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LCTC

TEACHING SCIENCE

Knowledge, Language, Pedagogy

Legitimation Code Theory

ROUTLEDGE

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TARGETING SCIENCE

Successfully integrating mathematics into science teaching

Karl Maton

… like trying to hit a bullet with a smaller bullet, whilst wearing a blindfold, riding a horse.
– 'Scotty', in *Star Trek*, 2009

Introduction

In his *Opus Majus* of 1267, Roger Bacon described mathematics as 'the door and key' to science. This has become an axiom of educational research into science and its constitutive disciplines.[1] Mathematics is widely heralded as the 'backbone' of science (Bing and Redish 2009: 1) and 'deeply woven' into its practice and teaching (Redish and Kuo 2015: 562). Science textbooks are shown to exhibit greater use of mathematics than those of other disciplines (Lemke 1998, Parodi 2012). Learning the 'appropriate application' of mathematical skills is said to be 'a key part of the hidden curriculum in science' (Quinnell *et al.* 2013: 814). Accordingly, the ability to integrate mathematical and scientific knowledge is viewed as an important sign of student progress (Redish 2017). In short, teaching and learning mathematical knowledge is a central issue for science education. However, just as widely acknowledged is that integrating mathematics into science teaching poses persistent problems. Studies regularly proclaim that many students struggle with mathematics in science lessons and consequently become discouraged from continuing in science (Meli *et al.* 2016). Even students who have chosen further studies in science 'often seem to see "maths" as a separate subject, a necessary evil … rather than an integral part of the discipline' (Quinnell *et al.* 2013: 811).

A key aspect of the problem is said to lie with fundamental differences between the two disciplines. Scholars emphasize that mathematics when used within science has 'a different purpose – representing meaning about physical systems rather than expressing abstract relationships. It even has a distinct semiotics … from pure mathematics.' (Redish and Kuo 2015: 563). Integrating mathematics into science education is thus not a simple matter of adding mathematical content to science lessons. Indeed, differences between the disciplines are such that students encountering a concept located within both bodies of knowledge often fail to recognize that they are exploring similar ideas in different contexts (e.g. Planinic *et al.* 2012). As a result, students may succeed in mathematics classes but 'fail to use those same tools effectively' in science classes, leaving educators ' distressed and confused' (Redish 2017: 25).

A challenge facing science education is thus to identify and develop teaching practices that select, recontextualize and integrate mathematical knowledge in ways that support the learning of scientific knowledge. Unsurprisingly, this has been the subject of a significant body of research. The resulting work has generated suggestive ideas for learning specific mathematical skills for particular scientific problems. However, there is no overarching or integrating model for successful pedagogic integration. The problem 'remains unsolved' (Redish and Kuo 2015: 561). This chapter contributes towards the creation of such a model. It does so through by offering a fresh approach that complements existing frameworks by bringing to light issues that have hitherto been sidelined.

We begin by highlighting how research into science education offers significant insights into how students learn scientific ways of knowing but either neglects the forms of knowledge being taught or, where knowledge is discussed, treats 'science' and 'mathematics' as self-evident and unchanging. We argue that these constructions of the problem help underpin its persistence by ignoring knowledge, backgrounding teaching, and glossing over how 'science', 'mathematics' and their interrelations vary across contexts and change through the course of education. We then outline a framework that can complement existing approaches by bringing these issues into the picture. We introduce concepts from the Autonomy dimension of Legitimation Code Theory that conceptualize one aspect of the forms taken by knowledge practices and allow research to capture changing relations between changing forms of knowledge. Specifically, *autonomy codes* reveal the organizing principles underlying different knowledge practices, *autonomy pathways* trace changes in relations between knowledge practices over time, and *targets* embrace the contextual nature of what is viewed as 'science' and 'mathematics'. We illustrate the value of these concepts in analyses of science classrooms drawn from a major study of secondary schooling. These show that different autonomy pathways taken by teachers enable or constrain the integration of mathematics into classroom science. We conclude by reflecting on the potential of the concepts of *autonomy codes*, *pathways* and *targets* to bring knowledge into the picture and to connect specific instances of pedagogic practice together within a general model of pedagogic integration.

Studying mathematics in science: blind spots

To integrate mathematics into classroom science requires teaching practices that appropriately select ideas from one body of knowledge (mathematics) and recontextualize that selection within a second selection of ideas from another body of knowledge (science). A key issue is thus how different teaching practices shape the forms taken by ideas from those bodies of knowledge when they are brought together. Put simply, the question is: what teaching practices enable or constrain the integration of mathematical knowledge into scientific knowledge? These statements may seem unnecessary. However, studies of science education typically sideline both teaching practices and changing forms of knowledge. These blind spots arise from three assumptions that pervade the field: that knowledge equates to knowing, that education equates to learning, and that 'science' and 'mathematics' are self-evident.

Knowing and learning

The first assumption is that 'knowledge' comprises mental processes of understanding that reside 'in the heads of persons' (von Glasersfeld 1995: 1). Reflecting this 'subjectivist doxa' (Maton 2014: 3–14), research focuses almost exclusively on cognitive and affective ways of knowing. Knowledge as an object of study in its own right – one taking particular forms which have effects for bringing that knowledge together with other forms – is left out of the picture. Put another way, the assumption is that to analyze knowledge one must analyze ways of knowing. Rather than distinguish between students' dispositions and what they are learning, as a precursor to exploring relations between knowing and knowledge, the only concern is the former. This assumption that knowledge is nothing but knowing is typically accompanied by a second assumption: that education is nothing but learning. When studying 'ways of knowing', research overwhelming focuses on student interactions, such as when solving a scientific problem. Teaching is rarely centre stage, if considered at all. Each of these assumptions thereby takes part of the picture for the whole.

This focus on learning and ways of knowing is illustrated by studies using the 'resources framework' (e.g. di Sessa 1993, Hammer 2000, Redish 2014, 2017), an influential approach to physics education research. The framework explores 'how our students think' (Redish 2014), such as 'the student's perception or judgement (unconscious or conscious) as to what class of tools and skills is appropriate to bring to bear in a particular context' (Bing and Redish 2009: 1). The concern is how 'cognitive resources' are 'activated in response to a perception and interpretation of both external and internal contexts' (Redish and Kuo 2015: 573). Thus student perceptions are central – what their perceptions may be about, the forms taken by knowledge itself, is not analyzed. This subjectivism is thoroughgoing – everything is psychological. For example, the term 'epistemology' is used to refer not to intersubjectively shared field-level knowledge practices but rather to personal frames of individual understanding (e.g. di Sessa 1993, Hammer and Elby 2002). Accordingly, disciplines such as mathematics are viewed as comprising 'ways of knowing' (Redish 2017) and studies of mathematics in science explore how students solve problems in order 'to model their thinking' (Bing and Redish 2009: 2).

