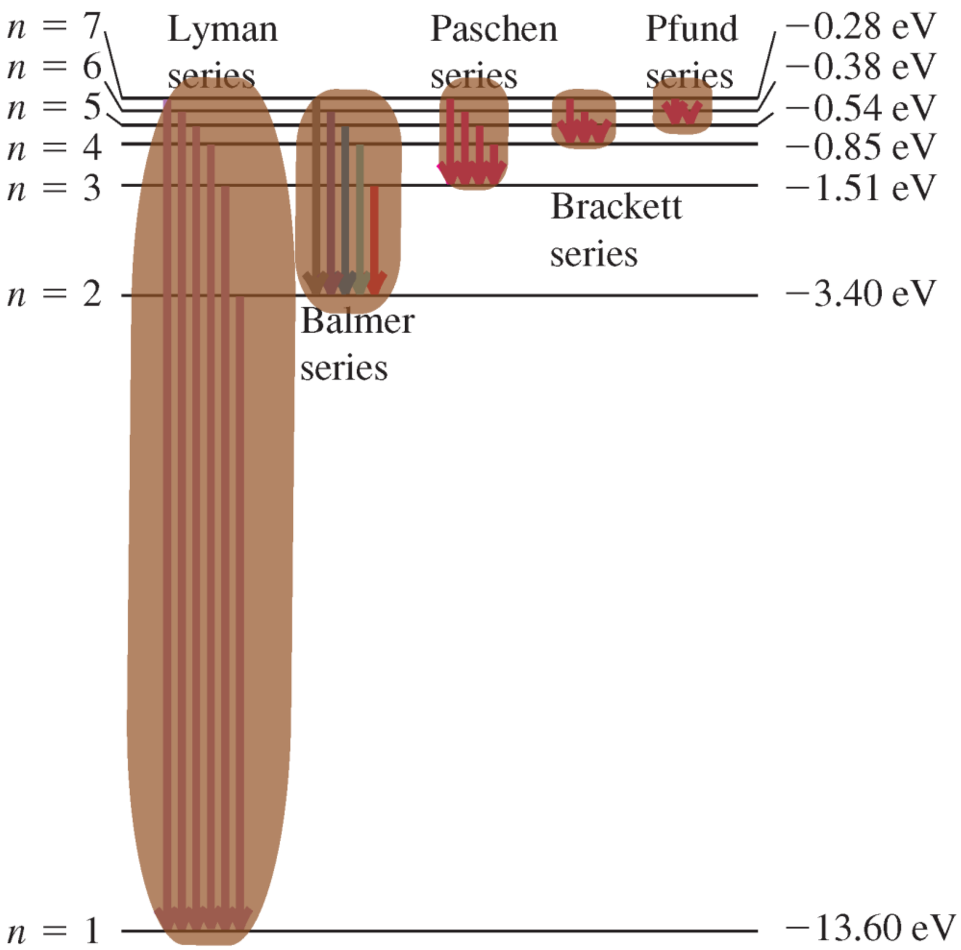
Figure 5.17 Graph of energy levels in a hydrogen atom
(Axis in red, Points in blue)
(Young and Freedman 2012: 1303)

In addition, the presents a narrative image with the series of arrows indicating a number of Vectors. Each Vector arrow (highlighted yellow in 5.18 below) emanates from a point on the graph. These points thus function as Sources (highlighted in pink). Additionally, each Vector moves toward another point on the graph, which function as Goals (highlighted in green). As all points except two (the top and the bottom) have both a Vector emanating from it and a Vector moving toward it, these function as both a Goal and a Source. 5.18 illustrates this reading below.



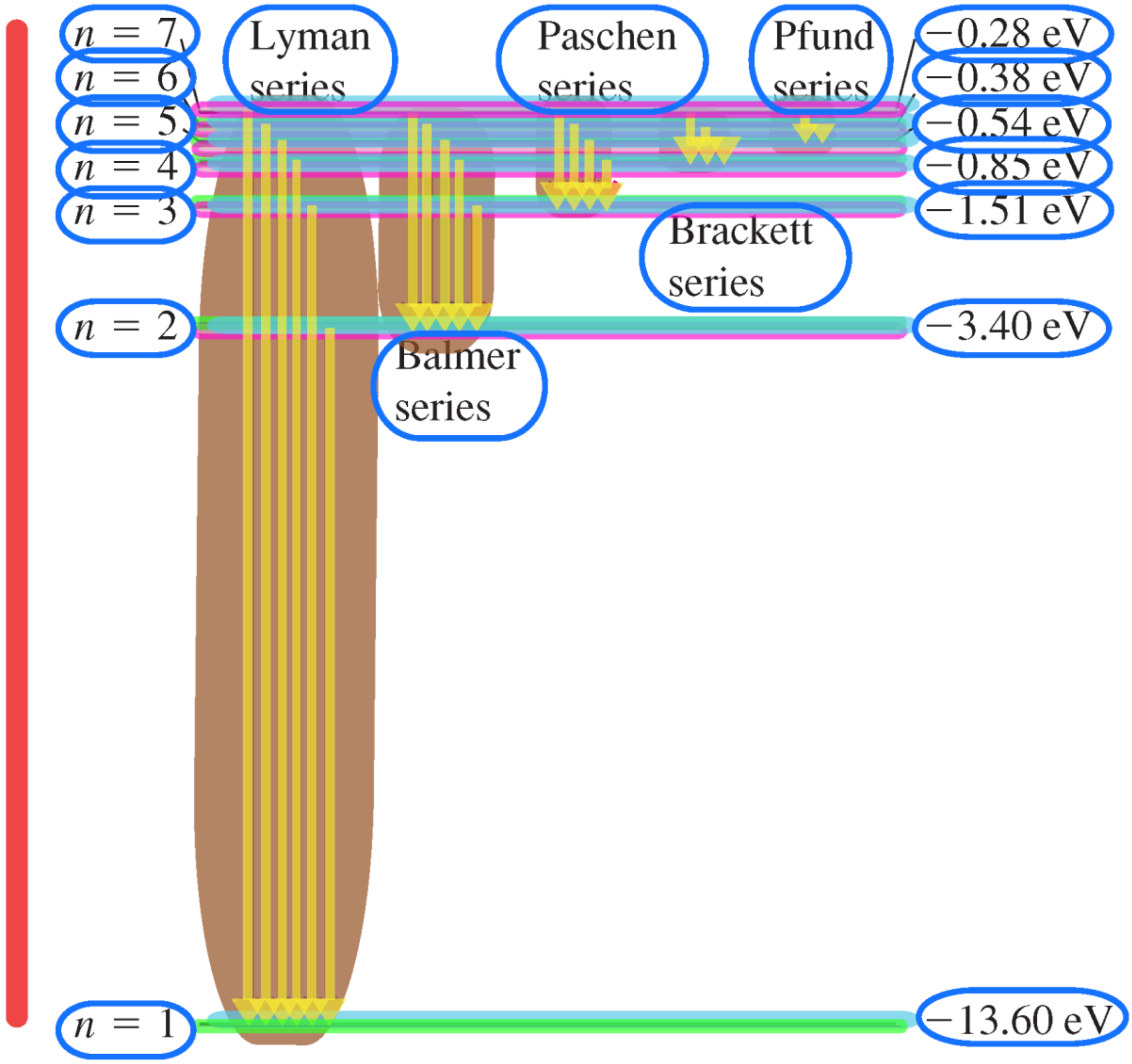
5.18 Narrative analysis of an energy level diagram of the hydrogen atom (Vectors in yellow, Sources in pink, Goals in green) (Young and Freedman 2012: 1303)

The twenty Vectors and their respective Goals and Sources in this image realise a large number of activities. Each activity corresponds to a transition from a particular energy level to another energy level. As there are twenty Vectors, but only seven points from which a Vector can emanate or transition toward, many activities share the same beginning or end point. This forms the basis for the image to present a classification taxonomy. Each Vector is grouped according to its end point (its Goal, $n = 1$, $n = 2$ etc.) and is labelled as a type of *series*. Those moving toward $n = 1$ are labelled the *Lyman series*, those moving toward $n = 2$ are labelled the *Balmer series*, those moving toward $n = 3$ are labelled the *Paschen series* and so on. The end result is a classification taxonomy with three levels of delicacy. The most general superordinate arises from the fact that each arrow is structured the same way and is labelled as part of a series. It suggests that at some level all of the transitions (arrows) are of the same type (they are electron transition lines). At the second level, the image presents five sub-types of transition lines according to their end-point, each of which are labelled. The five subtypes of transition lines are the Lyman transition lines, the Balmer transition lines, the Paschen transition lines, the Brackett transition lines and the Pfund transition lines. Finally the third level within each subtype of transition line includes the specific transitions distinguished by their starting point. The Lyman transition series includes six lines, the Balmer includes five, the Paschen includes four and so on. In all, the image realises a three level classification taxonomy that includes twenty-six nodes. This classification analysis is shown in 5.19. In the interest of readability, only the second level groups of transition lines are highlighted.



5.19 Classification analysis of an energy level diagram of the hydrogen atom (Superordinate in brown) (Young and Freedman 2012: 1303)

This three-level, twenty-six node classification taxonomy supplements the twenty activities being realised, the seven points on the graph and the nineteen labels. The image thus encodes a large degree of meaning for what may, at first glance, look like a relatively simple diagram. The full analysis, highlighted to show classification, activity and graphs, and with each label circled (showing synonymy with language and mathematical symbolism) is given by 5.20. The highlighting helps to make explicit the semantic density of the image.



5.20 Full analysis of an energy level diagram of the hydrogen atom (Superordinates in brown, Vectors in yellow, Sources in pink, Goals in green, Axis in red, Points in blue, Labels circled) (Young and Freedman 2012: 1303)

Once each dimension of meaning is highlighted, the entire image is coloured or circled. Each highlight realises a distinct structure in field. This is relatively typical of many images in physics. They provide a means to synoptically integrate meaning, with little extra information given that is superfluous to the technical meaning of the field. Just like mathematics is geared toward construing ideational meanings, so are images in physics. The full analysis also highlights the number of relations presented in the image by offering a path from the array to the activities to the classification taxonomy. The points on the array function as the beginning and end points for activities involving electrons transitions (though the electrons aren't shown).[54] Through the similarity in end-points, the image organises the arrows into a classification taxonomy. The different types within this taxonomy are then labelled, allowing these field-meanings to be discussed in language. By presenting an array, a taxonomy and a series of activities, this image realises much of the field-specific meaning associated with hydrogen atom electron transitions in a single snapshot.

