

Academic formalisms: toward a semiotic typology

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Highly technical discourse regularly uses a range of formalisms for organising its knowledge. Resources such as mathematics, system networks, tree diagrams and nuclear equations occur through varied disciplines and are crucial components of the texts students need to read and write to be successful in these areas. For the development of literacy pedagogy that can be embedded in disciplinary teaching and learning, an understanding of how these formalisms work and why they occur is vital. To date, however, multimodal discourse analysis informed by systemic functional linguistics (hereafter SFL) has not provided comprehensive descriptions of academic formalisms and the work that has been done has focused on specific resources in specific disciplines (such as mathematics; O'Halloran 2005, Doran 2017). This chapter extends this work by exploring a set of formalisms used in linguistics and physics to ascertain their role in building technical knowledge, and to highlight some of the properties they share across disciplines.

We begin by looking at some phases of technical discourse across physics and linguistics. In the first, a university physics text-book describes the behaviour of gases (Young and Freedman 2012: 591-592; bold and italics in original):

Measurements of the behaviour of various gases lead to three conclusions:

1. The volume V is proportional to the number of moles n . If we double the number of moles, keeping pressure and temperature constant, the volume doubles.
2. The volume varies *inversely* with the absolute pressure p . If we double the pressure while holding the temperature T and the number of moles n constant, the gas compresses to one-half of its initial volume. In other words, $pV = \text{constant}$ when n and T are constant.
3. The pressure is proportional to the *absolute* temperature. If we double the absolute temperature, keeping the volume and number of moles constant, the pressure doubles. In other words, $p = (\text{constant})T$ when n and V are constant.

These three relationships can be combined neatly into a single equation, called the **ideal-gas equation**:

$$pV = nRT \quad (\text{ideal-gas equation})$$

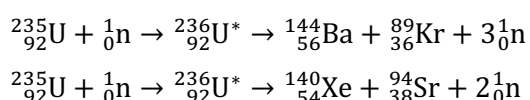
Text 1. Young and Freedman (2012: 591-592)

For outsiders to physics, this text is likely impenetrable. It is written for those with significant training in physics and is introducing a highly technical component of its overall knowledge structure – the ideal-gas equation. What we will be concerned with in this chapter is why the text introduces the mathematical equation $pV = nRT$ at the end of this phase, rather than just continuing with language.

Later on in the book (pg. 1465), when introducing the nuclear reactions of nuclear fission (used in nuclear power and weapons) and nuclear fusion (used by the sun to produce energy), there is a shift from language to a different type of formalism referred to as nuclear equations. In the following excerpt, the nuclear equation (indented in the text) does not synthesise what comes before (as in the previous example), but is used as a point of departure for a discussion of kinetic energy in fission:

Fission Reactions

You should check the following two typical fission reactions for conservation of nucleon number and charge:



The total kinetic energy of the fission fragments is enormous, about 200 MeV (compared to typical α and β energies of a few MeV). The reason for this is that nuclides at the high end of the mass spectrum (near $A = 240$) are less tightly bound than those nearer the middle ($A = 90$ to 145). Referring to Fig. 43.2 [not shown], we see that the average binding energy per nucleon is about 7.6 MeV at $A = 240$ but about 8.5 MeV at $A = 120$. Therefore a rough estimate of the expected *increase* in binding energy during fission is about 8.5 MeV – 7.6 MeV = 0.9 MeV per nucleon, or a total of $(235)(0.9 \text{ MeV}) \approx 200 \text{ MeV}$.

Text 2. Young and Freedman (2012: 1465)

The nuclear equations here indicate a series of events in the process of nuclear fission. Stepping through the first equation, it says that Uranium-235 (${}^{235}_{92}\text{U}$) collides with a neutron (${}^1_0\text{n}$) to produce (\rightarrow) a highly energised Uranium-236 atom (${}^{236}_{92}\text{U}^*$) which then decays (\rightarrow) into three distinct components: a Barium-144 atom (${}^{144}_{56}\text{Ba}$), a Krypton-89 atom (${}^{89}_{36}\text{Kr}$) and three neutrons ($3{}^1_0\text{n}$). The nuclear equation distils this quite technical event into a single symbolic formula.

Moving from physics to linguistics, we see similar shifts. In his seminal description of English transitivity, Halliday (1967:39-40), describes a set of process types in English before encapsulating the description in a system network:

Two of the three process types are each associated with only one participant, non-directed action with actor and ascription with attribuant; structurally, that participant is the subject in each case. The third, directed action, is associated with two participants, actor and goal, either of which may be the subject. The four examples could thus be grouped as follows:

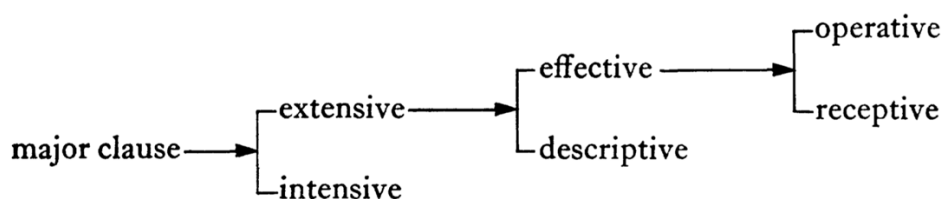
Process types:

	(S=actor)	(S=goal)
directed action	(i)	(i)
	she washed the clothes	the clothes were washed
non-directed action		(iii)
(S=actor)		the prisoners marched
ascription		(iv)
(S=attribuant)		she looked happy

Let us now represent these in terms of grammatical features of the clause using the following labels:

<i>extensive</i>	clause with 'action' process-type
<i>effective</i>	clause with 'directed action' process-type
<i>operative</i>	clause with 'directed action', subject as actor
<i>receptive</i>	clause with 'directed action', subject as goal
<i>descriptive</i>	clause with 'non-directed action' process-type
<i>intensive</i>	clause with 'ascription' process-type

These features may be organized in systems ordered in delicacy as follows:–



Text 3. Halliday (1967:39-40)

Here, the description of process types is reworked in language until it is synthesised into the culminating system network. Read from left to right, the network indicates that there are two types of major clause: extensive and intensive. Within extensive there are two further types, effective and descriptive, and within effective there are two further types, operative and receptive. The system network synthesises these types of clause in one diagram.