Research using the 'resources framework' offers valuable insights into how students learn ways of knowing. However, learning is not the sum of education, and ways of knowing are not the sum of disciplines. Much remains missing from the equation. Such studies could provide a powerful basis for understanding the integration of mathematics into science lessons if they were complemented by analyses of teaching and analyses of knowledge. However, frameworks for bringing these issues into the picture are lacking in the wider field. Currently, whatever approach they are using, studies of science education tend to reduce knowledge to knowing and education to learning, generating blind spots in the overall field of vision.[2] For example, studies of mathematics in science education draw on such frameworks as 'thinking dispositions' to suggest that attributes such as curiosity can help students shift from 'rigidity of mind' to 'fluid thinking' that ostensibly supports successful integration (Quinnell *et al.* 2013). Similarly, studies adopting a 'cognitive blending framework' analyze how students draw on 'mental spaces' when combining physical and mathematical knowledge (Bing and Redish 2007). Thus, everything lies in the mind of the beholder. Similarly, teaching is sidelined. The implications of studies of student learning for how mathematics should be taught in science often take the form of afterthoughts, as if teaching is merely an epiphenomenon of learning. Typically, such implications simply comprise calls for the integration of mathematics into science in teaching (e.g. Meli *et al.* 2016; Planinic *et al.* 2012), leaving unsaid what teaching practices would support that integration. In short, the widely shared focus on ways of knowing (rather than also knowledge) and on learning (rather than also teaching) limits current understanding of how pedagogic practices enable or constrain the integration of mathematics into science within classrooms.

Knowledge as self-evident and invariant

Knowledge is not entirely absent from discussions of mathematics in science. As mentioned earlier, differences between their purposes and 'semiotics' are said to contribute to student difficulties. Typically, science is described as condensing additional meanings into numbers and symbols and as requiring a different approach to interpreting mathematical results, reflecting its relationship with the external world (e.g. Bing and Redish 2009, Redish and Kuo 2015). Such attributes are also highlighted in discussion of the 'affordances' of mathematical representation for science (e.g. Fredlund *et al.* 2012). In systemic functional linguistics, work has shown how mathematics 'multiplies' meanings in relation with language and images (Lemke 1998), offering additional resources for construing scientific knowledge (O'Halloran 2010). More recently, Doran (2018) has brought together systemic functional linguistics with Legitimation Code Theory (LCT) to identify key mathematical genres in school physics and explore the role these play alongside language and images as physics progresses through school. Studies using LCT on its own have also explored forms of scientific knowledge (including mathematical symbols) involved in teaching and assessment in terms of differences in their complexity and context-dependence (Georgiou *et al.* 2014; Blackie 2014; Conana *et al.* 2016).

What is left open, however, is the question of identifying 'mathematics' and 'science'. To analyze integration in a classroom requires determining which knowledge is 'mathematics', which knowledge is 'science', and when a specific idea or practice has been recontextualized from one into the other. However, existing studies typically describe content, such as the formula or problem being discussed by students, and simply state what is 'mathematical' and what is 'scientific' (e.g. Meli *et al.* 2016), as if self-evident. Alternatively, discussions of the nature of 'science' or 'mathematics' make claims about each discipline as a whole, as if homogeneous and unchanging. That these ways of constructing knowledge are problematic flows from two uncontentious commonplaces. First, however distinctive the bodies of knowledge populating their *intellectual fields* might be (and this is debatable), the manifestations of 'science' or 'mathematics' *in a specific classroom* cannot be assumed. Which knowledge from an intellectual field is selected, recontextualized and enacted as curricula, and which knowledge from a curriculum is selected, recontextualized and enacted in classroom pedagogy varies geographically, institutionally and through the stages of education.[3] Put simply, the knowledge practices comprising 'science' and 'mathematics' are not necessarily the same in two classes in a school, let alone in different schools, years of study, states or countries. Second, at what stage of education specific knowledge practices from 'mathematics' are integrated into and become 'science' is not universal. What has already been integrated into 'science' in one classroom may remain separate 'mathematics' in another classroom. In short, what is 'science' varies between classroom contexts and changes through education, the 'mathematics' drawn on when teaching 'science' varies and changes, and the degree to which that 'mathematics' has been transformed into 'science' also varies and changes. Classroom 'science' and 'mathematics' are two variable and changing bodies of knowledge whose interrelations are themselves situational and mutable. They are anything but self-evident, homogeneous or unchanging – thus our liberal use of quote marks. To simply state that specific ideas are 'science' or 'mathematics' is to beg the question of how that is determined.

Addressing the question is not easy. It requires avoiding a false dichotomy between essentialism and relativism that continues to bewitch education research (Maton 2014: 1–22). On the one hand, universalizing claims about 'science' and 'mathematics' without a limiting context (such as 'science in this classroom') can lead to essentialism that treats their properties as homogeneous and invariant. On the other hand, insistence on the contextual limits of any definitions (or offering no more than 'science in this classroom, at this moment') can slide into relativism that treats 'science' or 'mathematics' as an endless flux. The former generates overgeneralized claims that are unhelpful for analyzing empirical data; the latter leads to the banal conclusion that subject areas are constructed, contested and fluid, paralyzing the possibility of analysis. As yet, research into science education has not steered a course between this Scylla and Charybdis. However, without facing squarely the question of distinguishing knowledge practices, empirical studies can only continue creating a series of context-bound models of specific instances. The wider issue of pedagogic integration will remain unsolved.

Seeing into the blind spots

Assuming that knowledge is only knowing, that education is only learning, and that disciplinary knowledges are self-evident generates blind spots. It is difficult to develop pedagogic practices that integrate mathematics into science lessons so long as teaching practice and both forms of knowledge are not analyzed. Existing insights into how students learn ways of knowing thus need to be complemented by: (i) studies of teaching practice; (ii) concepts that make visible the forms of knowledge practice being taught and learned; and (iii) a means of enacting those concepts that captures the variant and contextual nature of 'science' and 'mathematics'.[4] The first requires a shift of empirical focus. The second can be addressed by drawing on Legitimation Code Theory, a framework that reveals the organizing principles of knowledge practices. The third is trickier – it is akin to how Scotty described transwarp beaming in the motion picture *Star Trek*: 'like trying to hit a bullet with a smaller bullet, whilst wearing a blindfold, riding a horse'. It needs to account for two changing phenomena (what is 'science' and what is 'mathematics') whose relations are also changing (through different degrees of separation and integration). We now turn to a means for doing so.

Autonomy

Legitimation Code Theory or 'LCT' is a framework for researching and shaping practice. It begins from the notion that there is more to what we say or do than what we say or do. In other words, the meanings of practices are not exhausted by their content; practices are also 'languages of legitimation' or criteria for measuring achievement (Maton 2014). In short, what we say or do express principles of legitimacy or 'legitimation codes'. LCT comprises several *dimensions* or sets of concepts that explore different aspects of legitimacy (Maton 2016). Central to each dimension are concepts for analyzing the organizing principles underlying practices, dispositions and contexts as a particular species of 'legitimation code'. In terms of our needs here, these concepts bring knowledge into the picture by revealing the organizing principles generating its various forms. The dimension most directly relevant to exploring integration is Autonomy, which focuses on relations between sets of practices (such as subject areas) and conceptualizes their organizing principles as *autonomy codes*. We shall first define the concepts, then discuss how they are enacted using *translation devices* and *targets*. For reasons that become clear, we begin rather abstractly, before concretizing the meanings of concepts.