Images clearly hold great power for organising the knowledge of physics. In LCT terms, this offers physics strong semantic density by allowing a significant spectrum of phenomena to be encapsulated and vast swathes of technical meaning to be combined. Further, they offer a large range of semantic gravity. They can present empirical measurements or generalised theory, and illustrate a pathway between both. In this way, they complement the gravitation of mathematical quantifications by affording a tool for levitation (weakening of semantic gravity). Finally, they can be labelled by both mathematics and language, and thus allow the meanings developed in one resource to be expanded in another. In sum, images allow meaning to be related and proliferated while maintaining contact with the empirical world.

5.3 Field and the knowledge structure of physics

Mathematics, images and language all display a powerful utility for physics. They each utilise their own ways of meaning, their own types of texts and their own functional organisation to construe physics' intricate and multifaceted knowledge. This chapter has considered images and mathematics from the perspective of field in order to develop a

[54] These transitions are not movements in space – the electron does not go up or down within the atom – rather they represent changes in energy of the electron. This meaning can only be garnered through the relation between the graphical array (showing energy) and the narrative-based activities.

means of comparison with each other and with language. Viewing each resource from the same vantage point allows a consistent basis for understanding the disciplinary affordances (Fredlund et al. 2012) images, language and mathematics exhibit for physics. The chapter has shown that while each resource demonstrates its own functionality, many of the same dimensions of field can be realised across language, mathematics and image. For example, language and image can both realise taxonomy, language and mathematics can both construe types of implication and mathematics and image can both organise differing levels of generality. This shared possibility for meaning has implications for the broader stratal framework proposed in this thesis as it suggests a common stratum of register realised by language, mathematics and image may be productive for understanding their complementarity. Further to this, by examining each resource in relation to field of physics, we can approach the knowledge structure of physics in a more holistic way. We can see its potential for shifting semantic gravity and developing semantic density, and we can recognise the influence this has on the breadth of empirical phenomena integrated in the discipline. The final chapter will be focused on these two themes of disciplinary affordance and knowledge structure. First, it will overview the disciplinary affordances of language, image and mathematics in relation to the field of physics. Second, it will consolidate the work presented in this thesis to characterise role of each resource for the knowledge structure of physics as a whole. And finally, it will look to the broader implications of this thesis for our understanding of knowledge and semiosis.

CHAPTER 6

Multisemiosis and the Knowledge Structure of Physics

Physics organises its knowledge structure through language, mathematics and image. This thesis has explored the meaning-making potential of these resources and the role they play in construing this knowledge. As part of this exploration, it has also developed a detailed description of mathematics that brings out its specific functionality, and has illustrated Systemic Functional descriptive principles that allow theoretical categories to be derived and tested.

This final chapter consolidates the models developed in this thesis and considers some of the broader ramifications of this research. Section 6.1 focuses on multisemiosis in relation to the knowledge of physics. First, it brings together the descriptions of mathematics, language and image to examine the disciplinary affordances (Fredlund et al. 2012) for each resource in organising the knowledge of physics. Second, it considers physics as a whole and views its knowledge as a unified structure. Following this, Section 6.2 considers the broader descriptive issues this thesis has raised. In particular, it considers claims of the pervasiveness of metafunctionality across semiosis and the role of the connotative semiotics of genre and register in unifying multisemiotic descriptions. Finally, Section 6.3 looks ahead to the research avenues highlighted by this thesis in relation to developing a broader semiotic typology and a more comprehensive understanding of physics.

6.1 Knowledge and multisemiosis in physics

As the previous chapters have shown, physics displays the features needed for a hierarchical knowledge structure. It can both develop abstract theory and explicitly relate this theory to the empirical world. In construing this knowledge structure, mathematics, images and language all play crucial roles due to their specific semiotic affordances. This section brings the threads together from the previous chapters to present a multisemiotic picture of the knowledge structure of physics.

6.1.1 Disciplinary affordances of language, mathematics and images

Language, mathematics and images are consistently used in physics schooling and research. Based on the discussion in the previous chapters, we are now in a position to suggest why. By looking from field, we can see the types of meanings each resource specialises in and the types of text that organise these meanings. As we have seen, the multifaceted nature of physics' knowledge is such that no resource by itself adequately construes it all; where one resource falls short, another takes over. The result is a complementarity, whereby the interplay of language, mathematics and image expound and expand the field of physics.

Mathematics specialises in two field-specific relations: building large and interconnected implication complexes and measuring specific instances from generalised theory. First, mathematics establishes vast networks of interdependency that constitute much of the theoretical underpinning of physics. Each statement specifies only a small subset of these networks; however the symbols within statements regularly condense large sets of relations in related implication complexes. These implication relations can be made explicit through derivations, allowing the field to progress by expanding the range of phenomena it integrates. Second, mathematics enables movements in generality from established theory to empirical measurements. Through quantifications, generalised statements predict and describe specific instances, which in turn correspond to distinct field-specific meanings. These shifts in generality allow physics to maintain contact with its empirical object of study and ensure that evolving theory makes testable predictions about the physical world.

Mathematics' grammar is uniquely devoted to realising both implication complexes and levels of generality. Its logical component builds large univariate structures and establishes covariate relations between each symbol in a statement. This gives rise to large implication complexes that link much of physics' technical meaning. Its operational component distinguishes between numbers and pronumerical symbols. The distinction means that mathematical symbolism grammaticalises differences in generality; pronumerical symbols realise higher generality while numbers realise lower generality. The logical and operative components of mathematics' grammar thus display powerful functionalities.

They do not, however, cover all aspects of the field. Aside from implication and generality, mathematics is limited in the field meanings it can realise. Although it can indicate small classification taxonomies through subscripts (for example E_i and E_f suggest two types of E) and can present arrays by arranging numerical measurements arranged into tables of various sizes (illustrated in Table 6.1 from a university textbook), these strategies are relatively marginal in the data under study. The subscript grammar only allows a two-level taxonomy to be specified and large arrays are more commonly displayed in graphs. In essence, mathematics is primarily devoted to realising implication complexes and movements in generality; it does not establish elaborated classification or composition taxonomies, nor does it present activity sequences.

$Z =$ atomic number (number of protons)
$N =$ neutron number
$A = Z + N =$ mass number (total number of nucleons)

Nucleus	Z	N	$A = Z + N$
$^{1}_{1}H$	1	0	1
$^{2}_{1}H$	1	1	2
$^{4}_{2}He$	2	2	4
$^{6}_{3}Li$	3	3	6
$^{7}_{3}Li$	3	4	7
$^{9}_{4}Be$	4	5	9
$^{10}_{5}B$	5	5	10
$^{11}_{5}B$	5	6	11
$^{12}_{6}C$	6	6	12
$^{13}_{6}C$	6	7	13
$^{14}_{7}N$	7	7	14
$^{16}_{8}O$	8	8	16
$^{23}_{11}Na$	11	12	23
$^{65}_{29}Cu$	29	36	65
$^{200}_{80}Hg$	80	120	200
$^{235}_{92}U$	92	143	235
$^{238}_{92}U$	92	146	238

Table 6.1 Arrays of atomic, neutron and mass numbers of various atoms (Young and Freedman 2012: 1441)

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In contrast, images are more elaborate in the field-specific meanings they can realise. Diagrams display activity sequences in addition to organising classification and composition taxonomies. Graphs present large arrays and offer a counterpoint to the movements in generality shown by mathematics. Moreover each of these assemblies of relations can be illustrated in a single snapshot. The power of images is not, however, boundless. Images do not make a clean distinction between expectancy and implication sequences and they do not efficiently encode the large implication complexes condensed in mathematics. Nonetheless, their wide-ranging functionality is a possible explanation for their ubiquity throughout most facets of physics.

Through image and mathematics, most of the field-specific meaning of physics can be realised. Implication complexes, activity sequences, classification and composition taxonomies, arrays and generality are all covered. The question then arises, why use language? An exhaustive discussion of this would traverse most of the history of linguistics. But one reason we would highlight here is that whereas both image and mathematics are restricted in the range of meanings they can show, language can in some sense realise them all.[55] As has been extensively documented (e.g. Halliday and Martin 1993; see also Chapter 2) language can realise elaborate composition and classification taxonomies, as well as long and intricate activity sequences. Furthermore, it can distinguish expectancy from implication sequences (a distinction notably absent in images). In addition, it can provide a less technical yet more accessible precursor to the implication complexes of mathematics. This can be seen in Halliday's classic example *braking distance increases more rapidly at high speeds* (1998: 225). In this example *braking distance* and *speed* are being related through a dependency relation, much like would happen in mathematics. However this dependency is not precisely specified. Language is thereby establishing an implication relation, but not fully specifying what it is. This understanding also allows us to reinterpret Zhao's (2012) 'causation' taxonomic relations between terms such as *force* and *acceleration* (see Chapter 2, Section 2.2) as implication relations specified through language.

As well as taxonomy, implication and expectancy, language can also indicate differing levels of generality through generic and specific reference (Martin 1992: 103). For example, <u>*electrons*</u> *in circular motion are accelerating* (generic reference) and <u>*this electron*</u> *is accelerating* (specific reference) differ in the generality of the electron they are specifying; the former indicates a more general form of the latter. As Martin (1992: 103) argues 'generic reference is selected when the whole of some experiential class of participants is at stake rather than a specific manifestation of that class.' Finally language can display contrastive meanings that to a certain extent realise arrays. This is illustrated by:

[55] A possibly more powerful reason is that language appears to offer greater variation in tenor and mode than both mathematics and image. For example it allows a larger range of negotiation and appraisal options crucial for realising tenor relationships, and being both spoken and written it can either accompany or constitute the social action. Detailed tenor and mode considerations are beyond the scope of this thesis however.

> The observation that atoms are stable means that each atom has a *lowest* energy level, called the **ground level**. Levels with energies greater than the ground level are called **excited levels**.
>
> (Young and Freedman 2012: 1297-8, original emphasis)

Among other things, this text establishes a classification taxonomy of energy levels with the subtypes *ground level* and *excited levels*. The basis for classifying these levels arises from their position in an array of energy levels. The *ground level* has the lowest energy, while each *excited level* has a higher energy. Although not specified in this extract, each excited level (known as the $n = 2$ *level*, $n = 3$ *level*, $n = 4$ *level* etc.) also corresponds to a particular amount of energy. In distinguishing between the ground level and the excited levels, the language establishes an energy level array, albeit a small and relatively imprecise one.

To some extent therefore, language can realise each type of field-specific meaning needed in physics: expectancy, implication, composition, classification, generality and array. However as shown for implication complexing and arrays, it does not necessarily achieve this with the same precision as mathematics and image. If physics requires more elaborated and interconnected implication relations, it can turn to mathematics; if it requires more intricate and detailed arrays, it can introduce images. Inverting Lemke's (2003) formulation, we can therefore view language as straddling the more specialised functionalities of mathematics and image; language is the Jack of all trades, but only the master of some.