In another example from linguistics, after explaining context-sensitive grammars and exploring some example sentences, Lyons (1967) chooses to represent an example *the chimpanzee eats the bananas* through a tree diagram. A similar diagram to this is drawn in Figure 1.

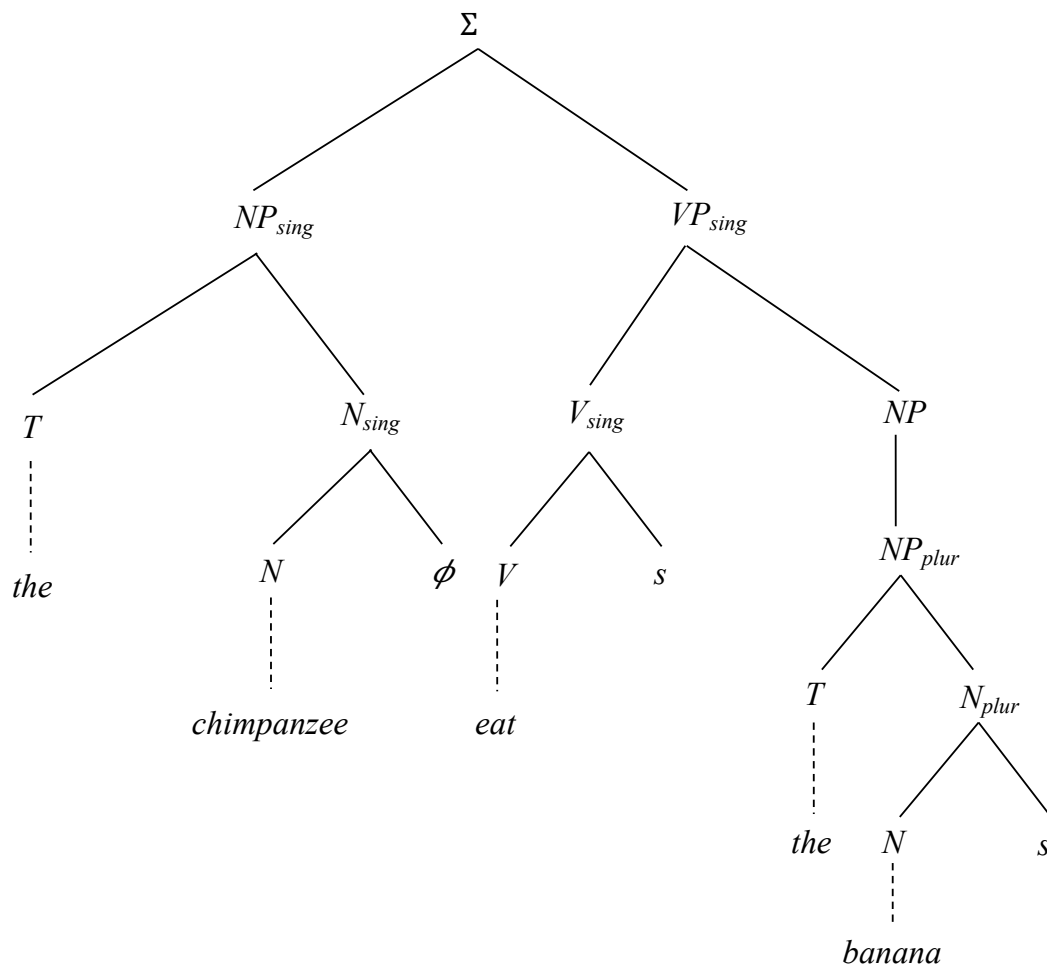


Figure 1. Tree diagram

Read from top to bottom, this says that a sentence (Σ) includes a singular noun phrase (NP_{sing}) and a singular verb phrase (VP_{sing}). Looking at just the NP_{sing} branch on the left, the tree also specifies that the noun phrase includes an article (T) and a singular noun (N_{sing}), which in turn includes a noun stem (N) and a zero suffix (\emptyset). Finally, the dotted line indicates that the noun is substituted with the word *chimpanzee*.

These four formalisms all occur in highly technical discourse and are used to synthesise key aspects of the knowledge of their disciplines. But at first glance it is not entirely clear exactly why they are used or how we can understand them in terms of their meaning-making. To see these as a simple short-hand for language – some sort of abbreviation or acronym – is to miss the functionality of these resources for distilling large amounts of intersecting technical meaning (Martin 1993: 252). Neither does it account for their role in deriving new technical meanings (Doran 2017, 2018a; Martin 2013). The fact that formalisms are used so

extensively across different disciplines suggests they form a crucial component of much academic discourse. But the fact that there are many diverse types of formalism – system networks, tree diagrams, nuclear equations and mathematics being only four of innumerable relatively discipline-specific formalisms – suggests they each maintain their own functionality for their own particular disciplinary context. This chapter is concerned with how to describe these formalisms, taking into account what they all share as formalisms used to organise technical knowledge, and how they all differ in terms of their specific functionality. In broad terms, this chapter takes a step toward developing a semiotic typology of academic formalisms.