Autonomy codes

The dimension of Autonomy begins from the simple premise that any set of practices comprises constituents that are related together in particular ways. Constituents may be actors, ideas, institutions, machine elements, body movements, etc.; how they are related together may be based on explicit procedures, tacit ways of working, mechanisms, unstated orthodoxies, etc. The concepts of 'autonomy codes' explore how practices distinguish their constituents and their ways of relating from those of other practices. Put another way, the concepts examine how practices establish different degrees of insulation around their constituents and the ways those constituents are related together. These are analytically distinguished as:

- *positional autonomy* (PA) between constituents positioned within a context or category and those positioned in other contexts or categories; and
- *relational autonomy* (RA) between the relations among constituents of a context or category and the relations among constituents of other contexts or categories.

Each may be stronger (+) or weaker (−) along a continuum of strengths, where stronger represents greater insulation and weaker represents lesser insulation. Stronger positional autonomy (PA+) indicates that constituents of a context or category are relatively strongly delimited from constituents associated with other contexts or categories (strongly insulated positions); and weaker positional autonomy (PA–) indicates where such distinctions are less demarcated (weakly insulated positions). Stronger relational autonomy (RA+) indicates that the ways of relating constituents together are relatively specific to a set of practices (autonomous principles), and weaker relational autonomy (RA–) indicates that the ways of relating may be drawn from or shared with other sets of practices (heteronomous principles).

As shown in Figure 2.1, positional autonomy and relational autonomy are visualized as axes of the *autonomy plane*. Varying their strengths independently (PA+/−, RA+/−) generates four principal autonomy codes:

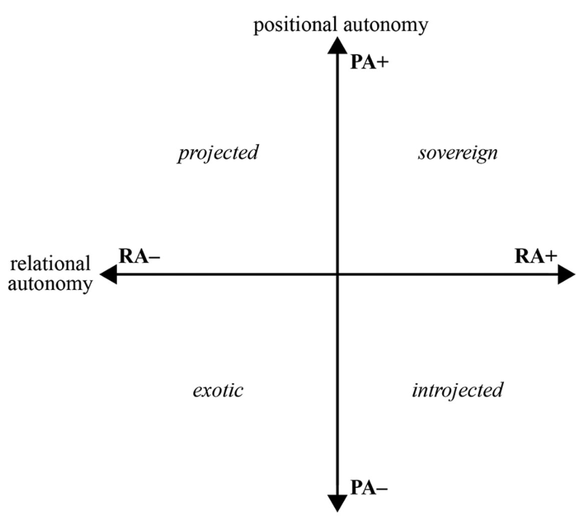

FIGURE 2.1 The autonomy plane (Maton 2018: 6)

- *sovereign codes* (PA+, RA+) of strongly insulated positions and autonomous principles, where constituents are associated with the context or category and act according to its specific ways of working;
- *exotic codes* (PA−, RA−) of weakly insulated positions and heteronomous principles, where constituents are associated with other contexts or categories and act according to ways of working from other contexts or categories;
- *introjected codes* (PA−, RA+) of weakly insulated positions and autonomous principles, where constituents associated with other contexts or categories are oriented towards ways of working emanating from within the specific context or category; and
- *projected codes* (PA+, RA−) of strongly insulated positions and heteronomous principles, where constituents associated with the specific context or category are oriented towards ways of working from elsewhere.

These concepts help address the need to make visible the forms of knowledge being taught and learned. Put simply, the four codes state that what matters are: internal practices and principles (sovereign codes); other practices and principles (exotic codes); other practices turned to intrinsic purposes (introjected codes); and internal practices turned to other purposes (projected codes). To explore processes that occur through time, such as classroom practice, one can analyze the different *pathways* traced around the plane by successive autonomy codes. There is an unlim-ited number of potential pathways (see Maton 2018). In this chapter we discuss the two pathways illustrated in Figure 2.2: *one-way trips* that begin in one code and end in another code; and *tours* that begin in one code, move through other codes, and return to their originating code. We shall show that autonomy tours in teaching practice enable, and one-way trips constrain the integration of 'mathematics' into 'science'. However, before doing so there remains the question of defining

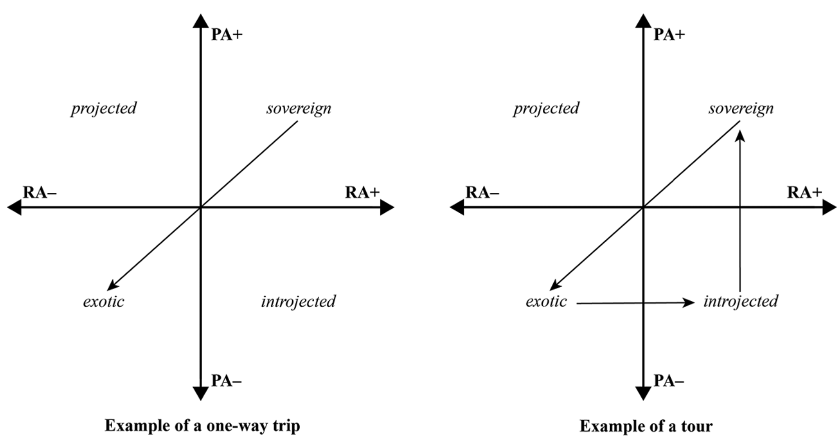

FIGURE 2.2 Examples of two autonomy pathways

'science' and 'mathematics' in a way that systematically embraces their variant and contextual nature. This is achieved through *translation devices* and *targets*.

Translation devices and targets

Thus far we have described 'autonomy codes' in abstract terms and without specific examples. This allows the concepts to be enacted across a wide range of diverse phenomena. Doing so offers the possibility of reaching beyond descriptions of specific instances of classroom practice to generate a general model of pedagogic integration. However, it also means one must be clear how the concepts are manifested within a specific object of study. In LCT this is achieved through 'translation devices' that relate concepts to data (Maton and Chen 2016). Table 2.1 is a *generic translation device* that relates autonomy codes to all forms of data. The device divides the continua of strengths for positional autonomy and relational autonomy into categories of progressively finer-grained levels of delicacy, from categories for stronger/weaker (*target/non-target*) through subcategories, use of which depends on the analysis.

To activate the device one asks: what constituents (practices, beliefs, ideas, actors, etc.) and what principles (purposes, aims, ways of working, etc.) are considered constitutive of *this* context or category, here, in *this* space and time, for *these* actors? This gives a 'target'. As shown in Table 2.1, target constituents embody stronger positional autonomy and all other, non-target constituents embody weaker positional autonomy; similarly, target principles embody stronger relational autonomy and all other, non-target principles embody weaker relational autonomy. These categories can be divided into subcategories by asking which target constituents and principles are considered *core* and which *ancillary* to the context or category, and which non-target constituents and principles are considered *associated* or *unassociated* with the target. Asking the same basic questions again generates a third level comprising *inner* and *outer* forms of core and ancillary targets, and *near* and *remote* forms of associated and unassociated non-targets.