Table 6.2 consolidates the field based affordances of each resource and indicates the typical genres or types of image that are focused on developing each of these meanings in physics (linguistic genres are taken from Martin and Rose 2008). As the table shows, many field-based meanings can be elaborated in each of image, language and mathematics systems. However each resource construes these meanings through their own grammars, genres and methods of organisation. This breadth of realisation provides a range of ways for knowledge to be organised in physics.

		Mathematics	**Language**	**Image**
Activity	**Expectancy**		*Yes* through procedures	*Sequences* through diagrams (No clear expectancy and implication distinction)
	Implication	*Complexes* through derivations	*Sequences* through explanations	
Taxonomy	**Composition**		*Yes* through compositional reports	*Yes* through diagrams
	Classification	*Minimal* (specified by subscripts)	*Yes* through taxonomic reports	*Yes* through diagrams
Array		*Yes* through tables	*Yes* (no clear genre as yet apparent)	*Yes* through graphs
Generality		*Yes* through quantifications (high to low generality)	*Yes* (no clear genre as yet apparent)	*Yes* through graphs (low to high generality)

Table 6.2 Field affordances of language, mathematics and image for physics

6.1.2 The knowledge structure of physics

The survey of mathematics, image and language undertaken in this chapter reveals their capacity for organising the knowledge of physics. Although each resource brings its own ways of meaning and its own types of texts, their complementarity enables the overall knowledge structure of physics. Through their interaction, technical meanings built in one resource can be shared and consolidated among the others. In this way, physics can utilise the specific disciplinary affordances of each resource to evolve its understanding of the material world. As suggested in Chapter 5, for physics to be characterised as a hierarchical knowledge structure, it should display the capacity to develop ever more integrative and general propositions and to generate relatively unambiguous empirical descriptions. In LCT terms, this involves building relatively strong semantic density across a wide range of physical phenomena at the same time as managing a large range of semantic gravity that tethers theory to its empirical objects of study. This necessitates tools that can expand theory and consolidate data, and is achieved through the interplay of mathematics, language and image.

Language is the first resource to develop technicality in physics schooling. It arranges technical terms through part-whole and type-subtype relations and thereby establishes deep taxonomies of composition and classification. In addition, it organises these terms into long sequences of activity that are primarily related through cause and effect. As decades of SFL research on science has shown, these activities can be packaged as single participants through both grammatical metaphor and activity entities, which can in turn function as technical terms in the field (Halliday and Martin 1993, Martin and Veel 1998, Halliday 2004, Hao 2015). Each technical term is precisely related to its surrounding composition and classification taxonomies, and its specific activity sequences. As more technical meaning is built on top of existing technicality, the constellation of meaning in physics becomes increasingly integrated while the range of phenomena accounted for expands. Language thus construes a significant degree of semantic density for physics and establishes a solid base upon which images and mathematics can extend this knowledge.

Through their interaction with language, images garner technical meaning which can then be further related to a large number of other meanings in a single snapshot. Single images often make explicit multiple levels of taxonomies and large activity sequences, and thus clearly illustrate large segments of the knowledge constellations of physics. By presenting broad swathes of technical meaning across activity and taxonomy, images can display synoptic glimpses of the overall semantic density of physics. In addition to reflecting this complexity, images also contribute semantic density over and above that created by language. On the one hand, images establish further relations between different types of meaning by presenting multiple activities and taxonomies overlaid onto each other. On the other, images more readily arrange technical meanings into indefinitely gradable arrays. This adds a further set of meanings to the field and elaborates the constellation of physics knowledge with which each instance of technicality is interconnected. Images thus both illustrate and magnify the semantic density of physics.

Finally, mathematics amplifies physics' semantic density by adding elaborated implication complexes. Like images, mathematics garners its initial technical meaning primarily from language. Technical symbols are then related to an indefinite number of other symbols in mathematical statements. Due to the enormous combinatory potential of mathematics, these implication complexes become indefinitely large, resulting in each symbol invoking a complex assembly of meanings. Each symbol thereby holds significant semantic density for physics. Compounding this, derivations utilise these invoked implication complexes to build further technical meanings. Mathematics thus arranges large constellations of meaning in physics and embeds a tool for expanding these constellations. It both presents and constructs the strong semantic density of physics.

Working together, the interaction of images, mathematics and language fosters both strong semantic density and strong condensation. The constellations of physics involve deep composition and classification taxonomies, long activity sequences, intricate implication complexes and multidimensional arrays, with many technical entities invoking all at once. Once meaning is built in one resource, it can be shared with another to expand and amplify physics' constellation of meaning, and then be returned over and again as further meaning is consolidated and condensed. This process of give and take enables meanings to proliferate and theoretical understandings to be integrated. It allows an expanding range of phenomena to be incorporated into generalised theory, and thus lays the foundation for constructing physics' hierarchical knowledge structure.

Complementing this, the multimodal realisation of physics also provides pathways for theory to connect with its physical object of study. That is, in addition to condensation, it provides tools for gravitation (strengthening semantic gravity) and levitation (weakening semantic gravity). On the one hand mathematics allows movement from generalised theory to numerical instance through quantifications. This decrease in generality ensures theory can be tested by data and data can be predicted by theory. Mathematics' gravitation thus ensures the proliferation of theory maintains relevance for the physical world. On the other hand, graphs offer a path from measured data to abstracted theory by generalising patterns from observed instances. This complements gravitation by providing a tool for levitation. Through mathematics and images, physics can thus move from theory to data and from data to theory; the empirical object of study can be used to develop theory and the theory can be used to predict and describe the object of study.

Through language, mathematics and images, the knowledge of physics undergoes condensation, gravitation and levitation. The division of labor across these resources lays the platform for physics' hierarchical knowledge structure by incorporating tools for creating general propositions and theories, and integrating knowledge across a range of phenomena (Bernstein 1999). Moreover, by creating a pathway between theory and data, these resources provide the means for physics to strengthen its epistemic relations between its knowledge and its object of study (Maton 2014). Through its use of mathematics, language and image, physics can broaden the horizons of knowledge and ensure that knowledge keeps in touch with the physical world.

6.2 Semiotic description

Chapters 2 and 3 proposed a series of principles for developing descriptions in Systemic Functional Semiotics. These principles took the paradigmatic and syntagmatic axes as theoretical primitives from which the larger theoretical architecture of semiotic resources can be derived. Utilising these principles allows claims such as the pervasiveness of metafunctions to be examined, and the rank/nesting and stratal frameworks to be justified. Chapters 3 and 4 used these principles to build a model of mathematics' grammar and a description of the genres of mathematics and language. These models highlighted significant differences between mathematics and language in terms of their metafunctional organisation and level hierarchies, but also the potential utility of genre as a unifying stratum for the two resources. These observations have far-reaching implications for Systemic Functional Semiotics.

6.2.1 Metafunctions and levels in mathematics

By beginning with axis, the grammatical description in Chapter 3 was able to derive a metafunctional model for mathematics. However this derivation showed that the metafunctional organisation of mathematics was distinct from that of language. This should not be surprising. If different resources are consistently used in conjunction with each other, it is reasonable to assume that they maintain some sort of distinct functionality. In Systemic Functional theory, this functionality is characterised most broadly in a resource's metafunctional organisation. Since Kress and van Leeuwen (1990) it has been generally assumed (with some notable exceptions such as van Leeuwen 1999 for sound) that all of semiosis is organised with respect to three metafunctions: ideational, interpersonal and textual (sometimes renamed and sometimes interpreted as four if dividing the ideational metafunction into the logical and experiential component). In Chapter 2, it was argued that this assumption risked simply transposing categories from the Systemic Functional description of English onto other resources, and in doing so, homogenising the semiotic landscape. By deriving metafunctions from axis, this thesis has offered an avenue for testing this claim of metafunctionality.

It was found that there does indeed appear to be a metafunctional organisation in mathematics, but that this organisation is not that proposed in most studies. The mathematical system has been shown to be principally organised around recursive systems and realised by univariate structures. Interpreted metafunctionally, much of its variation occurs in the logical component. This logical component gives rise to two levels based not on constituency (as for the rank scale of English), but on univariate nesting. The logical component is complemented by a second component based on a multivariate structure. This component does give rise to a constituency-based rank scale, and concerns itself with the unary operations that modify symbols. For this reason, it was termed the operational component. Finally, mathematics involves a third component, termed the textual component, that is concerned with information flow.

Importantly for our general understanding of semiosis, there is no evidence for an independent component in mathematics comparable to the interpersonal metafunction in language. The operational, logical and textual components exhaust the meaning making resources in the register of mathematics under study, and so there is no reason to propose another component. Moreover, no features appear to be realised by a prosodic structure and mathematics tends not to make meanings that are generally associated with the interpersonal metafunction in language, such as MOOD and ATTITUDE. When it does, it either necessarily needs language (such as the shift in speech function associated with *Let x=2*) or the element denoting this meaning is wholly dependent on systems in the logical component and thus is part of the logical component (such as the Relator ≠ notionally giving negative polarity).

Mathematics, therefore, does not conform to a metafunctional organisation involving ideational, interpersonal and textual components. Rather it includes just two metafunctions: ideational (including logical and operational) and textual. By deriving metafunctions from axis, the description provides a counterexample to the assumption that all semiotic resources display ideational, interpersonal and textual functionality. This demonstrates that metafunctional organisation is resource-specific. In terms of Halliday's (1992b) distinction between theoretical and descriptive categories, metafunctions are not theoretical categories generalisable for all semiosis, but rather descriptive categories that must be justified internally with respect to each semiotic system under study.

This has significant ramifications for semiotics in the Systemic Functional tradition. If mathematics does not display an interpersonal component, but rather is largely organised around an ideational metafunction, is it possible that there are other semiotic resources that do not display an ideational component and rather revolve around an interpersonal metafunction? Or are there resources that do not involve metafunctional organisation at all, along the lines of van Leeuwen's (1999) suggestion for sound? These questions can only be answered if metafunction is not taken as a theoretical primitive, but as a derivable descriptive category.

Similarly, the level hierarchies in mathematics are not like those in any resource yet described. The grammar of mathematics involves both an obligatory rank scale and an obligatory nesting scale that arise from the different structural realisations. The univariate organisation within the logical component arranges the nesting scale, while the multivariate organisation within the operational component arranges the rank scale. An interaction such as this between two different hierarchies within the grammar is significantly different to the single rank scale and optional layering generally considered for language. However it may not be unique. In discussing the broader field of semiotic typology, Section 6.3.2 below will suggest that the interaction of univariate and multivariate hierarchies may in fact be a regular feature of academic formalisms.