The description in this chapter arises from three broad research programs. The first has to do with a long running educational linguistics concern with exploring the nature of knowledge across disciplines – understandings which are fundamental to the development of linguistically informed pedagogy and curriculum, embedded in disciplines (Rose & Martin 2012). This action research program has been considerably enhanced through dialogue between Systemic Functional Linguistics (SFL), which underpins this chapter, and a sociological framework known as Legitimation Code Theory (LCT) (Maton et al. 2016, 2019, Martin et al. 2019, Maton and Doran 2017). In work arising from this dialogue, it is clear that disciplines that utilise a range of formalisms tend to be associated with the sciences (e.g. physics and chemistry) or social sciences (e.g. linguistics and psychology), in comparison with the humanities (e.g. cultural studies and art history), which tend not to use formalisms (Parodi 2012, Lemke 1998).¹ In terms of Legitimation Code Theory (LCT), this marks a broad distinction between the *knowledge codes* tending to underpin the sciences and the *knower codes* that are more often prevalent in the humanities (Maton 2014). From this perspective, we can ask why academic formalisms tend to be used more in knowledge codes than in knower codes.

The second research program relevant to this chapter is the expanding field of multimodal semiotics. This research is concerned with the full range of meaning-making resources used in human communication, and how they come together to organise our lives. From the perspective of multimodality, academic formalisms are pervasive in particular registers; but

¹ Although this is a strong tendency, a clear exception to this is the formal logic used in some branches of philosophy.

as noted above, aside from mathematical symbolism (see Lemke 1998, 2003; O'Halloran 2005; Doran 2017, 2018a, b, 2020), they have not yet received a great deal of attention. These semiotic resources have been considered more language-like when compared resources such as images, animation, architecture etc. (cf. Lemke's 2003 *mathematics in the middle* interpretation of mathematics in relation to language and image in science, and O'Halloran's 2005 *language-based* approach to mathematics). Questions about the inherent similarity and difference among the meanings afforded by different modalities of communication have yet to be resolved, in part because an adequate theory of broader semiosis is yet to be developed. So from this perspective, we can ask about the specific functionality of academic formalisms, and their joint communicative affordances.

The final research program contextualising this chapter is the focus on Systemic Functional language description and typology explored in this book. In particular, it addresses the question of what we mean by *functional* language typology (see Martin and Quiroz this volume), and how we describe languages in ways that enable us to compare and contrast their functionality. In the spirit of the previous two research programs, this concern with functional language typology can be expanded into asking how we can go about developing a functional *semiotic* typology of academic formalisms in particular, and semiosis in general.

With reference to these three research programs, this chapter will explore four academic formalisms – system networks and tree diagrams in linguistics, and nuclear equations and mathematical symbolism in physics. It will address both their grammatical organisation and the technical meanings they organise. The analysis will focus on two main points. First, each formalism construes a distinct set of meanings that are crucial for the development of knowledge in the discipline they are used in. Importantly, the particular meanings that each formalism construe is discipline specific. Unlike language that can organise an enormously wide range of different ideational, interpersonal and textual meanings for use across a range of registers and genres, each formalism is much more specialised – targeting one or two types of ideational meaning. We will explore this through an evolving model of field in SFL presented in Doran and Martin (2020). Second, although each formalism construes different meanings, they are all able to iterate these meanings to an indefinite extent. That is, no matter the particular meanings they construe, each formalism can make as many of these meanings as necessary for purposes of knowledge building. This potential for repetition underpins the general functionality of academic formalisms, as it enables highly complex and elaborated

meanings to be integrated relatively efficiently. Iteration is enabled by the grammatical organisation of each resource. Although at first glance each formalism appears very different, the need for repetition of field-specific meanings means that they are all organised around a particular type of structure that Halliday (1965) has referred to as a univariate structure, which he defines as repetitions of a single variable.

These two points, specialisation and iteration, will shape the chapter. We begin by focusing on the differences between these formalisms in terms of the particular field-specific meanings they construe. Following this, we will come back to their similarities in terms of their ability for iteration and the grammatical organisation enabling this.

Academic formalisms and field

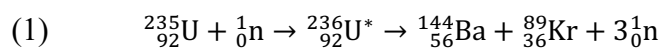
Systemic networks, tree diagrams, nuclear equations and mathematical symbolism all construe different types of meaning. To get a sense of their differences, we will step through each from the perspective of *field*, which can be viewed in common sense terms as considering the ‘content’ meanings of educational knowledge. More strictly speaking, in SFL terms, field is a resource for construing phenomena – for building knowledge across all walks of life.

In terms of Doran and Martin (2020), system networks and tree diagrams both construe relatively static relations between *items*. Beginning with system networks, as noted above these networks make distinctions in terms of type and subtype. In the network in Text 3 above, extensive and intensive are types of major clause, while effective and descriptive are types of extensive clause and so on. This organisation of clauses into types and subtypes is known as a *classification* taxonomy.

Tree diagrams also organise taxonomic relations between items. However whereas system networks organise their meanings into type-subtype relations, tree diagrams organise them into part-whole relations – a *composition* taxonomy. In Figure 1 above, for example, singular noun phrases (NP_{sing}) and singular verb phrases (VP_{sing}) are not types of sentence (Σ), but rather are *parts* of the sentence. Similarly, suffixes such as $-\emptyset$ are $-s$ are not a type of noun (N) or verb (V), but are parts of these word classes. In this sense, the two linguistic formalisms of

system networks and tree diagrams both construe taxonomic relations, but one construes taxonomies of classification while the other construes those of composition.²

Whereas the two linguistic formalisms construe a relatively static perspective, nuclear equations in physics symbolise a dynamic perspective on unfolding events. Each arrow in these equations construes a shift from one set of particles to another. In terms of field, rather than establishing taxonomic relations between items, nuclear equations realise unfolding *activities*. In equation (1), the nuclear reaction is organised in terms of two activities, each specified by \rightarrow :



The first activity, ${}_{92}^{235}\text{U} + {}_0^1\text{n} \rightarrow {}_{92}^{236}\text{U}^*$, specifies that ${}_{92}^{235}\text{U} + {}_0^1\text{n}$ (Uranium-235 and a neutron) shifts to ${}_{92}^{236}\text{U}^*$ (an excited uranium-236 atom). The second activity ${}_{92}^{236}\text{U}^* \rightarrow {}_{56}^{144}\text{Ba} + {}_{36}^{89}\text{Kr} + 3{}_0^1\text{n}$ specifies that the excited Uranium-236 atom splits into ${}_{56}^{144}\text{Ba} + {}_{36}^{89}\text{Kr} + 3{}_0^1\text{n}$ (a Barium-144 atom, a Krypton-89 atom and three neutrons). By placing both of these activities in a single equation, the formalism indicates that they form moments of a broader activity, which is known in language as nuclear fission. This means in Doran and Martin's (2020) terms that the full equation involving multiple arrows realises a *momented activity*.