TABLE 2.1 Generic translation device (Maton 2018: 10)

PA/RA	*1st level*	*2nd level*	*3rd level*
+ ↑	*target*	*core*	*inner*
			outer
		ancillary	*inner*
			outer
	non-target	*associated*	*near*
			remote
		unassociated	*near*
↓ −			*remote*

Activating the device allows us to aim directly at the problem of defining 'science' and 'mathematics'. Rather than shying away from knowledge practices varying across contexts, the notion of 'target' makes that the starting point for analysis. This is best shown by a concrete example. Here we draw on a major study of how secondary school teachers select, assemble and enact knowledge in their classroom practice when teaching at Stage 4 (Years 7–8) and Stage 5 (Years 9–10) in three secondary schools in New South Wales, Australia.[5] Data comprised videorecordings of lessons across whole units of study (6–8 hours each), interviews with teachers, all teaching materials, and student artefacts. In this chapter we shall discuss examples of classroom practice by two science teachers in Year 7 of secondary schooling.[6]

To enact 'target' first one considers *whose* view of the target to begin from, as other agents in a context (such as students in a classroom) may have different targets. A target is always *someone's* or *something's* conception of what makes a context distinctive and thus in our analyses always accompanied by possessives (e.g. his/her/their targets). Here, reflecting our concern with teachers' practices, we focus on their targets. Second, one must consider *what level* of their targets to examine. LCT concepts can be enacted at all levels of analysis; for example, a teacher's targets may include an entire curriculum, a unit of study, a lesson, a task, and so forth. Reflecting our concern with how teachers attempt to integrate mathematics into science to meet the needs of the Stage 4 curriculum, our specific translation device has curriculum stage as its first level and unit of study as its second level.[7] As summarized in Table 2.2, in interviews and pedagogic materials the teachers identified their target content (PA+) as the Stage 4 science syllabus in New South Wales and their target purpose (RA+) as teaching students that content. Put simply, here: *positional autonomy* conceptualizes where the ideas expressed in classroom practice are drawn from, the Stage 4 science syllabus (PA+) or elsewhere (PA–); and *relational autonomy* conceptualizes the purposes for which they being expressed, teaching and learning that science syllabus (RA+) or other purposes (RA–). Interviews and pedagogic materials further identified the teachers' *core* targets (++) as the specific science unit being taught, with other units in Stage 4 science considered *ancillary* targets (+).

TABLE 2.2 Simplified specific translation device for this analysis

PA/RA	*1st level*	*In this analysis:*	*2nd level*	*In this analysis:*
+	*target*	New South Wales Stage 4 syllabus for subject area	*core*	specific unit in target
↑			*ancillary*	other topics or years in target
↓	*non-target*	other contents or purposes	*associated*	other educational knowledge
–			*unassociated*	knowledge from beyond education

(Their *inner-core* targets comprised the content points they created for each specific lesson). In terms of non-targets, teachers viewed other educational knowledge (such as other subjects or other Stages and levels of education in science) as *associated* (–) with their target, and knowledge from beyond education as more distanced or *unassociated* (– –).

By 'targeting' analysis we can identify 'science' as it is constructed in the specific context under study, avoiding universalizing essentialism. By translating that particular set of empirical ideas and practices into 'autonomy codes', we can move beyond context-bound, endlessly varying descriptions of difference, avoiding relativism. We can *both* embrace the specificities of each context *and* compare practices across different contexts, capturing the endless forms most wonderful that are 'science'. Moreover, targeting 'science' allows us to analyze the movement of ideas and practices between subject areas as they are recontextualized and integrated. Given that the 'target' depends on the object of study, no single idea, practice, belief, etc. is always and everywhere the same code. A practice may be moved around the plane; for example, in our discussion of an autonomy tour below, 'graphing' is successively constructed as an exotic code (as mathematics content for mathematical purposes), an introjected code (mathematics content for learning science), and a sovereign code (science content for learning science).

We can now begin to explore what teaching practices enable or constrain integration of 'mathematics' into 'science'. To illustrate how, we shall analyze classroom practices by two teachers (mentioned above) from Year 7 schools teaching the same unit from the same state curriculum. The difference between the examples lies in the autonomy pathways traced by their teaching practice. In the first example, the teacher fails to integrate mathematics into science. He leads students on a *one-way trip* out of 'science' into an activity he describes as 'maths' that remains segmented from his target knowledge. In the second example, a different teacher takes students on an *autonomy tour* that integrates non-target 'mathematical' knowledge about creating graphs into her target 'science' knowledge about Earth's seasons.

One-way trip from science 'to do some maths'

Our example of teaching that fails to integrate 'mathematics' into 'science' comprises a distinct phase of activity spanning an entire lesson of over 50 minutes. The teacher's core target for the wider unit, as later described in an interview, is:

> to teach them [students] about the universe and our solar system and what's beyond Earth. Some of them didn't quite understand the relationships in the universe so we have to make them clearer for them.... How we get night and day or how you get the different seasons.

This reflects a 'sub-strand' of the state curriculum for Year 7 science entitled 'Earth and space sciences', which is 'concerned with Earth's dynamic structure and its place in the cosmos' (NESA).[8] In the lesson discussed here, the teacher tells students they are going to make sense of the scale of the solar system, but quickly shifts classroom practice into using numbers to calculate percentages, detached from learning about the science content. This takes students on a *one-way trip* from the teacher's sovereign code into an exotic code, a pathway that does not return to his target content or target purpose. After 52 minutes he draws the 'maths' activity to a close by declaring: 'I know it's confusing'.

Pathway into 'a lot of numbers'

The teacher begins the lesson by showing students a short YouTube video entitled 'The smallest to the biggest thing in the universe'. Starting from hypothesized entities in quantum physics (such as strings), the video zooms outwards through ever-larger phenomena to end with the known galaxy. He then segues to the activity that will consume the rest of the lesson:

TEACHER So, as you saw, some of those distances and some of those sizes don't really mean a lot to us, because we just can't fathom the distances involved, okay? So some of the other distances, especially in our solar system, are the same. So what we're going to do is, we're going to put the distances and the sizes relative to Earth. Okay? So we're going to put all the planets and the distance to the Sun and we're going to make them relative to the Earth.

At this point, the intended classroom practice is to explore content about the solar system (stronger positional autonomy) for the purpose of understanding the solar system (stronger relational autonomy); i.e. within the teacher's core target – deep inside his sovereign code.