The function-level matrix for mathematics is reproduced in Table 6.3 below. The full system networks for each level in the grammar and for genre are presented in Appendix B.

Nesting	**Rank**	**Logical**	**Textual**	**Operational**
statement		STATEMENT TYPE REARTICULATION COVARIATION COVARIATE MULTIPLICITY		
symbol		EXPRESSION TYPE	THEME THEME ELLIPSIS	UNARY OPERATION SPECIFICATION
	element			ELEMENT TYPE

Table 6.3 Function-level matrix for the grammar of mathematics

6.2.2 Genre and register as unifying semiotics

Chapter 4 used genre to unify language and mathematics. It showed that despite quantifications and derivations primarily utilising mathematics, they also regularly used language across various stages. Conversely, it showed that some primarily linguistic genres such as reports sometimes involve mathematics. In addition, it illustrated that both linguistic and mathematical genres regularly interact with each other using the same complexing relations (the exception being that mathematical genres cannot project other genres; they can, however, be projected). In order to account for this, a common stratum of genre was proposed that is realised by both mathematics and language. Like the categories developed in the grammar, justification for this stratum was based on the axial principles presented in Chapter 2. First, the complexing relations between linguistic and mathematical genres showed that they could be unified in a single system. Second, the relation between mathematical genres and mathematics' grammar was shown to be one of abstraction rather than constituency. And third, the notion of genre as a connotative semiotic, and thus a semiotic system in its own right, positioned it theoretically in such a way that it could be realised by multiple denotative semiotics (language and mathematics) (c.f. Matthiessen 2009).

Although both language and mathematics maintain their own distinct systems, they can be unified as realising a common stratum of genre. By doing this, the notion of genre description as mapping a culture's meaning potential (Martin 2000, Martin and Rose 2008) can be strengthened, as broader configurations of meaning outside language are encompassed. This offers a powerful tool for understanding bimodal mathematics and language texts, as well as the disciplines that use them. However it remains to be seen whether a common stratum of genre can be deployed to encompass other semiotic resources such as images, demonstration apparatus and film etc. As mentioned above, the common system of genre realised by mathematics and language is based on the paradigmatic relations between the two (in particular their complexing relations) and the fact that many genres included in the system can in principle be realised by both mathematics and language. To date, this kind of paradigmatic model linking image or any other resource with language and mathematics has not yet been proposed. Indeed, Bateman's intricate studies of genre in highly multimodal documents (2008, see also Hiippala 2015) highlights the significantly increased complexity when moving from the linear text-flow of written language and mathematics to the varied reading paths available in images and larger layouts.

The study of images in Chapter 5 does indicate the possibility of classifying certain types of images as genres. Graphs in particular are a prime candidate. They maintain a regular structure, they construe specific dimensions of field (arrays and generality) and they commonly involve linguistic and mathematical elements as labels on axes and lines. However there remain a number of issues before graphs can be networked as part of a unified stratum of genre. The first is how graphs would fit in the overall system. The relations between graphs, and linguistic and mathematical genres would need to be explored, both in terms of their possible complexing relations and the paradigmatic oppositions they enter into. Second, without a comprehensive grammar of graphs, it is difficult to determine whether the regular configurations of meaning associated with graphs are better described as grammatical patterns or as large scale genres. And finally, although graphs seem a relatively clear-cut image-type, it is not so simple to distinguish between other types of image. In Chapter 5, graphs were opposed to a very broadly defined category of 'diagrams'. These diagrams displayed immense variation, and so it remains to be seen whether diagram remains as a distinct category or whether it groups together a number of wholly different categories. Before we can propose a unified stratum of genre that incorporates language, mathematics and images, each of these issues must be worked out. As for grammatical description, we must be wary of assuming categories such as genre without systemic and structural motivation. Nonetheless, a unified stratum of genre does offer a potentially productive avenue for understanding the highly multimodal texts regularly seen in physics and other academic discourse.

Similarly, Chapter 5 showed that the register variable of field provides a useful insight into the similarities and differences in meaning realised by mathematics, language and image. Throughout the chapter, it was shown that many apparently similar meanings can be realised by different resources. For example, the same compositional taxonomy can be realised in a single image or through a stretch of language; the same range of generality can be realised through mathematics or through graphs; and the same sets of implication relations can be realised through language or mathematical symbolism. Although the construal of each meaning varies somewhat when realised by different resources, there is nonetheless an affinity among these meanings. For example, presenting a composition taxonomy in an image can be considered up to a point as simply another way of presenting the same composition taxonomy in language. By proposing a shared stratum of register (field in particular) these similarities can be accounted for despite their alternative realisations. At this stage, however, this formulation is simply a useful heuristic. Without detailed systems for register, we cannot definitively test such a model using the axial principles put forward throughout this thesis. How we understand the interaction of language, mathematics, image and indeed any other resource in relation to its context is still a matter for research and debate. Nonetheless, if genre and register could be shown to incorporate diverse realisations across multiple semiotic resources, it would provide a rich model for understanding context and the broader meanings that constitute our culture.

6.3 Looking forward

Based on the issues described above, the research in this thesis has highlighted a number of avenues for further research. First, looking at physics from the perspective of mathematics, image and language has given only a glimpse of its multifaceted knowledge. Physics involves numerous other resources such as demonstration apparatus, gesture and other symbolic formalisms that all likely form crucial components in building its knowledge. Second, the descriptions developed throughout have highlighted the need for a genuine semiotic typology that compares both the functionality and form of a wide range of resources. Such a typology would allow an understanding of what types of resources are used in what contexts, and would give significant insights into the growth and development of resources across our culture. Each of these threads will be commented on briefly here.

6.3.1 Understanding physics

As we have seen, the discourse of physics is complex. Before understanding the rich complexity of physics knowledge and discourse, it was necessary to develop the descriptive model of mathematics and interpret images in terms of field. These descriptive models allow an understanding of the potential of these resources in construing physics' knowledge. But there still remains significant work to understand how they are used in interaction in the diverse contexts of the classroom, student work, textbooks and in research. As a step toward this, Chapter 4 presented a broad map of the development of mathematics through schooling. This map offers a starting point for understanding how mathematics is used in education and shows the changes students need to deal with when progressing from primary school through to university. However to develop a fuller picture of physics, there are a number of further research paths that need to be taken.

First, to build a more comprehensive picture of the development of physics knowledge, similar mappings are needed for both images and the language of physics. Scientific language has been studied relatively extensively within SFL (see Chapter 2), with the growth of grammatical metaphor and technicality steadily building through the years. However there has yet to be a comprehensive tracking of the ontogenetic development specifically within physics. Images, on the other hand, have not had the same amount of attention. It is clear that long before mathematics is introduced, physics uses simple images to build its knowledge. These simple images primarily comprise single activities that illustrate a specific physical phenomenon (such as a push or a pull). Through high school, other relations such as composition become more prevalent, and graphs are gradually introduced. Later on in high school and university more complex images appear with multiple structures mapped onto a single image. Toward the end of undergraduate university, graphs appear to be the predominant images that are used, with other structures (regularly of classification) arising from patterns in the plot. What is clear is that images are used throughout schooling from early primary school to the end of undergraduate university. Like mathematics, understanding the use of images will give insights into the types of knowledge students are expected to learn and will build a fuller picture of the progression of physics education.

The second avenue for further research involves a deeper understanding of mathematics. The descriptive model built in this thesis focused on a restricted register of algebra used in high school physics. When moving into university physics, however, calculus increasingly forms a crucial component of its discourse. As discussed briefly in Chapter 3, calculus utilises the multivariate structures that modify symbols. These modifiers, such as the differential $\frac{dy}{dx}$ or the integral $\int$, encode large sets of relations that are applied to many technical symbols. To include calculus, it is therefore likely that the operative component of the grammar will need to be considerably expanded. In addition, as calculus involves its own procedures for solving problems, it will require further development of the model of genre given in Chapter 4. Looking further afield, mathematics as a discipline involves numerous distinct formalisms that are used in various subfields (indeed Bernstein 1999 and O'Halloran 2007a consider it a horizontal knowledge structure made up of multiple discrete languages). The elementary algebra described in this thesis is only the tip of the iceberg; mathematics is a semiotic resource deserving study in its own right.

Finally, a broad map of mathematics, image and language in physics offers a general perspective on the development of physics. But to understand how knowledge is built day-to-day in schooling we must look closely at classroom discourse. Classrooms involve innumerable episodes of knowledge building that aggregate to form an ever more comprehensive disciplinary understanding. When looking at physics classrooms, however, two issues become immediately clear. First, it is not sufficient to simply characterise each semiotic resource in isolation. Mathematics, language and image interact, combine and collaborate to produce meaning greater than the sum of their parts. Understanding knowledge requires more than simply understanding the intrasemiosis of each resource; it also requires a deep appreciation for the complex intersemiosis from which new meanings continually emerge. This is by no means a new idea; indeed it has been a concern since the earliest days of multimodality (e.g. Lemke 1998). However, we are still a long way from a comprehensive model of intersemiotic meaning.

The second issue for knowledge building in physics classes is the sheer number of semiotic resources involved in every lesson. In addition to spoken language, written language, mathematics and images, physics knowledge is regularly construed through gesture (e.g. the ‘right hand rule’ of electromagnetism), nuclear equations (e.g. the fission reaction involving Uranium 235: ${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{236}_{92}\mathrm{U}^{*} \rightarrow {}^{144}_{56}\mathrm{Ba} + {}^{89}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n}$), demonstration apparatus and numerous others. If we continue with the broad assumption that semiotic resources are used because they maintain some sort of unique functionality, then a comprehensive understanding of physics must take each of these into account. Characterising each of these resources, modelling their intersemiotic interactions and explaining their role in organising knowledge is a long term research focus that requires a continuing commitment to description and theoretical development.

6.3.2 Towards a semiotic typology

The final avenue for future research I will consider here involves the development of a robust semiotic typology. Just like language typology, semiotic typology would compare and contrast the form and function of semiotic resources to understand the parameters of variation and their contexts of use. This would greatly aid future description and theoretical modelling by providing a map of the range of semiotic resources used in a culture, and potentially offer a point of departure for future description. Whereas typological similarities across languages tend to be explored with respect to genetic and areal factors, similarities between semiotic resources are likely to be more associated with shared contexts of use. If multiple semiotic resources are used in a culture, this is likely so due to the different functions they play. Similarly, if resources evolve with similar functionalities, it is likely this is due to similarities in their context of use.

This can be illustrated by comparing mathematics with two other academic formalisms: the nuclear equations used to show nuclear reactions and decays in physics (e.g. ${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{236}_{92}\mathrm{U}^{*} \rightarrow {}^{144}_{56}\mathrm{Ba} + {}^{89}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n}$) and the system network notation used in Systemic Functional Linguistics. Each of these resources formalise relations between technical meanings in their respective areas and are only used in certain (more or less specific) academic fields. That is, they are a marker of vertical (academic) discourse and they encapsulate in a relatively economical way highly complex and often abstract theory.