In addition to these activities, nuclear equations also realise a set of compositional relations. The symbols on either side of the arrows (${}_{92}^{235}\text{U} + {}_0^1\text{n}$, ${}_{92}^{236}\text{U}^*$, ${}_{56}^{144}\text{Ba} + {}_{36}^{89}\text{Kr} + 3{}_0^1\text{n}$) specify the particles involved at each step of the activity. At the highest level, the expression ${}_{92}^{235}\text{U} + {}_0^1\text{n}$ specifies that in the system at this step of the activity, there is a Uranium-235 atom (${}_{92}^{235}\text{U}$) and a neutron (${}_0^1\text{n}$); in the second step, the system comprises just a single Uranium-236 atom (${}_{92}^{236}\text{U}^*$); and in the third step, the system comprises a Barium-144 atom, a Krypton-89 atom and three neutrons (${}_{56}^{144}\text{Ba} + {}_{36}^{89}\text{Kr} + 3{}_0^1\text{n}$).

² The tree diagram does make some small distinctions in classification through subscripts on the symbols, such as N_{sing} and N_{plur} distinguishing between singular and plural nouns, however importantly, this cannot be repeated indefinitely in the way the branches showing composition can.

Whereas the linguistic formalisms realise taxonomic relations and nuclear equations realise activities, the symbols in mathematics construe a third type of meaning modelled by Doran and Martin (2020) as *properties*. This accounts for the fact that unlike the elements in the linguistic formalisms or the arrows in the nuclear equations, each symbol in mathematical equations is gradable and/or measurable. In $pV = nRT$, for example, we are able to have more or less volume or pressure (symbolised by V and p) and are able to measure these numerically as, say, $V = 0.0224 \text{ m}^3$ or $p = 1.013 \times 10^5 \text{ Pa}$. In system networks on the other hand we are not able to have more or less *extensive* or *intensive*; ³ neither can we have more or less of the symbols NP_{sing} or *-s* in tree diagrams, or more or less \rightarrow (glossed as ‘becomes’) in nuclear equations. Properties are gradable and potentially measurable meanings, and are crucial components of many academic fields. ⁴

Symbols are organised into mathematical statements through a set of relations such that if a change in the value of one symbol occurs, then this will affect all other symbols. For example in $pV = nRT$, assuming all else stays the same, if p increases, then V must decrease or the product of n , R and T must increase (or both). Doran and Martin (2020) refer to relations

³ In some system networks, descriptions have involved cline systems that show gradations between ‘more’ or ‘less’ of a feature, such as van Leeuwen’s (2009) ‘parametric’ systems for voice quality, which would here be described in terms of property. In these instances, the network is no longer realising a classification taxonomy. There have also been attempts at complementing the typologies of system networks with topologies, generally shown through cartesian planes (e.g. Martin and Matthiessen 1991 for various areas of English lexicogrammar, and Martin and Rose 2008 for genre). However in general, most system networks conform to the description in this chapter.

⁴ More specifically, mathematical symbols are instances of *itemised* properties (Doran and Martin 2019; Doran 2019). Properties in general describe elements that are gradable, such as *warm : warmer; big : bigger*. Itemised properties are also gradable, but are construed as items in and of themselves, either through grammatical metaphor such as *warmth*, or technicalisation of a dimension such as *temperature* (Hao 2020a, b). For example *warm* is a property and can be graded, but when discussed as *temperature*, it is an itemised property that can be both graded and classified into *absolute temperature, relative temperature* etc. This distinction does not affect the argument in this chapter.

between properties as *interdependency* relations.⁵ Each mathematical statement sets up a vast network of interdependency relations that construe the possibilities of the physical systems they describe.

The overview so far gives a sense of each formalism in terms of the main meanings they realise in field, as shown in Table 1.

Formalism	Field meanings realised
System networks	classification taxonomies
Tree diagrams	composition taxonomies
Nuclear equations	momented activities and composition taxonomies
Mathematics	interdependencies between properties

Table 1. Field meanings realised by academic formalisms

Importantly for our understanding of these formalisms, these meanings constitute the vast majority of the possible meanings each can make. For example, although mathematics can make very small distinctions in classification through subscripts on symbols (e.g. x_1, x_2, x_3 etc.), this is minor compared to its ability to realise interdependencies; and algebraic mathematics has no way of construing composition or activity (Doran 2018a). Similarly, although tree diagrams construe composition, any construal of classification is relatively minor. Looking more broadly, although most of the formalisms have some small variations that may organise informational prominence (giving textual meaning), such as the vertical arrangement of system networks, the choice of the left or right side of the equation in mathematics and the ordering of particular symbols in nuclear equations, they do not appear to show any variation that can be considered ‘interpersonal’ in terms of organising dialogue, evaluation or nuanced social relations of any sort; seen from a linguistic perspective, they do

⁵ In Doran (2018a) these relations were called implication complexes and were described as being realised by co-variate structures. With the revision of the model of field in Doran and Martin (2019), this is now better described in terms of inter-dependency relations.

not display relatively independent variation that suggests meanings of MOOD or (Halliday and Matthiessen 2014), nor do they suggest any APPRAISAL or NEGOTIATION (Martin and White 2005), nor do they indicate variations in STATUS or CONTACT (Martin 1992) (see Doran 2018b for detailed argumentation concerning this in mathematics). All this is to say that each formalism is heavily devoted to a very specific set of field-relations. This explains the relatively mundane observation that we do not converse in mathematics, nor share our feelings with system networks. In general terms then, although the particular meanings they construe differ, each formalism share this feature of construing only a very constrained set of field-specific meanings

In the next section we will show that this is not all they share. In addition to having a relatively constrained set of meanings they realise, these formalisms share the ability to iterate these meanings through a highly recursive grammatical organisation.