The teacher then directs students to 'draw up a table' of 'seven columns and 10 rows' and shows a PowerPoint slide of a table, to which he adds two column titles by hand on the whiteboard, reproduced here as Table 2.3. He tells the class to 'copy down this information if you haven't already got it'. After reminding students they

TABLE 2.3 Table provided by teacher for activity

	Radius (km)	*Distance from the sun (km)*	*Time to orbit around the sun*	*Time taken to turn once on its axis*	*Diameters as % of Earths*	*Distance as % of Earths*
The Sun	695800					
Mercury	2439.7	57910000	88Ed	58d15h30m		
Venus	6052	108200000	224.7Ed	116d18h0m		
Earth	6371	149600000	365.25Ed	1d		
Mars	3390	227900000	686.97Ed	1d0h40m		
Jupiter	69911	778500000	12Ey	9h56m		
Saturn	58232	1433000000	29Ey	10h39m		
Uranus	25362	2877000000	84Ey	17h14		
Neptune	26422	4503000000	165Ey	16h06m		

had written down diameters of planets and their distances from the sun in a previous lesson, he explains:

TEACHER I want you to add this information, these two [points to third and fourth titled columns], because these two are relative to the Earth. … Since we've already got the information just do four columns, because we're going to do some maths.

That the teacher declares 'we're going to do some maths' does not by itself indicate a shift beyond 'science' into non-target knowledge. For example, 'maths' could refer to procedures or ideas he has previously integrated into his target – already scientized mathematics, so to speak. Similarly, calculating percentages is not necessarily beyond his target. As we shall see in our second example, no practice is always a specific code. To identify the autonomy code, one must leave aside assumptions of what is 'science' or 'maths' and begin from the teacher's target. This he described, and teaching materials revealed, as the Stage 4 science syllabus. A strand of this syllabus entitled 'science inquiry skills' includes for Year 7: 'Summarise data, from students' own investigations and secondary sources, and use scientific understanding to identify relationships and draw conclusions based on evidence'.[9] So, calculating diameters and distances from the sun of planets as percentages of those of Earth could *potentially* sit within the teacher's target. However, the syllabus emphasizes that such 'science inquiry skills' give students 'the tools they need to achieve deeper understanding of the science concepts' – they must be 'closely integrated' with learning the 'science knowledge' outlined in a strand entitled 'Science understanding'. This strand includes having 'students view Earth as part of a solar system, which is part of a galaxy, which is one of many in the universe, and explore the immense scales associated with space'. Thus, whether calculating percentages lies within the teacher's target depends on whether he integrates its content or purpose with viewing Earth as part of a solar system and exploring the immense scales of space. As the teacher stated at the outset, this was his intention. However, as we shall see, in practice he does not relate the activity to any such 'science understanding'.

Instead, as illustrated by Figure 2.3, the teacher quickly shifts the task into an exotic code in which non-target content is used for non-target purposes. He weakens positional autonomy by disconnecting the contents of the table from his target topic. For example, he describes its contents as 'information' five times in just the first minute: 'copy down this information … just use your information … add this information … you've already got the information … we've already got the information'. At the same time he weakens relational autonomy by describing the purpose as 'to do some maths' without relating this either to procedures previously integrated into 'science' or to learning new syllabus content.[10] For example, when responding to questions from students, he states:

TEACHER Just do the last two columns and then add two more because we're going to do some maths in the last two.

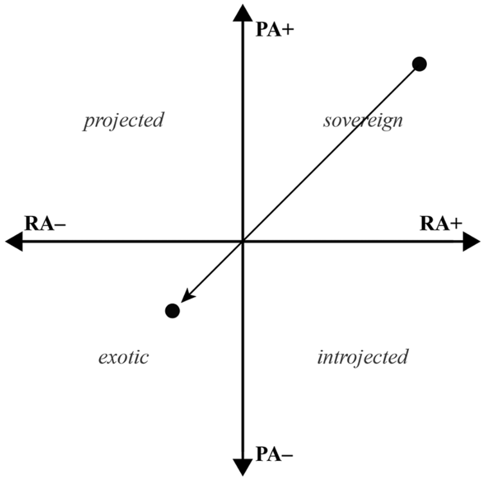

FIGURE 2.3 Shift from sovereign code to exotic code

Following his instructions, some students copy numbers from the table while a series of students ask the teacher which numbers they should copy. This concern with numbers as only numbers continues when, after eight minutes, he addresses the whole class:

TEACHER Alright, do we know how to work out the percentage for ….? So which one do we have to divide by? [Student name], what do you think? To work out Mercury, the percentage compared to Earth? What do you think we'd have to do?

STUDENT Divide it by a hundred?

TEACHER No, no, no. Alright. What we do [draws on whiteboard: $\frac{\text{Mercury}}{\text{Earth}} \times 100$]. Alright, so distance percentage [pointing to last column title] is the distance from the sun. Okay? So to work out the percentage, you divide each of the planets by the Earth's diameter.

Over the next 15 minutes the teacher repeats similar instructions to a series of individual students, each time describing what 'information' must be multiplied or divided to 'give you a percentage'. When he mentions the names of planets, the teacher is referring to specific empty cells in the table – shown by physically pointing to the cell – rather than to planets. More often, the teacher refers to the content as 'information' or 'number', such as:

TEACHER Divide that number [pointing to the table] by that number [pointing] for the distance; that number [pointing] by that number [pointing] for the diameter. Alright? That's what you're supposed to be doing.

The content is from neither the teacher's target of the syllabus nor his core target of learning about the solar system; the purpose is to 'work out the percentage' or 'do maths'. The knowledge being expressed thus remains within an exotic code (Figure 2.3).

During the course of the activity some students ask questions that represent opportunities to relate the activity to the issue of grasping the scale of the solar system. For example, one student asks whether people are made of 'planks', another asks what is smaller than a 'string' (both mentioned in the earlier video), and a third asks 'Are we made of stardust?'. The teacher's responses – 'No', 'Didn't you watch the video?', and 'What do you think?', respectively – do not connect to target content or turn the questions to his target purpose.

After 38 minutes the teacher asks students to call out numbers for cells in the column 'Diameters as % of Earths'. He then raises the question: 'What do these percentages actually mean?'. This is an opportunity to strengthen relational autonomy by turning these numbers to the purpose of viewing Earth as part of a solar system or exploring the immense scales involved, and an opportunity to strengthen positional autonomy by connecting the 'information' to knowledge about the solar system. A student suggests 'A lot of numbers', an answer that accurately reflects the exotic code characterizing the activity. The teacher leaves his own question unanswered. The class then repeats the pattern outlined above: students make calculations (for the 'Distances' column), the teacher repeats similar instructions to students, and numbers are solicited from the class. Classroom practice stays in an exotic code. The activity is ended after 52 minutes by the teacher saying 'I know it's confusing' and announcing that they will look at 'day and night' in the next lesson.

Summary: 'That's maths!'

The autonomy pathway traced by this lesson represents a one-way trip out of the teacher's target of 'science' in order 'to do some maths'. As portrayed by Figure 2.3, the knowledge expressed in classroom practice shifts from a fleeting sovereign code to a very long stay in an exotic code. As shown by the times given above, almost the entire 'science' lesson is 'maths'. The teacher could have chosen to conduct this activity inside his sovereign code by closely integrating the numeric activity with his syllabus target. Instead, he chooses to project the activity as beyond his target, as doing 'maths' to 'work out the percentage'. As we discuss below, this code shift is not necessarily antithetical to integrating this 'maths' into 'science'. At any point during the lesson, the teacher could strengthen positional autonomy by connecting to his target content or strengthen relational autonomy by turning non-target content (calculating percentages) to his target purpose. Instead, he keeps classroom practice in the exotic code: the content remains numbers and calculations, and the purpose remains using numbers to calculate other numbers. Thus, the shift to an exotic code does not integrate 'maths' into 'science'. Late in the lesson, in response to a student declaring 'This is hard, sir', the teacher replies 'That's maths! We still have to do maths in science'. However, this 'maths' is not 'in science' and so knowledge of calculating percentages remains strongly segmented from knowledge of the solar system.