When looked at grammatically, they also seem to share a strikingly similar structural organisation. We will explore this briefly here. As described in Chapter 3, mathematics maintains a two-level univariate structure where elements at each level are indefinitely iterative. As well as this, each symbol can take a restricted set of modifiers that conform to a multivariate structure. Considering nuclear equations first, we can see this resource displays a very similar organisation.[56] At its minimum, a nuclear equation includes two expressions related by a Relator (usually →).[57]

(6:1)

^{238}U	$\rightarrow$	$^{234}\text{Th} + \alpha$
expression	Relator	expression

Like mathematics, these expressions are indefinitely iterative; there can be any number in sequence. The following shows three expressions:

(6:2)

$^{235}_{92}\text{U} + ^{1}_{0}\text{n}$	$\rightarrow$	$^{236}_{92}\text{U}^{*}$	$\rightarrow$	$^{144}_{56}\text{Ba} + ^{89}_{36}\text{Kr} + 3^{1}_{0}\text{n}$
expression	Relator	expression	Relator	expression

And in a university physics textbook (Young and Freedman 2012: 1465), the beta decay of Xenon 140 is described using an equation with five expressions:

(6:3)

$$^{140}_{54}\text{Xe} \xrightarrow{\beta^-} {}^{140}_{55}\text{Cs} \xrightarrow{\beta^-} {}^{140}_{56}\text{Ba} \xrightarrow{\beta^-} {}^{140}_{57}\text{La} \xrightarrow{\beta^-} {}^{140}_{58}\text{Ce}$$

The indefinitely iterative nature of expressions in these equations suggests that, like mathematical equations, nuclear equations are best considered as having a univariate structure at their highest level.

(6:4)

[56] Note that despite their apparent structural similarities and some shared graphical symbols (e.g. +), mathematical equations and nuclear equations are distinct. Also, everything discussed here is tentative and requires a more in depth examination.
[57] All examples are from Young and Freedman (2012).

$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n}$	$\rightarrow$	$^{236}_{92}\mathrm{U}^{*}$	$\rightarrow$	$^{144}_{56}\mathrm{Ba} + ^{89}_{36}\mathrm{Kr} + 3^{1}_{0}\mathrm{n}$
1		2		3

Within each expression, the above equations illustrate that there may also be an indefinite number of symbols (linked by +). For example:

(6:5)

$$^{144}_{56}\mathrm{Ba} + ^{89}_{36}\mathrm{Kr} + 3^{1}_{0}\mathrm{n}$$

The combination of symbols into expressions can therefore also be seen as a univariate structure. This means that there are two distinct levels based on univariate structures, and thus that these nuclear equations display a similar nesting scale to mathematics.

When looking at the individual symbols the structural similarities continue. First, each symbol may be modified by certain elements. For example the final symbol in (5) above, $3^{1}_{0}\mathrm{n}$, involves four elements: 'n' indicates the particle (a neutron), the preceding superscript (^{1}) indicates the mass number (the number of protons and neutrons in the particle, in this case just one), the preceding subscript ($_{0}$) indicates the atomic number (the number of protons, in this case zero) and the full-size number (3) indicates the number of particles. Second, none of these elements are iterative. It is not possible, for example, to write $^{1\,1}\mathrm{n}$ or $_{2\,36}\mathrm{Kr}$. Further, only the element indicating the symbol (e.g. 'n' or 'Kr') can occur by itself; all the other elements are modifiers.

As each element performs a distinct function and none are indefinitely iterative, the symbols in nuclear equations display a multivariate structure, just like those in mathematics. In addition, each element within the symbol is constrained by what can realise it: the element indicating the particle can only be realised by a close set of chemical and particle symbols (such as 'n', 'Kr', 'U' etc.) and the preceding modifiers can only be realised by numbers. The multivariate structure thus gives rise to a rank below that involves a distinction between particle symbols and numbers. Nuclear equations show the same structural organisation and level hierarchies that mathematics does. They involve a two-level nesting scale interacting with a two-level rank scale.

Looking outside of science into linguistic formalism, we see a similar pattern. System networks formalise paradigmatic relations in Systemic Functional Linguistics, and so maintain a similar function to mathematics and nuclear equations in that they encapsulate highly intricate and technical relations between elements of the field. Despite being an image, system networks also display a remarkably similar structural organisation to both mathematics and nuclear equations.[58] They are primarily organised around different layers of univariate complexing, with a small multivariate component at the lowest level. To see this, we can begin with a single system:

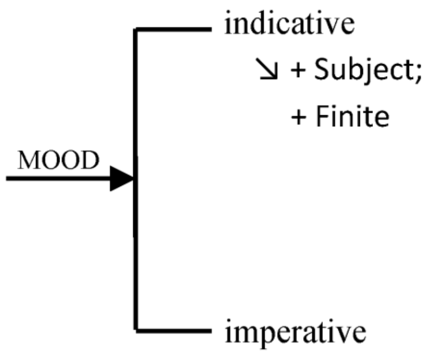

Figure 6.1 Simple system with two choices

A minimal system is made up of two choices (in this case labelled indicative and imperative).[59] Both choices have the same status; their position higher or lower is meaningless (at least ideationally). They thus perform the same function. In addition, a system can in principle contain an indefinite number of choices. Figures 6.2 and 6.3 show systems from Chapter 3 with three and five choices respectively.[60]

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[58] It is here that we need to heed Bateman's (2011) warning about taking semiotic resources to be the same or different at face value. Although system networks appear at first glance to be images, their structure and contexts of use closely resemble symbolic formalisms such as mathematics. Whether this is enough to distinguish system networks as being distinct from other images is a matter for debate.

[59] In this description, a choice includes both a feature (shown in lower case) and its realisation rule marked by ↘. This distinction will become important below.

[60] As we are only looking at single systems, any wiring to dependent systems have been deleted and so some of the choices in Figure 6.3 appear unmotivated. This does not, however, affect the argument being developed.

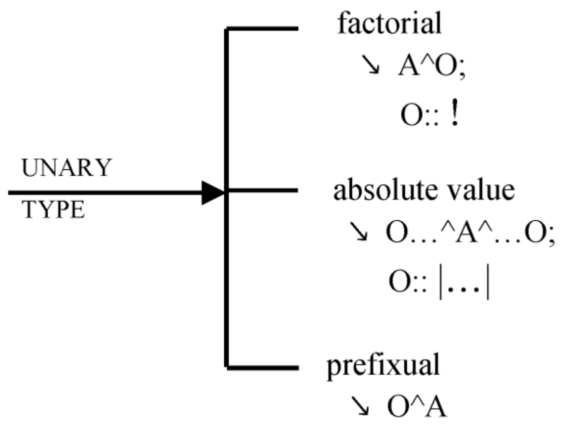

Figure 6.2 Single system with three choices

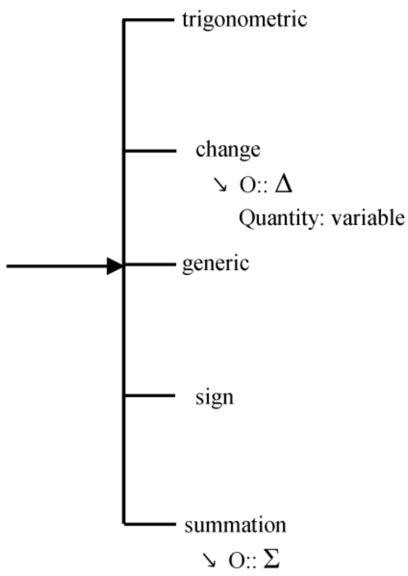

Figure 6.3 Single system with five choices

As each choice performs the same function with the same status and there may be any number of choices in a system, a system may be considered a choice complex. That is, a system is organised through an obligatory univariate structure (more specifically, a paratactic univariate structure), with the square bracket indicating the relation between each choice.

In order to increase the generalising power of system networks, systems tend to only include two choices (Martin 2013). Any further distinctions are given by dependent systems that are one further step in delicacy. An example of this is shown in Figure 6.4. In this network, the least delicate system involves two choices: indicative and imperative. Two further systems then emanate from both the indicative and imperative choices.

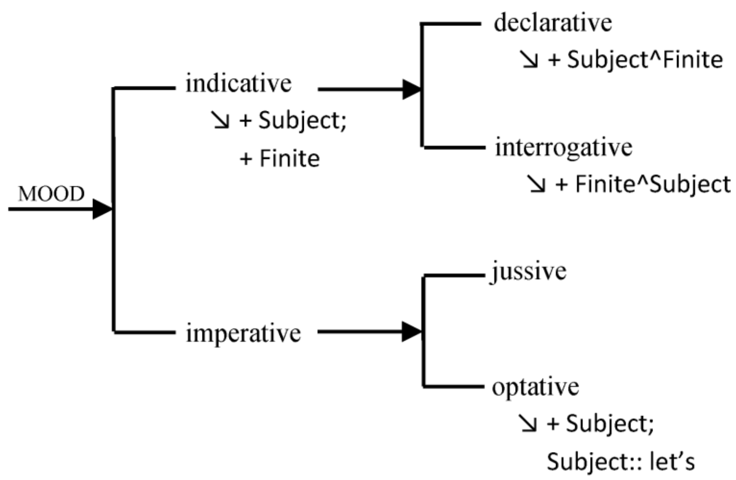

Figure 6.4 Two sets of dependent systems

Adding further systems that emanate from individual choices is a form of univariate layering. A system (a univariate structure involving two choices, e.g. declarative vs interrogative) is grouped together under a single choice (indicative), with this choice entering into a higher level system with its own univariate structure. There are thus layers of univariate structures within other univariate structures. The level of delicacy in system networks can therefore be interpreted as the degree of layering in the formalism.

System networks regularly involve a large amount of layering (see Appendix B for examples of networks with up to five steps in delicacy, and Matthiessen 1995 for very delicate networks for English). However there is one further type of univariate complexing that dramatically increases the potential of system networks. This is the potential for simultaneous networks indicated by braces (curly brackets). Braces allow an indefinite number of systems to be related through an 'and' relation. Figure 6.5 shows an example of a network with three simultaneous systems.