The grammatical organisation of academic formalisms

So far we have seen that each formalism construes very particular field-specific meanings. This section will explore these formalisms in terms of their overall grammatical organisation (building upon the final chapter of Doran 2018a). In particular, we will show that they all maintain a highly iterative grammar that enables an indefinite repetition of their field-specific meanings. As defined by Halliday (1965) univariate structures involve indefinite iteration of the same structural relation. In English, for example, a typical univariate structure involves the complexing of groups or clauses where multiple elements of the same type are presented in sequence:


(1) *Dylan was the first to go, then (2) Frank left, and then (3) Josie went home.*

Here, each of the clauses perform a similar function to the others in terms of their temporal unfolding, and importantly, an indefinite number of clauses can be added into this sequence. This is in contrast to a typical multivariate structure that involves distinct functions that can only occur once. The functional organisation of English TRANSITIVITY structures exemplifies a multivariate structure (Halliday and Matthiessen 2014):

<i>Twenty-two working-class northern clubs</i>	<i>formed</i>	<i>the Northern Rugby Football Union</i>
Actor	Process	Goal

Here, the Actor, Process and Goal each perform distinct functions in terms of their potential grammatical patterns and can only occur once (we cannot have two Processes in a single clause, two Actors or two Goals, for example). This is quite distinct from the patterns we will see below in the formalisms where most options can occur an indefinite number of times. Starting with system networks, we will step through each formalism in turn, building up our picture of how they work.

System Networks

System networks are organised around sets of choices placed in systems, with the minimum network being a system with two choices as in Figure 2. Here the choices are shown through the square ‘or’ bracket '['. This indicates that in the system of MOOD either indicative or imperative is a possibility. As discussed in the previous section, from the perspective of field, each choice realises an item in a classification taxonomy. Note that for this description, a choice includes both the feature, shown in lower case (indicative or imperative) and its realisation statement – marked by  and including functions (+Subject; +Finite).⁶

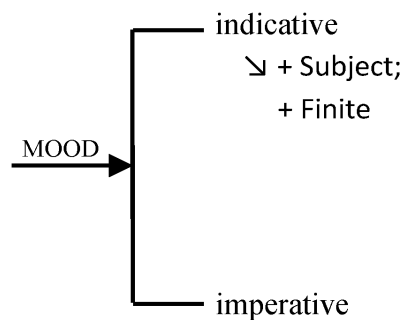


Figure 2. Minimal system network

⁶ For reasons of space, we will not explore realisation statements in detail here. See the final chapter of Doran (2018a) for a more in-depth discussion.

In such systems, each choice plays the same role; their vertical ordering higher or lower is not meaningful, ideationally speaking.

Systems can be expanded to involve more than two choices, as in Figure 3:

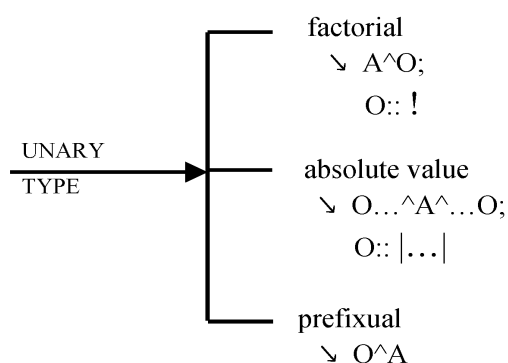


Figure 3. System with three choices.

In principle, systems can be expanded indefinitely to involve any number of choices. This opens the potential for networks to realise an indefinitely *broad* classification taxonomy with innumerable subtypes. Grammatically speaking, as each choice plays the same role and there can be any number of them, these systems involve univariate structure organised as a complex of choices.⁷

Although in principle systems may include any number of choices, in practice they tend toward only two. This enable generalisations to be captured at multiple levels of delicacy. Any further choices at a greater delicacy occur by adding systems to the right of each choice in the network, as in Figure 4.

⁷ It must be stressed that here we are talking about the structure of the system network formalism itself, not the component of language the system network describes (which can be either multivariate or univariate).

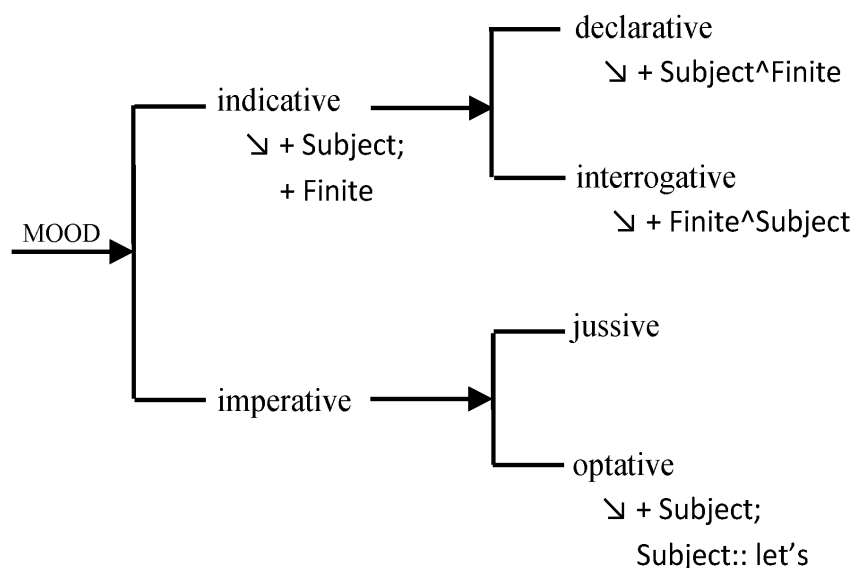


Figure 4. System network with two levels of delicacy.