Autonomy tour integrating 'mathematics' into 'science'

To illustrate how 'mathematics' can be integrated into 'science' we turn to a different teacher at a different school but teaching the same unit ('Earth and space sciences') at the same level (Year 7 secondary school). The example begins in the second lesson of a unit on the causes of Earth's seasons, as students transform their results from a practical experiment into graphs. In the first lesson students had conducted an experiment to explore the effect on temperature of the angle at which sunlight strikes the Earth's surface. In groups, students used a lamp to represent the sun, and a wooden block to represent the Earth. Varying the angle of the lamp to the block (15, 30, 60 and 90 degrees), they recorded the temperature of the block at different times (initial, 2.5 minutes, 5 minutes) from an attached thermometer. Prior to the experiment each student had written a hypothesis of whether increasing the angle would increase, decrease or have no effect on the temperature. The second lesson directly builds on this activity. The teacher begins by setting out her (inner-core) target:

TEACHER What we will be doing today is looking at those results, graphing the results and then talking about what it is that we were actually trying to model.

Over the next 35 minutes the teacher leads students on an autonomy tour through those activities: from her sovereign code (discussing their results), through an exotic code (recapping 'graphing rules'), and an introjected code (applying those rules to graph their results), before returning to her sovereign code (by relating the resulting graphs to Earth's seasons). As a result, the graphing activity becomes integrated into 'science'.

A tour through 'graphing'

The teacher begins by recounting the experiment and then solicits students' overall findings:

TEACHER So looking at your results there, who can give me a statement about what their results did?

STUDENT As the angle of the block increased, the temperature increased.

TEACHER Fantastic. I love that. That's a really great statement. Did someone get something different in their results?

The teacher thus begins deep inside her sovereign code. Both content (experiment modelling a factor in Earth's seasons) and purpose (to learn about the results) are located within her *inner-core target* for the lesson. After discussing the findings of several students, she announces:

TEACHER Here [on the whiteboard] is your table that you should have had drawn up from the last lesson. We are going to graph… I want you to think about the graphing rules and start getting yourself ready for graphing.

As we emphasized, no activity is intrinsically a specific autonomy code. 'Graphing' is not necessarily non-target – 'graphing' can be mathematical or scientific. To determine autonomy codes we must consider the teacher's target: the Stage 4 science syllabus. A strand entitled 'science inquiry skills' includes for Year 7: 'Construct and use a range of representations, including graphs, keys and models to represent and analyze patterns or relationships in data'.[11] Thus, graphing is potentially within the teacher's target. However, as discussed in the previous example, the syllabus describes 'science inquiry skills' as giving students 'the tools they need to achieve deeper understanding of the science concepts' by being 'closely integrated' with learning the 'science knowledge' outlined in the syllabus strand 'Science understanding'. This strand includes 'how changes on Earth, such as day and night and the seasons, relate to Earth's rotation and its orbit around the sun'. Thus, whether graphing lies within the teacher's target depends on whether she integrates its content or purpose with learning about Earth's seasons. Here, we shall show that she begins by separating graphing in terms of both content and purpose, then turns it to purpose, before connecting its content.

This tour begins with the teacher recapping her 'graphing rules' separately from Earth's seasons. Continuing on from the preceding classroom quote, she says:

TEACHER So, who can remind me about what the rules are for graphing?
STUDENT Y versus X.
TEACHER Y versus X. How do we know which one goes where?
STUDENT The independent variable goes on one side.
TEACHER The independent variable goes on one of them. Yes, that's good.
STUDENT And the dependent variable …
TEACHER … goes on the other one. The thing that is the most regular, which is usually your IV [independent variable], goes on the X, and your DV [dependent variable] goes on the Y.

This recap embodies: weaker positional autonomy (PA–), as these 'graphing rules' are not related to Earth's seasons; and weaker relational autonomy (RA–), as the purpose is recapping the 'graphing rules' rather than learning about Earth's seasons. As portrayed in Figure 2.4, the teacher has shifted from deep inside her sovereign code to just inside an exotic code.[12]

Thus far, classroom practice traces the same pathway as the previous example. However, where that teacher remained within an exotic code for the entire lesson, this teacher does not stay for long. She quickly shifts the class into a third code by repurposing the knowledge of 'graphing rules':

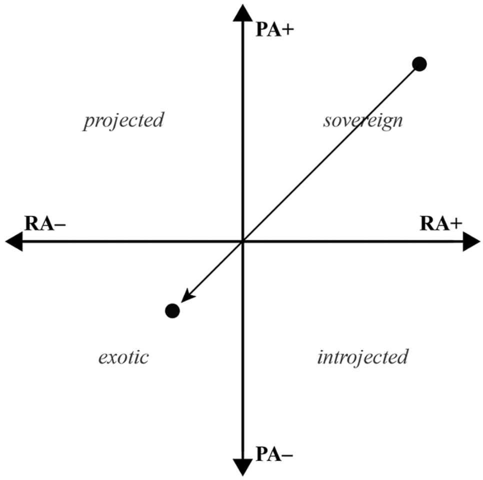

FIGURE 2.4 Shift from sovereign code to exotic code

TEACHER Now, in *this* experiment, who can tell me – there's a little problem. Have a look at our data. Can you tell me which one goes on the X and which one goes on the Y?

One student suggests 'the angle' should go on the Y-axis, another suggests the X-axis, and the teacher asks students for the locations of 'temperature' and 'time'. After a student exclaims 'Wait! What?', the teacher explains the problem:

TEACHER So in this experiment we've got three sets of data, okay? So, this one's going to kind of break the rules a tiny bit. The easiest way for us to do this is that you're going to have [...] 'time' on the X, 'temperature' on the Y, and four different lines. The four lines you're going to draw is one line for 15 degrees, one line for 30 degrees, one line for 60 degrees and one line for 90.

The content of discussion – rules about locating variables on axes – remains weakly integrated with what the experiment reveals about Earth's seasons and so embodies weaker positional autonomy. However, the purpose is to create a graph that can show this knowledge, embodying stronger relational autonomy. As portrayed in Figure 2.5, this shifts classroom practice into an introjected code.[13]

This introjected code is maintained throughout the graphing activity. While students apply the adapted 'graphing rules' to their results, the teacher alternates between addressing the whole class and advising individual students; for example, to the class:

TEACHER All right! Along the X-axis, there will be three values: the X-axis has your time on it. There will be a time for five minutes, there will be a time for two and a half, and there will be a time for 'initial', which we can call zero, zero minutes. Okay? …

Then (continuing straight on), she looks at a student's workbook and asks:

TEACHER Why is this word here?
STUDENT 'Angle'.
TEACHER We are not doing 'angle' like that.
STUDENT Oh, whoops!
TEACHER Just follow what's going on here. This is the X.
STUDENT Okay.
TEACHER Okay? So 'temperature' does not belong there. X along here is 'time'. Y along here is 'temperature'.