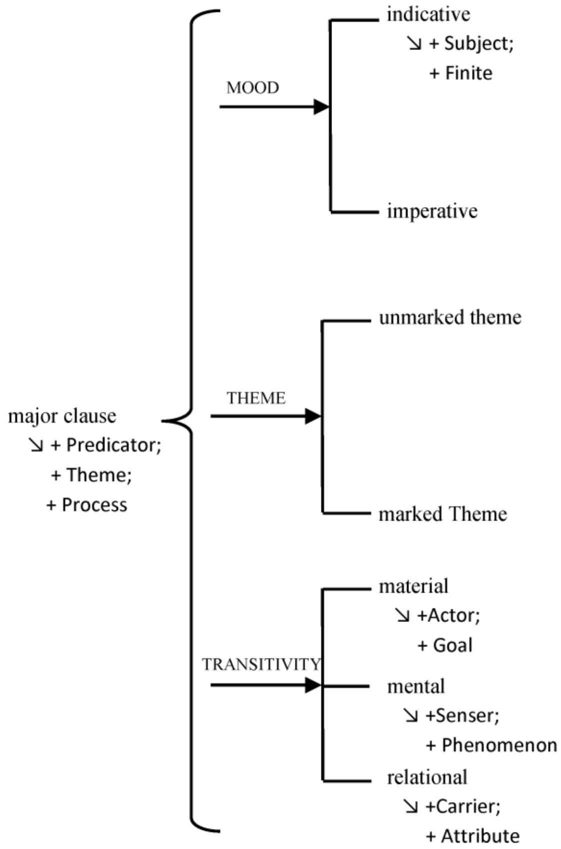

Figure 6.5 Three simultaneous systems

Like choices within systems, systems linked by braces can be indefinitely iterative. And again, each system performs the same function and is of the same status; their position in the brace is meaningless aside from potential information organisation. This means that they once more display a univariate structure. As choices complex into systems and systems complex into larger networks, entire system networks can therefore be read as large choice complexes. Just as the value of any symbol in a mathematical equation is entirely dependent on the symbols it is related to in the equation, the value of a choice in a system network is entirely dependent on its relation to other choices in the network.[61]

[61] And so, a century later, we return to Saussure's relational interpretation of valeur (1916).

Just like mathematics and nuclear equations, a system network at its minimum involves an obligatory univariate structure involving two elements, i.e. a system with two choices. This system can then further complex into larger networks through either simultaneous braces or univariate layering within each choice. The higher levels of system networks are thus, like mathematics and nuclear equations, organised univariately.

There is one final structural similarity between system networks and both mathematics and nuclear equations. This is a small multivariate component at a level below its univariate component. To see this, we can look at individual choices, such as that for declarative given in Figure 6.4:

declarative
↘ + Subject^Finite

The core of this choice is the feature declarative (indicated by lower case). A feature may occur on its own without any modification. In contrast, the ↘ + Subject^Finite that functions as a realisation rule cannot occur in a network on its own; it must modify a feature. A feature and its realisation rule thus perform two distinct functions. Further, features may not be repeated; there may not be multiple features within a single choice.[62] The relation between a feature and its realisation rule can therefore be interpreted as a multivariate structure. Like in mathematics and nuclear equations, this multivariate structure gives rise to another network of choices at the level below: only a *class* indicated by lower case can be a feature, and only a *function* indicated by initial upper case can occur as a realisation.

Despite their appearances, mathematics, nuclear equations and system networks all display remarkably similar organisations. At their highest levels, they maintain multiple layers of potentially elaborate univariate complexing (both obligatory and optional). At their lower levels, these complexes relate individual elements that maintain a multivariate structure, which in turn gives rise to a two-level rank scale. Their obligatory hierarchies are summarised in Table 6.4 (optional layerings are not included).

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[62] Though there may be multiple realisations rules, e.g. + Subject; + Finite; Subject^Finite. All realisations, however, are grouped under a single linker ↘, and so these are best analysed together as performing a single function in relation to the feature, with the possibility for complexing within this function.

Mathematics		***Nuclear Equations***		***System Networks***	
Nesting	*Rank*	*Nesting*	*Rank*	*Nesting*	*Rank*
statement		equation		system	
symbol		symbol		choice	
	element		element		element (function vs class)

Table 6.4 Hierarchies in mathematics, nuclear equations and system networks

If we interpret these resources metafunctionally, they each appear to be organised primarily through the ideational metafunction. In particular, they involve considerably expanded logical components based on elaborate univariate structures. Although this is a very preliminary analysis, nuclear equations and system networks seem to also join mathematics in not displaying an interpersonal component. They do not include prosodic structures, nor do they construe meanings typically associated with the interpersonal metafunction. It appears, then, that each resource backgrounds interpersonal meanings in favour of expanding its logical component.

Considering the context of each resource's use, this analysis makes sense. Each resource has been developed as an academic formalism to explicitly relate technical elements in their field. As they are concerned with field-specific meaning, it is unsurprising that they are more concerned with ideational meanings than other meanings (following the register-metafunction relation where field tends to be associated with ideational meaning, tenor with interpersonal meanings and mode with textual meanings, Halliday 1978b). By expanding their logical components, each resource offers the potential for the field to relate an indefinitely large number of meanings. It thus presents a method through which their respective fields can construe increasingly expanding complexes of meaning.

Each resource does, of course, have distinctive properties. Mathematics shows significantly more layering within expressions than nuclear equations; and system networks involve complexing of their highest nesting (systems), where mathematics does not (there are no statement complexes). In addition, the specific relations and choices available within each resource are tailored to their own specific functionality. Moreover, when looked at from field, their similar grammatical architecture does not necessarily lead to similarities in the types of meaning they realise. System networks display large taxonomies of classification (declarative and interrogative are both types of indicative) and, possibly dues to the use of arrows in the system, they are often read as activity sequences (if you choose indicative, then you must choose either declarative or interrogative). Nuclear equations, on the other hand, more clearly show activity sequences. For example, ${}^{140}_{54}\mathrm{Xe} \xrightarrow{\beta^-} {}^{140}_{55}\mathrm{Cs} \xrightarrow{\beta^-} {}^{140}_{56}\mathrm{Ba} \xrightarrow{\beta^-} {}^{140}_{57}\mathrm{La} \xrightarrow{\beta^-} {}^{140}_{58}\mathrm{Ce}$, indicates a temporal unfolding whereby Xenon 140 undergoes beta decay to become Cesium 140, which then undergoes further beta decay to become Barium 140 etc. In contrast, as we have seen, mathematics realises large implication complexes with no suggestions of temporal unfolding.

Despite these differences, their strikingly similar architectures in comparison to that of language do indicate a possible recurring motif in academic formalism. As resources such as these are developed within specific disciplines to construe field-specific meaning, they will evolve to encapsulate meaning relevant to the field (i.e. ideational meaning). Moreover, as they occur alongside language, they will likely develop distinct organisations and functionalities to language; if not, they would be redundant (and would not evolve). By exploring semiotic resources along these lines, it is possible to ask whether a large logical component based on a univariate structure is a regular feature of academic formalism in certain fields. If so, this would give an insight into the boundaries of language for construing technical meaning. But to know this, we need much more detailed studies of a larger range of academic formalisms; we would need to know whether the various families of symbolic logic, chemical formulae, linguistic phrase structure notation, programming languages and numerous other idiosyncratic formalisms share similar organisations.

More broadly, we would need a genuine semiotic typology that compares and contrasts the functionality of semiotic resources. Such a typology would allow generalisations to be developed that would allow a fuller understanding of semiosis in our culture. It would also provide a platform from which descriptive studies can begin their explorations. This typology would need to be based on descriptions that consider each semiotic resource on its own terms, and develop according to explicit and shared descriptive principles. The model of mathematics developed in this thesis looks significantly different to the Systemic Functional model of language. But when considered in relation to resources that share similar contexts of use, they appear to show a number of affinities. A semiotic typology would make explicit these similarities and differences and take a step toward mapping the wide world of semiosis that we live in.

6.4 Physics, knowledge and semiosis

We began this thesis with the observation that physics is hard. It involves spoken language, written language, mathematics, images, nuclear equations, demonstration apparatus and gesture to construe a large and highly technical knowledge structure. By developing models of this knowledge structure and the resources that organise it, we have taken a step toward understanding how it works and how it is similar to or different from other disciplines. From these models, we can build educational programs that allow teachers to more effectively teach physics. And from this, we can make it easier for students to access this knowledge and the literacy practices associated with it.

Similarly, semiotics is hard. It has a vast and multifaceted object of study that encompasses all the meanings that constitute human culture. The last thirty years have seen a dramatic broadening of its horizons, with an appreciation of the multitude of ways of meaning that pervade every aspect of our lives. By taking these ways of meaning seriously and considering them on their own terms, we can take a step toward understanding how they work and how they are similar to or different from other meanings. From this, we can begin to understand how our culture organises both its knowledge and its ways of knowing.

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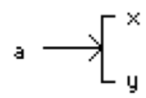
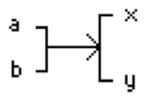
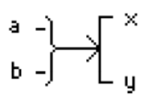
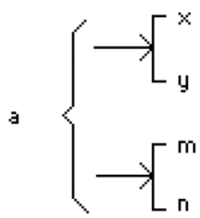
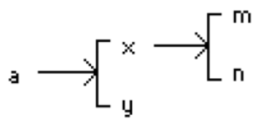
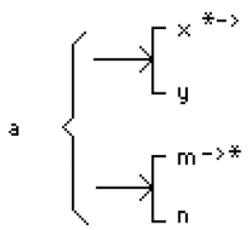
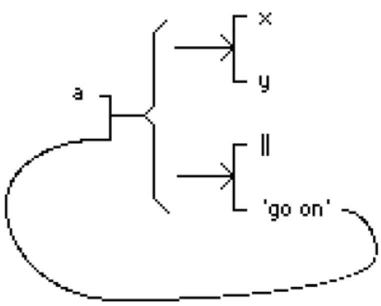
APPENDIX A

System Network Conventions

Systems reproduced from Matthiessen and Halliday (2009).

Realisation statements primarily reproduced from Martin (2013) with some adaptations.

<u>Systems</u>

	**system:** if 'a', then 'x' or 'y' -- abbreviated as 'a : x / y'
	**disjunction in entry condition:** if 'a / b', then 'x / y'
	**conjunction in entry condition:** if 'a' and 'b' (abbreviated as 'a & b'), then 'x / y'
	**simultaneity:** if 'a', then simultaneously 'x / y' and 'm / n'
	**delicacy ordering:** if 'a', then 'x / y'; if 'x', then 'm / n'
	**conditional marking:** if 'x', then also 'm'
	**recursive system (logical):** if 'a', then 'x / y' and simultaneously option of entering and selecting from the same system again

Realisation statements

A realisation statement consists of an operator, such as insert or conflate, and one or more operands, at least one of which is a grammatical function.

major type	operator	operand 1	operand 2	example
(i) structuring	insert (+) expand (( )) order (^) subclassification (X_y)	Function Function Function Function	- Function Function Function	+ Subject Mood (Subject) Subject^Finite $\text{Result}_{\text{Numerical}}$
(ii) layering	conflate (/)	Function	Function	Subject/Agent
(iii) inter-rank realisation	preselect (:)	Function	feature(s)	Subject: nominal group

One of the operands of "order" may also be a boundary symbol, as in # ^ Theme and Moodtag^#.