Here, each of the choices of indicative and imperative have two subtypes: declarative and interrogative for indicative, and jussive and optative for imperative. Networks can specify any degree of delicacy, meaning there is potential for indefinite iteration along this dimension of the formalism (alongside the potential for an indefinite number of features in systems noted above). This means there may be an indefinite number of layers of systems, with each system potentially including an indefinite number of features.

The indefinite expansion of choices within a system and the ability for indefinite layering through increased delicacy offer two paths for iteration in the grammar of system networks. From the perspective of field, this means that system networks enable the realisation of both extra *breadth* in taxonomies (indefinite numbers of items in a single layer of a classification taxonomy – i.e. an indefinite number of co-classes) and extra *depth* (an indefinite number of levels in a classification taxonomy – i.e. extra iterations of subclasses within subclasses).

There is one further means for complexing that dramatically increases the degree of field-specific meaning a network can realise. This is the possibility for simultaneous systems, where multiple systems can be cross-classified. This is shown in system networks through a curly bracket '{', that indicates an 'and' relation. In Figure 5, the network indicates that if a

major clause is chosen, then one must choose from each of MOOD *and* THEME *and* TRANSITIVITY. Thus a clause will take choices from each of these systems.

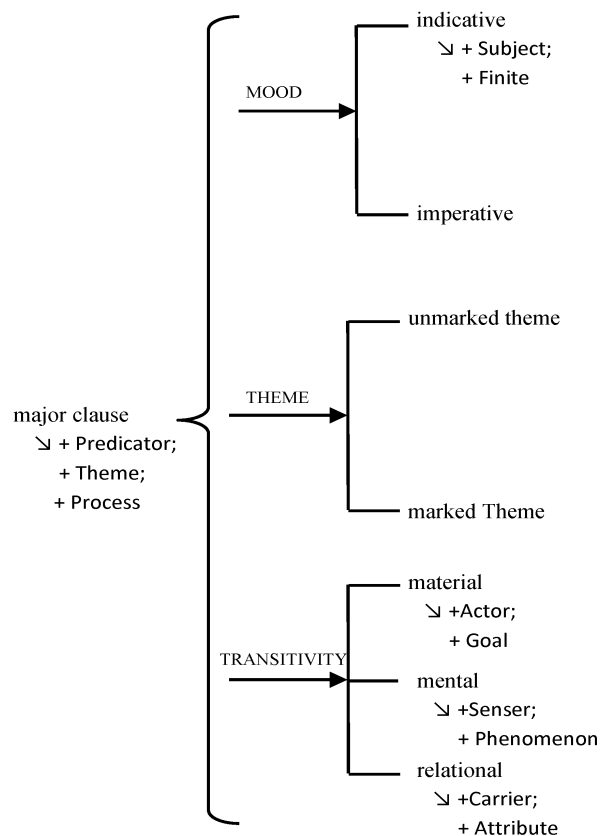


Figure 5. Simultaneous systems

As with the previous expansions, there can be an indefinite number of simultaneous systems. In terms of field, the use of simultaneous systems enables multiple classification taxonomies to be realised at once. Grammatically speaking, this again involves a univariate structure; there may be any number of simultaneous systems and each performs the same broad ideational function.

In Figure 5 these taxonomies are all relatively independent: each choice in THEME can occur with each choice in TRANSITIVITY with each choice in MOOD. But system networks also enable more specific interdependencies to be shown, as in Figure 6 (from further on in Halliday 1967).

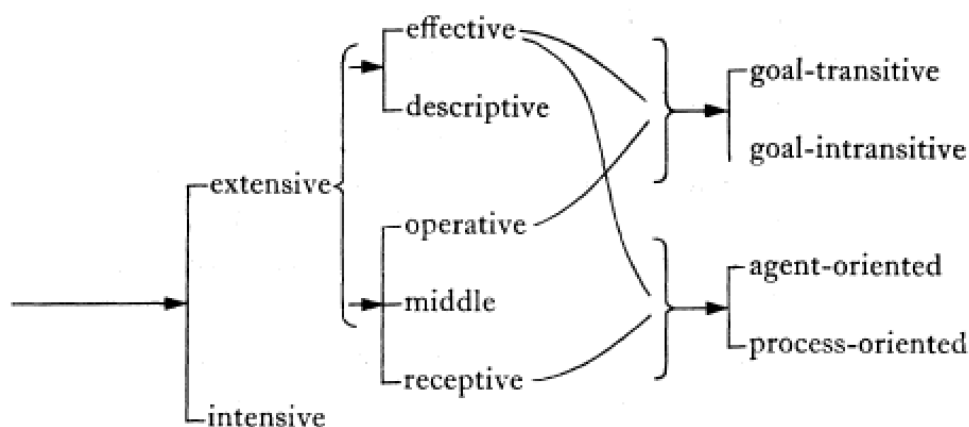


Figure 6. System network realising multiple interdependent classification taxonomies.

This system network sets up multiple, interdependent classification taxonomies. At its most general, it sets up a single taxonomy of extensive vs intensive. But within extensive, it realises two distinct taxonomies: one with two choices, effective and descriptive, and another with three choices: operative, middle and receptive. At a more delicate level, it specifies much more intricate interdependencies. For example the intersection of both effective from one taxonomy and operative from another open up a third taxonomy with two types (goal-transitive and goal-intransitive), while the intersection of effective and receptive allows for another taxonomy that distinguishes two further types (agent-oriented and process-oriented). System networks enable a precise realisation of these classification taxonomies in one synoptic snapshot. And its grammar enables there to be any number of these taxonomies that may be indefinitely deep or indefinitely wide (for good examples of large networks along these lines, see Halliday and Matthiessen 2014, especially Fig.4-13, and Matthiessen 1995). The functionality of system networks is summarised in Table 2.