Discussion continues along these lines for the next 12 minutes while students draw their graphs. As these quotes illustrate, the content of discussion involves locating variables on axes, setting ranges for variables, sizing the graph, using symbols, labelling, evenly spacing intervals, creating a key for symbols, and avoiding overlapping lines. Content is thus not related to what the results of the experiment might reveal about Earth's seasons: weaker positional autonomy. However, the purpose is to create graphs which help show what the experiment might reveal about Earth's season: stronger relational autonomy. As the teacher explains to the whole class: 'This is a better way of presenting the data than it is to look at a table. … Straight away when you look at this graph, you can see which one has increased in temperature fastest.' Graphing thus manifests here as an introjected code (Figure 2.5).

Once students have completed graphing, the teacher shifts classroom practice back to her sovereign code. Students write in their workbooks a 'conclusion' of

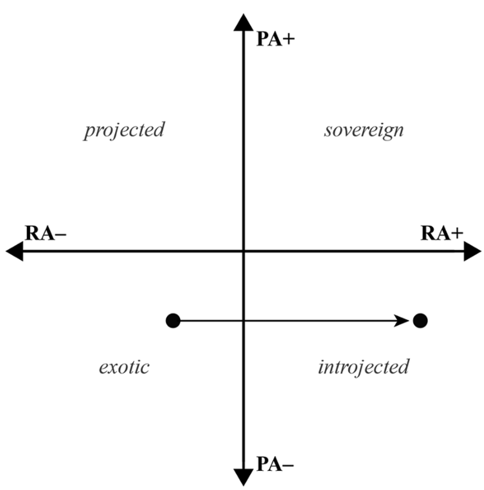

FIGURE 2.5 Shift from exotic code to introjected code

what their graphs show and whether this supports or refutes their previous hypothesis and a 'discussion' of whether their results were consistent and expected, what errors occurred, how they could improve their experimental design, and what they could do to test the idea further. The teacher then leads a discussion of the graphs that continues this concern with what they reveal about the focus of the experiment:

TEACHER What did we learn? [...]
STUDENT We learned that the steeper the angle, the hotter the temperature.
TEACHER Good. The steeper the angle of the block, we got a greater increase in our temperature. Who can tell me why? Why did it get hotter? …
STUDENT Because the core of the block is closer to the light.
TEACHER Good. The middle part of the block, as you increase the degrees, makes it closer to the light. Good.
STUDENT Because it's getting more direct rays when it's on a higher angle as opposed to when it's on …
TEACHER Good. When we have a higher angle, we have more of those light rays striking the block, and those light rays then can heat up the block more effectively than the ones that are just skimming over the top.

This shifts the content and purpose to exploring the results of the experiment. The graphing activity has now been integrated with the experiment. The teacher then consolidates this sovereign code to integrate the experiment into discussion of Earth's seasons. First, she emphasizes her target purpose – stronger relational autonomy:

TEACHER Okay, but what's the point in doing this? Are we really interested in whether or not blocks can heat up with a lamp?
STUDENT No!
TEACHER No? Who can remember the word I used to describe what this experiment was? Starts with an 'm'.
STUDENT A model?
TEACHER A model. Fabulous. This was a model. It was a model of the Earth and the sun.

Second, she explains differences between the model and reality and how those differences shape the experience of heat on Earth, content that embodies stronger positional autonomy:

TEACHER Does the Earth change its angle?
STUDENTS Yeah.
STUDENTS No!
STUDENTS It rotates.
TEACHER It rotates – good. When the Earth rotates the angle changes. … When the Earth rotates, we change the angle that the sunlight is striking the Earth.

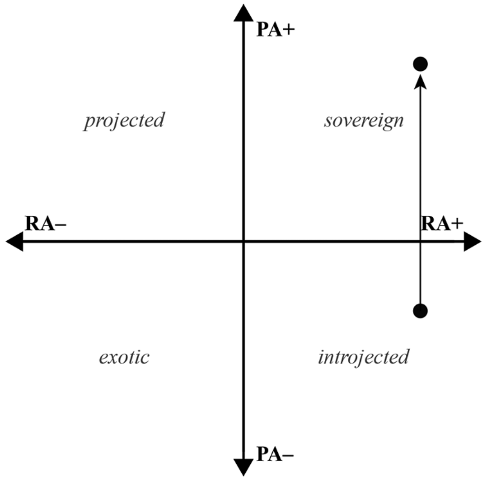

FIGURE 2.6 Shift from introjected code to sovereign code

The teacher then segues to an animation that shows the Earth rotating, sunlight striking the surface, and how this creates night and day. As portrayed in Figure 2.6, the teacher has shifted to content from the syllabus for the purpose of learning that syllabus – her sovereign code. The experiment and graphing activity are now integrated into the wider discussion of Earth's seasons.

Summary: separate, repurpose, integrate

Analysis of workbooks from this lesson suggests that students successfully translated their tables of experimental results into graphs and their graphs into conclusions about the effects of the angle of sunlight on temperature. This is no simple feat. Studies of science education widely report that many secondary school and university students struggle with understanding and interpreting graphs (Planinic *et al.* 2012). Indeed, the students here were *creating* graphs from data. Moreover, the teacher is also laying foundations for students' future learning. As she highlighted in an interview, her Year 7 students 'have no experience with graphing for science or they've got no experience with drawing tables for science – we've really got to teach that stuff in the beginning, because then we expect them to follow it through' subsequent years of school science.

In terms of the knowledge involved, this learning was supported by teaching which traced an autonomy tour from 'science' through graphing and back to 'science'. As shown by Figure 2.7, classroom practice went through:

(1) the teacher's *sovereign code* by discussing results of the experiment
(2) an *exotic code* when discussing 'graphing rules'

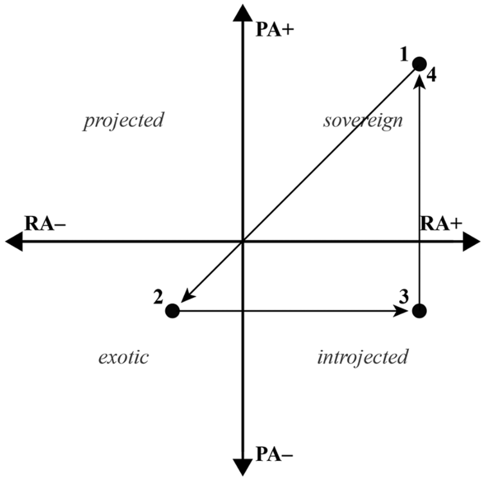

FIGURE 2.7 Autonomy tour with graphing results of an experiment

(3) an *introjected code* when adapting those 'rules' to graphing results of the experiment
(4) the teacher's *sovereign code* when translating graphs into conclusions about what was modelled by the experiment