The different types of realisation statement are outlined in more detail below:

(1) Presence of Functions in the structure: the presence of a Function in a Function structure is specified by inserting the Function into the structure; the operation of insertion is symbolised by "+"; e.g. + Subject, + Mood, etc.

(2) Functional constituency relations: two Functions may be related by constituency and to specify this constituency relationship in the Function structure one Function is expanded by the other; the expansion is symbolised by putting the expanding constituent Function within parenthesis, e.g. Mood (Subject), which means that Mood is expanded to have Subject as a constituent Function. A Function may be expanded by more than one other Function, e.g. Mood (Subject, Finite).

(3) Relative ordering of Functions and ordering relative to unit boundaries: two Functions may be ordered relative to one another in the Function structure and this relative ordering is symbolised by the "^"; e.g. Subject^Finite, Mood^Residue. The ordering may also be relative to the left or right boundary of a grammatical unit (represented by #), e.g. #^Theme and Moodtag^#.

A distinction can be made between sequencing Functions directly after one another, e.g. Subject^Finite, and sequencing Functions with respect to one another, e.g. Subject → Finite (meaning that the Subject comes before the Finite but that another Function, for example a Mood Adjunct, might intervene).

(4) Conflation of one Function with another: one Function from one perspective is conflated with a Function from another perspective, i.e. the two Functions are specific as different layers of the same constituent – they are identified with one another. Conflation is symbolised by "/"; for example, Subject/Agent means that Subject (interpersonal) and Agent (ideational) apply to the same constituent.

(5) Realisation of a Function in terms of features from the rank below: the realisation of a Function in a Function structure is stated by preselecting one of more features from the unit realising it; preselection is symbolised by ":", e.g. Subject: nominal group, Finite & Predicator: verbal group, etc.

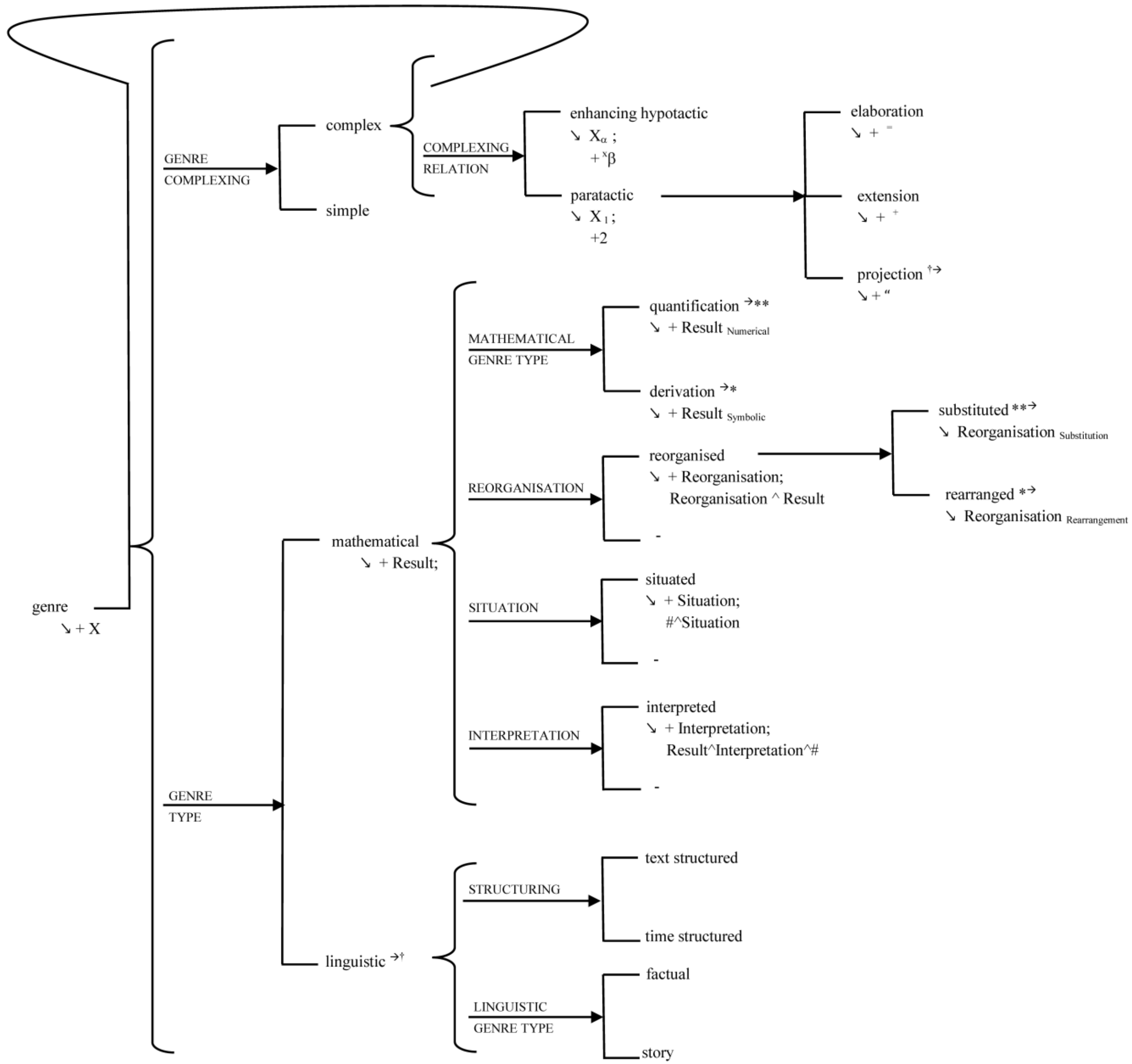
A distinction can be made between the realisation of a Function through a feature (formalised as Function: feature) and lexicalisation of a Function through a specific lexical item (e.g. formalised for Tagalog interrogatives as Q::*ba*). Embedding can be defined as the realisation of a Function through a feature from the same or higher rank.

(6) Subclassification of a Function: one Function can be subclassified into two or more different Functions, i.e. the subclassified Function is a less delicate generalisation of the Functions subclassifying it. Subclassification is indicated by a subscript, where the subclassifying Function is the subscript of the more general Function, e.g. $\text{Result}_{\text{Numerical}}$ indicates that Numerical subclassifies Result.