System networks

Univariate structures in the grammar	Field-specific meanings realised
choices in a system	breadth of a classification taxonomy
systems ordered in delicacy (layers of univariate structure)	depth of a classification taxonomy
simultaneous systems	number of classification taxonomies

Table 2. Grammatical organisation and field relations in system networks

Tree diagrams

Like system networks, tree diagrams are organised around a relatively small number of elements. However unlike system networks, tree diagrams have been taken up by a wide range of different schools of linguistics. This means there is not just one type of tree diagram; what is possible and what regularly occurs in this formalism varies across the field. For this reason, here we will focus only on general features that associated with basic phrase structure grammars (e.g. Chomsky 1957).

Constituency tree diagrams are organised around branches that emanate from nodes. A basic tree will involve a node with two or more branches, as in Figure 7:

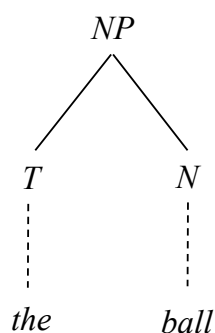
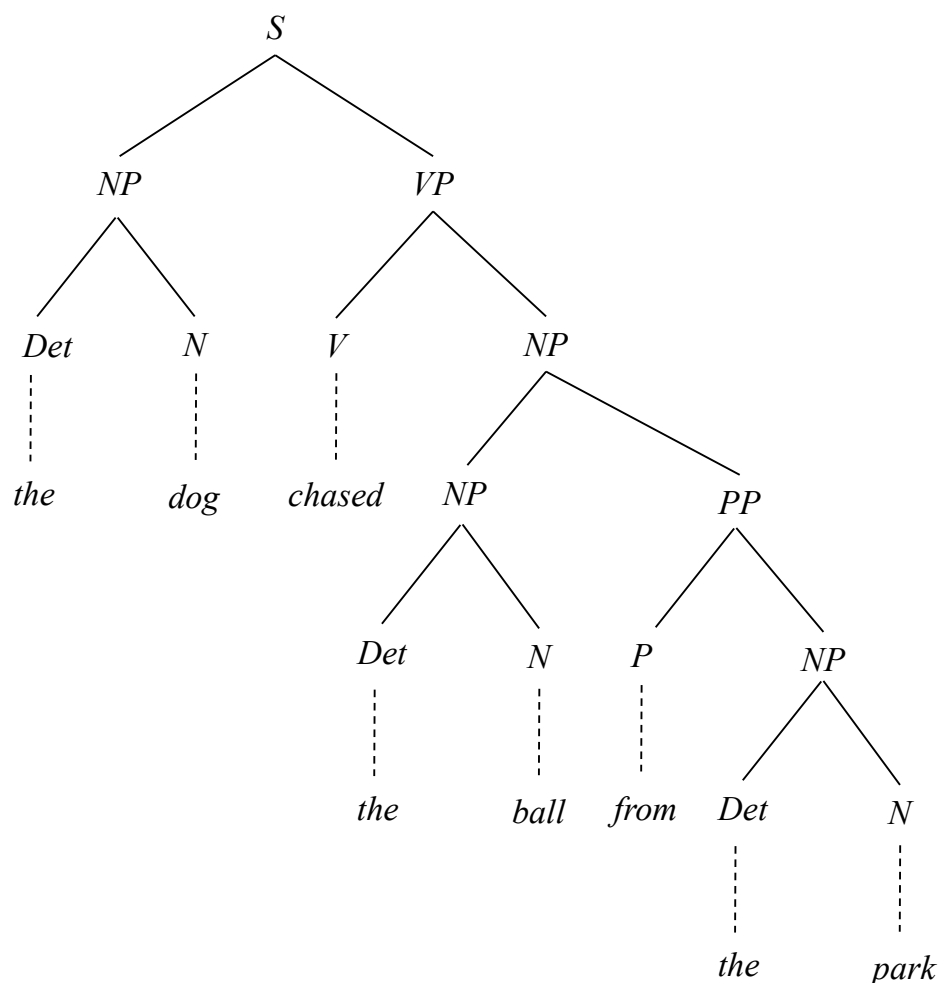


Figure 7. Basic tree diagram

This tree diagram is read as saying an *NP* is composed of a *T* and an *N*. The *T* is substituted lexically by *the* and the *N* is substituted by *ball*. As noted above, the relationship between the *NP* and the *T* and *N* is one of composition. Like system networks above, this relationship can

be repeated indefinitely for each branch.⁸ In Figure 8, for example, there are six levels of the composition taxonomy realised by a series of branches from *S* to the *Det* and *N* of *the park*.⁹



⁸ In contrast, the relation between *T* and *the*, and *N* and *ball*, known as lexical substitution, is not one of composition; *the* is not part of *T* and *ball* is not part of *N* here. Rather, it is better described in terms of an elaboration, where the lexical item (*ball/the*) and its grammatical category (*N/T*) are equated. Importantly for our description, this distinction between lexical substitution and the composition relations of the tree is paralleled by the fact that whereas the compositional relations can be repeated, lexical substitution cannot. In general, once a branch has had a lexical item substituted (e.g. *N* has become *ball*), one cannot substitute another lexical item for *ball* and then another and then another indefinitely.

⁹ For examples of how many iterations constituency trees can show, see those used in the early years of generative semantics, especially Ross and Lakoff (such as one reproduced in Harris (1993: 144) with no fewer than twenty-three tiers for the sentence *Floyd broke the glass*).

Figure 8. Tree diagram with increased depth

Iterating branches down the page realises increasing levels of *depth* in the composition taxonomy.