Given that graphing could be discussed in ways that locate it either within or beyond the teacher's target, one question this pathway raises is why she chose to leave her sovereign code. Students had learned the 'graphing rules' in previous lessons, so the teacher could have treated them as part of her target – as already integrated into 'science' – by relating them directly to the experiment's result. However, this strategy would not have captured the distinct nature of this experiment – that there are three variables. By constructing the 'graphing rules' as an exotic code, the teacher keeps that knowledge separate from the specific 'science' content, allowing her to connect to ideas that students have already learned in a way that highlights how graphing will be different here. By separating this 'mathematics' from the 'science', she is able to select ideas from the 'graphing rules', repurpose those ideas, and integrate their use into her target knowledge. Separation comprises a shift to an exotic code. Repurposing involves strengthening relational autonomy by turning the 'rules' to the purpose of graphing results from this experiment – an introjected code. Integration involves strengthening positional autonomy to translate the resultant graphs into knowledge about Earth's seasons – a sovereign code. In our previous example, leaving 'science' for 'maths' was a one-way trip that failed to integrate the 'maths' back into 'science'. Here, leaving 'science' was a precursor to successful integration through an autonomy tour.

Conclusion

A key challenge faced by research into science education is developing teaching practices that select, recontextualize and integrate mathematical knowledge to support the learning of scientific knowledge. We argued that existing approaches offer insights into student learning of ways of knowing but typically sideline teaching practice and the forms of knowledge being taught and learned. To help address these blind spots we offered a complementary approach centred on *autonomy codes*, *pathways*, and *targets*. The concepts of *autonomy codes* foreground the forms taken by knowledge practices expressed in classroom discourse, focusing on a key relevant feature for integration: their relations with other knowledge practices. Analyzing the *pathways* traced by successive codes reveals how different teaching practices enable or constrain the integration of 'mathematics' into 'science'. Using these concepts, we analyzed contrasting examples of classroom practice from secondary school lessons in science that suggest *autonomy tours* support and *one-way trips* obstruct the integration of 'mathematics' into 'science'.

One implication of the analysis is that pathways to successful integration are not necessarily direct. In the tour example, the separation of 'mathematical' ideas from 'scientific' knowledge was an important precursor to its subsequent integration. This speaks to an issue highlighted by science education research: student difficulties with recognizing they are exploring mathematical ideas when presented in scientific contexts (e.g. Planinic *et al.* 2012). Constructing the 'mathematics' knowledge as separate (exotic code) is an opportunity to highlight the specific constellations of meanings within which that knowledge is located and which underpins its 'mathematical' nature. Turning those ideas to a 'scientific' purpose (introjected code) and connecting those repurposed ideas to 'scientific' knowledge (sovereign code) recontextualizes the ideas within a new constellation of meanings. A tour thus offers the possibility of making explicit those constellational differences. It makes the knowledge visible to students. As a growing body of research is showing, not all tours involve this specific combination of autonomy codes, but all involve departing and returning. If classroom discourse remained within a sovereign code throughout, these constellational differences would not be made visible; and if classroom discourse did not return, the recontextualization of ideas between constellations would not be possible.

While 'autonomy codes' help bring knowledge practices into the picture, we should emphasize that the concepts are not limited to that focus – one can also analyze students' dispositions, interactions and changing understandings. In short, the concepts can be enacted to examine *both* forms of knowledge practices and ways of knowing. This would enable 'matches' and 'clashes' to be identified, supporting the development of appropriate pedagogic practices.

Autonomy codes are, of course, not the only feature of knowledge practices, and autonomy tours are not the only factor in integration. As mentioned earlier, a key issue highlighted by physics education research is that science involves 'learning to blend physical meaning into mathematical representations and use that physical meaning in solving problems' (Redish 2017: 25). This can be traced through autonomy pathways: the second teacher condenses 'mathematical' ideas with empirical meanings when repurposing 'graphing rules' (from exotic code to introjected code) and when relating the resultant graphs to explaining Earth's seasons (from introjected code to sovereign code). In contrast, the 'maths' of the first teacher remains disconnected from empirical referents. However, this changing attribute is not *directly* conceptualized by autonomy codes. For this one can draw on the LCT dimension of Specialization to conceptualize relations with empirical referents in terms of *epistemic relations* and to reveal that integration of 'mathematics' into 'science' involves processes of *ontic condensation* (Maton 2014: 175–84; Wolff 2017).

Nonetheless, *autonomy codes* offer a valuable start for bringing knowledge practices into view and *autonomy tours* may represent a key to pedagogically integrating mathematics into science learning. Using the notion of *targets* to enact these concepts resolves a major obstacle to generating a general model of pedagogic integration: the problem of defining 'mathematics' and 'science' when the constitutive features of each subject, and relations between them, vary across contexts. 'Targeting' these constitutive features in *translation devices* addresses the contextual and changing nature of whether practices are constructed as 'mathematics' or as 'science'. 'Targeting' also allows studies to translate the specificities of each empirical context into concepts capable of generating a general model. We can examine, for example, the role of 'autonomy tours' in integration, howsoever 'science' and 'mathematics' are defined. By targeting science in this way, we can indeed hit a bullet with a smaller bullet, while blindfolded and riding a horse.

Notes

1 Research into science education is divided into 'science education research' for schooling and disciplinary specialisms (such as 'physics education research') at university level. In our empirical examples from secondary schooling, 'science' is taught, but our argument is not limited to one discipline or level of education.

2 Work discussing 'disciplinary discourse' (e.g. Airey and Linder 2009) points towards knowledge but reduces this 'discourse' to representations of more fundamental 'ways of knowing', leading again to studies of student 'fluency' in ways of knowing – subjectivism returns.

3 In LCT this is to say that *production fields*, *recontextualization fields*, and *reproduction fields* have their own distinctive logics (Maton 2014: 47–52).

4 I must emphasize 'complemented': to replace studies of ways of knowing with analysis of forms of knowledge would be to continue taking part of the picture for the whole. Both knowledge and knowing are significant.

5 This study was funded by the Australian Research Council (DP130100481).

6 These examples were introduced in Maton (2018) and are more extensively analysed here.

7 In Chapter 4 of this volume I focus on how teachers integrate a multimedia object into a specific task, so each teacher's *target* is the lesson and their *core target* is the specific task.

8 During data collection the curriculum authority was the New South Wales Board of Studies. Though renamed the New South Wales Education Standards Authority (NESA), its syllabus remains the same at the time of publication. All 'NESA' quotes are from https://syllabus.nesa.nsw.edu.au/stage-4-content/.
9 All quotes in this paragraph are from NESA.
10 In Figure 2.3 the shift is to just inside the exotic code because content and purpose concern educational knowledge or *associated non-targets* (PA–, RA–).
11 All quotes in this paragraph are from NESA.
12 Both content and purpose may be non-target but still concern educational knowledge, so embody an *associated* exotic code.
13 The pathway in Figure 2.5 shifts to the far right, indicating extremely strong relational autonomy, because creating a graph for the experiment's results is within the teacher's *inner-core target* purpose.

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