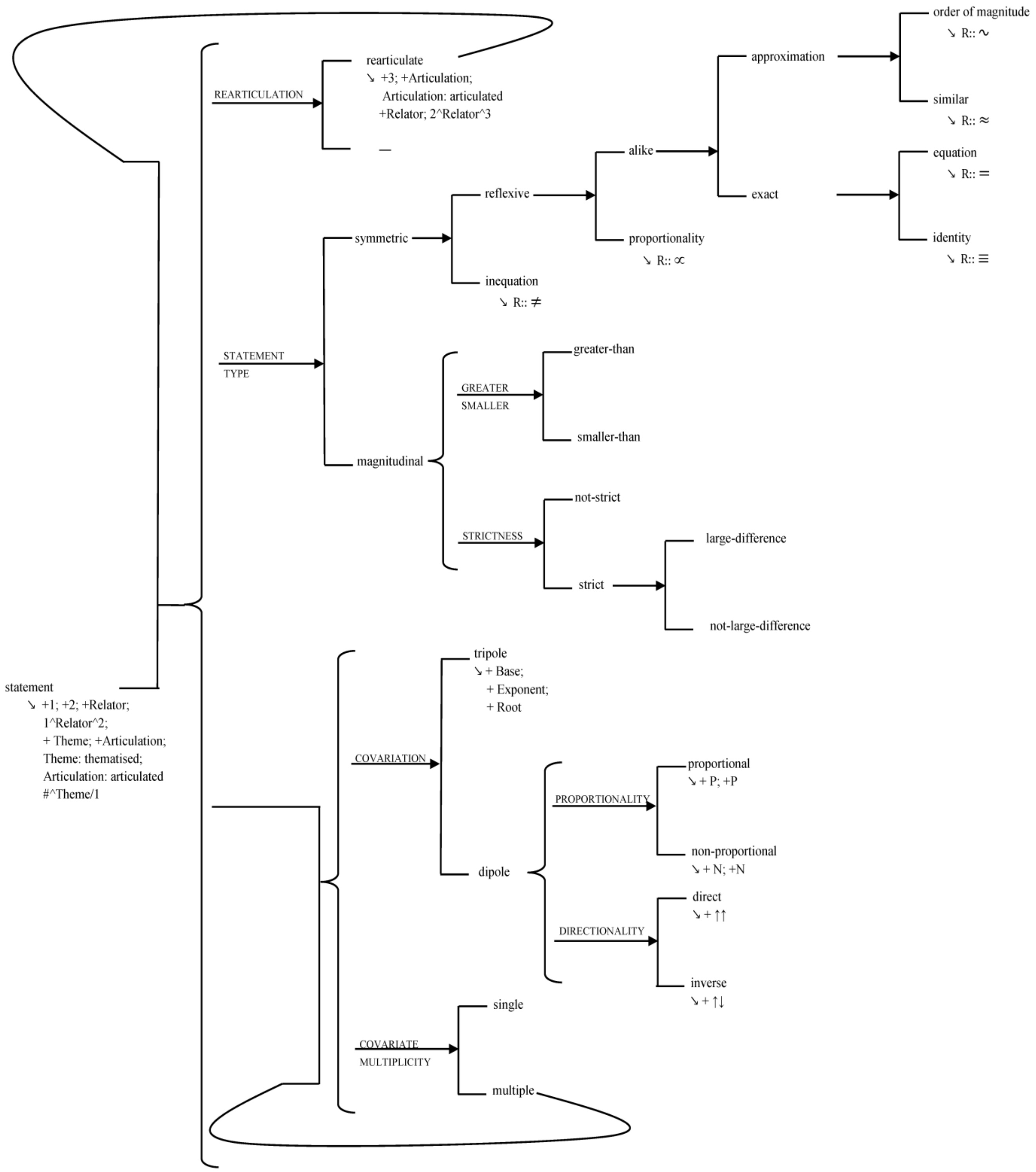
APPENDIX B

Full System Networks for Mathematics

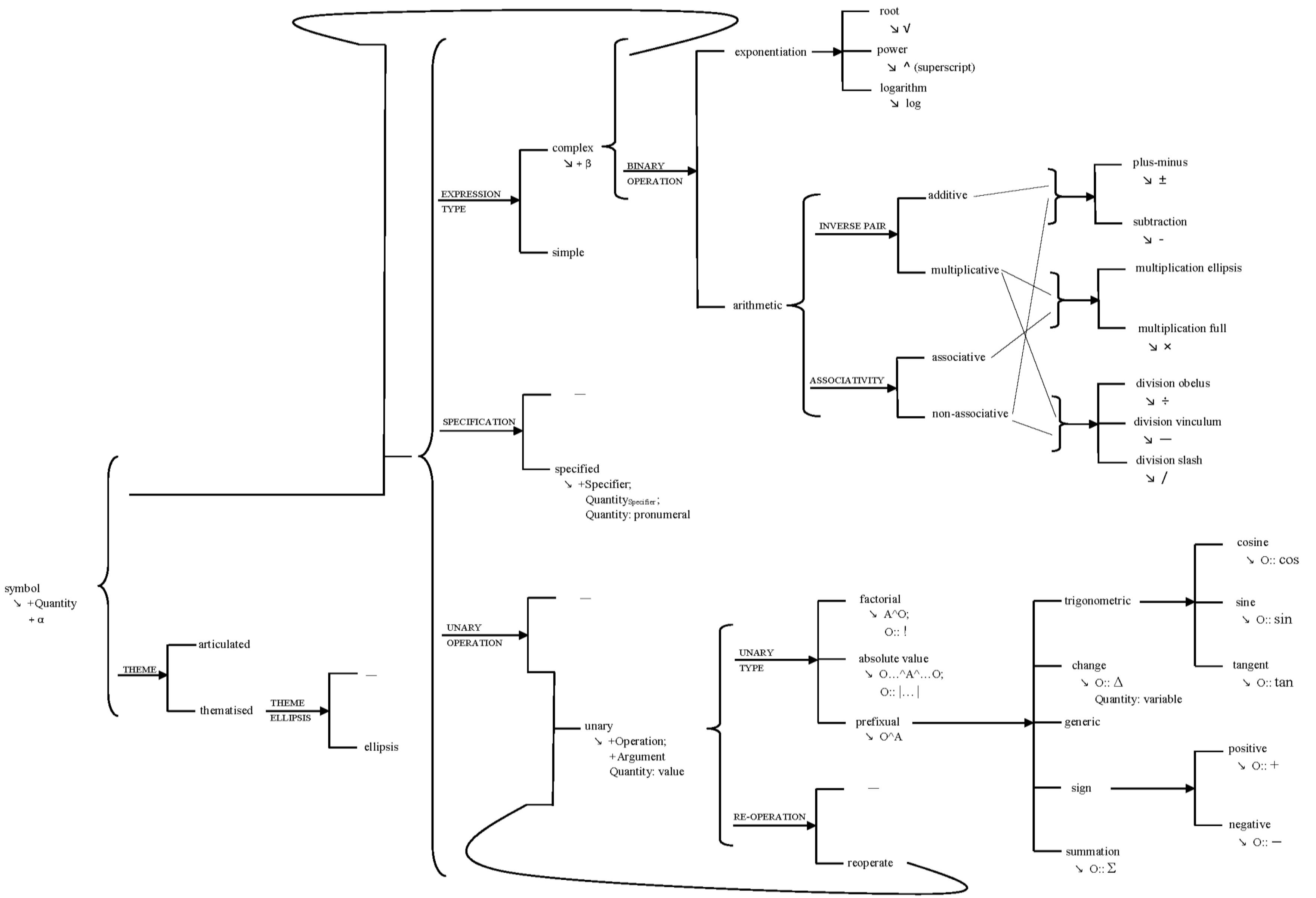
Genre


genre
↘ + X
GENRE COMPLEXING
complex
simple
COMPLEXING RELATION
enhancing hypotactic
↘ Xα ;
+ ˣβ
paratactic
↘ X1 ;
+2
elaboration
↘ + =
extension
↘ + +
projection †→
↘ + “
GENRE TYPE
mathematical
↘ + Result;
MATHEMATICAL GENRE TYPE
quantification →**
↘ + Result Numerical
derivation →*
↘ + Result Symbolic
REORGANISATION
reorganised
↘ + Reorganisation;
Reorganisation ^ Result
-
substituted **→
↘ Reorganisation Substitution
rearranged *→
↘ Reorganisation Rearrangement
SITUATION
situated
↘ + Situation;
#^Situation
-
INTERPRETATION
interpreted
↘ + Interpretation;
Result^Interpretation^#
-
linguistic →†
STRUCTURING
text structured
time structured
LINGUISTIC GENRE TYPE
factual
story

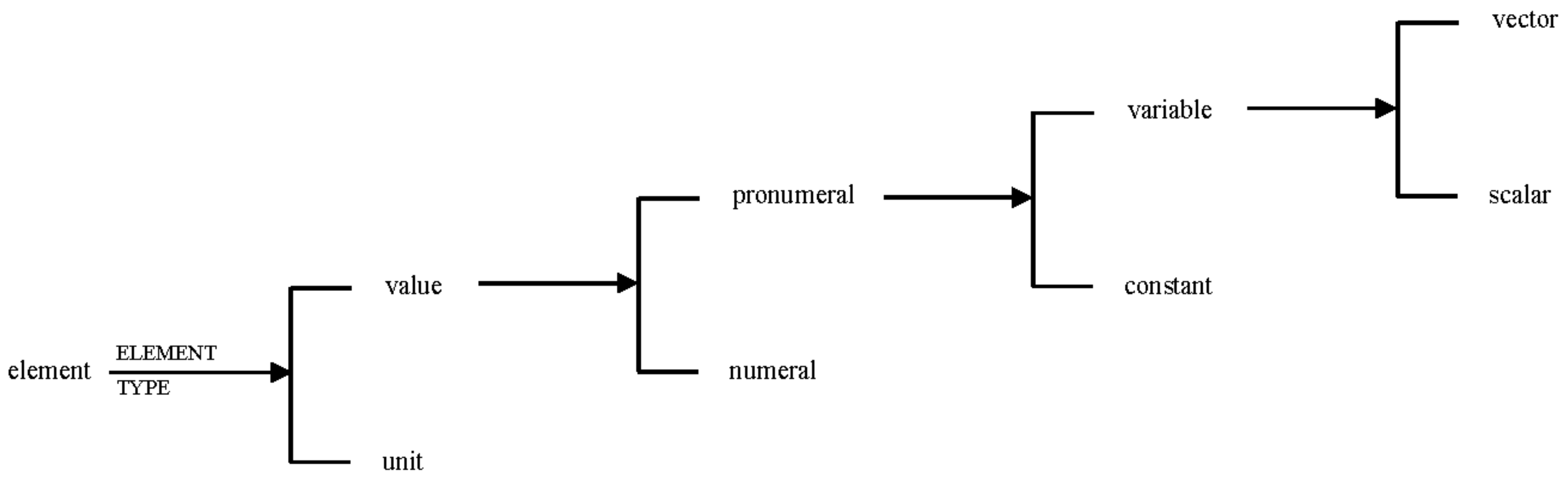
Grammar: Statement nesting



Grammar: Symbol rank/nesting



Grammar: Element rank



APPENDIX C

Details of corpus

	Data
Primary School (~6-12 years old)	• Excerpts from eight general science textbooks. ○ Focused forces and motion (classical mechanics)
Junior High School (~12-16 years old)	• Excerpts from five general science textbooks. ○ Focused on forces and motion (classical mechanics)

Senior High School
(~16-18 years old)
	• Excerpts from five physics textbooks.
	  ○ Four focusing on forces and motion (classical mechanics)
	  ○ One focusing on quantum physics
	• Excerpts from video and audio of one unit in a final year high school classroom.
	  ○ Includes sixteen classes focusing on quantum physics
	• Marked final exam student responses focusing on quantum physics, classical mechanics, special relativity and electromagnetism
	  ○ Seven exams ranging from high to low achieving responses

1st Year Undergraduate University	• Excerpts from one physics textbook
	  ○ Focusing on quantum physics
	• Excerpts from video and audio of one unit in an undergraduate lecture series.
	  ○ Includes twelve lectures on quantum physics
	• Marked final exam student responses focusing on quantum physics, fluid physics and electromagnetism.
	  ○ Twenty exams ranging from high to low achieving responses

Textbooks used in study

Primary School

Ardley, N. (1992) *Science for Kids: Movement*. London: Dorling Kindersley.

Bardon, K. (1991) *Exploring Forces and Structures*. Hove: Wayland.

Bentley, J. (2003) *Start Science: Forces and Motion*. London: Chrysalis.

Eason, S. (2009) *What are Forces and Motion?* Sydney: Franklin Watts.

Farndon, J. (2003) *Science Experiments: Motion*. New York: Marshall Cavendish.

Gordon, B. (2001) *Hands on Science: Over 40 Fantastic Experiments: Forces and Motion.*

Morgan, B. (2003) *Elementary Physics: Motion*. London: Brown Reference Group.

Riley, P. (2001) *Ways into Science: Push and Pull*. London: Franklin Watts.

Junior High School

Haire, M., Kennedy, E., Lofts, G. and Evergreen, M. J. (1999) *Core Science 1*. Milton: John Wiley and Sons. 114-139.

Haire, M., Kennedy, E., Lofts, G. and Evergreen, M. J. (2000) *Core Science 4*. Milton: John Wiley and Sons. 108-135.

Mau, J. (1999) *Science Australia 4*. Carlton: Curriculum Corporation. 2-24.

Shadwick, B. and Barlow, S. (2003a) *About Science 2*. Marrickville: Science Press. 207-219

Shadwick, B. and Barlow, S. (2003b) *About Science 4.* Marrickville: Science Press. 1-21.

Senior High School

de Jong, E., Armitage, F., Brown, M., Butler, P. and Hayes, J. (1990) *Heinemann Physics in Context: Physics One: Energy in Everyday Life: Movement and Electricity*. Port Melbourne: Heinemann Educational. 242-276.

Marsden, G. R. (2003) *Timesavers Senior Physics: Quanta to Quarks*. Peakhurst: Timesavers. a1-b44.

Warren, N. (2000) *Excel Preliminary Physics*. Glebe: Pascal. 110-149.

Wiecek, C., Zealey, B., Hynoski, M., Mathur, J. and Tatnell, I. (2005) *Physics in Context: The Force of Life. Preliminary*. South Melbourne: Oxford University Press. 178-246.

Williams, K. J. and Pemberton, J. (2001) *Spotlight Physics: Preliminary*. Marrickville: Science Press. 151-202.

First Year Undergraduate University

Young, H. D. and Freedman, R. A. (2012) *Sears and Zemansky's University Physics: With Modern Physics.* 13th ed. Boston: Addison-Wesley.