In the above examples the trees set up binary branches where no more than two branches occurred for each node (e.g. an *NP* split into two branches, a *Det* and an *N*). In some schools of linguistics, there can also be as many branches as needed for any level in the constituency hierarchy. That is, one can also expand indefinitely the *breadth* of compositional taxonomies, such as in Figure 9.¹⁰

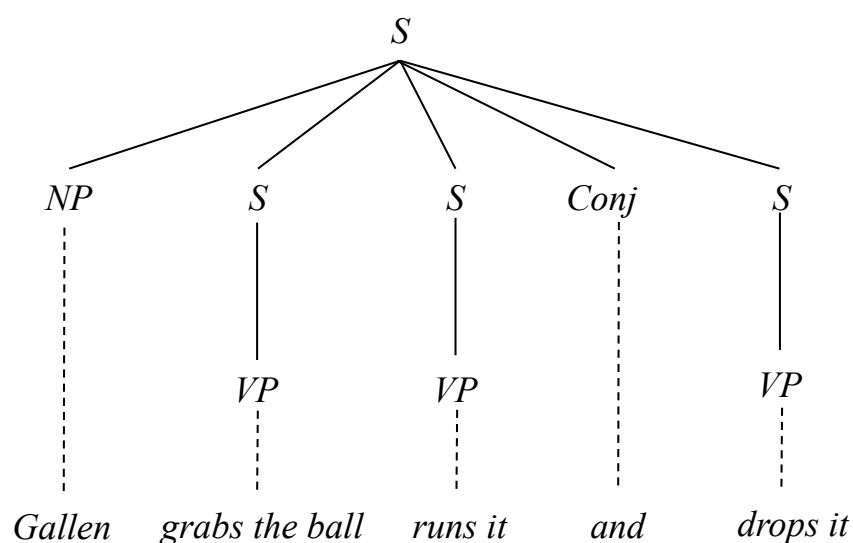


Figure 9. Tree diagram with increased breadth

Tree diagrams are thus organised around a univariate structure on two dimensions. First, they enable an indefinitely iterative number of levels in hierarchy, developing depth in the composition taxonomy. Second, depending on the subfield, they enable an indefinite number of branches to occur in each level, allowing increasing breadth of the taxonomy. These two possibilities for expansion are summarised in Table 3.¹¹

¹⁰ For examples of trees that significantly iterate the number of branches in each level – so emphasising breadth over depth – see the Appendix B of Fawcett (2000).

¹¹ The order of the lexical items at the bottom of the diagram also reflect, to a certain degree, their ordering in language. In this sense, the tree diagrams realise what Doran and Martin

Tree diagrams

Univariate structures in the grammar	Field-specific meanings realised
levels in the tree	depth of a classification taxonomy
branches in each level	breadth of a classification taxonomy

Table 3. Grammatical organisation and field relations in tree diagrams.

Mathematical symbolism

Moving from linguistic formalism to mathematical symbolism, there is a shift from construing taxonomic relations between items to establishing interdependencies between measurable properties. Like system networks and tree diagrams, mathematical symbolism can establish innumerable relations between properties through a highly univariate grammar at multiple levels (the overview given here is based upon the more detailed description given in Doran 2018a).

Here we will focus on mathematical symbolism most closely associated with elementary algebra, as often used in high school science (see Doran 2018a for details). A basic mathematical statement involves two expressions linked by a Relator. In equation (2) which describes *kinetic energy*, the energy of motion, the two expressions are K_E and $\frac{1}{2}mv^2$ and the Relator is =

$$(2) \quad K_E = \frac{1}{2}mv^2$$

(2019) call a *spatio-temporal property*; they specify the location of each lexical item in the sentence. Utilising both the vertical and horizontal axes enables the diagram to show these multiple field-specific meanings, depth and breadth of composition, and spatio-temporal properties. Indeed this is one of the main affordances of these types of images over phrase structure rules. However a discussion of the functionality of imagic vs symbolic formalisms is beyond the scope of this paper.

Ideationally speaking, the two expressions perform the same role. This can be shown by the fact that they can be swapped in order with only a change in the information organisation (i.e. ideationally, (2) and (3) are the same):

$$(3) \quad \frac{1}{2}mv^2 = K_E$$

This basic mathematical statement can be expanded to include more expressions and Relators, such as the following equations that have three and four respectively:

$$(4) \quad E_1 = E_2 = k$$

$$(5) \quad \lambda = \frac{v}{f} = \frac{3 \times 10^8}{104.1 \times 10^6} = 2.9\text{m}$$

In principle, statements can include any number of expressions linked by Relators and can generally be rearranged in any order without affecting their ideational meaning.¹² As there is the potential for indefinite expansion and each expression performs the same ideational function, we can again describe this area of mathematics' grammar as a univariate structure.

Although in principle there can be an indefinite number of expressions in a statement, for textual reasons, there is a strong tendency to have only two, with three or four occurring at times, and any more being unusual. To allow for further expansion of relations between symbols, then, there is a second avenue for indefinite iteration involving the number of symbols within an expression. As the left-hand side of equations (2), (4) and (5) show, it is common for one of the expressions to include only a single symbol. But in other expressions in these equations, there are many more. Every symbol that is added must necessarily be linked by an arithmetic operator such as + (addition), - (subtraction), ÷ (division) and ×

¹² This analysis is slightly complicated by inequations that use Relators such as > (larger than), < (smaller than), ≥ (larger than or equal) and ≤ (smaller than or equal to). These can also be rearranged without affecting their ideational meanings, however it requires a small alternation in the direction of the Relator. That is $y > 7$ construes the same ideational meanings as $7 < y$. See Doran (2018a: 80-88) for more detailed argumentation.

(multiplication).¹³ For example, the second expression in equation (5) $(\frac{v}{f})$ has two, while the second expression in (2) $(\frac{1}{2}mv^2)$ has five, and in the following equation, there are sixteen across three expressions:

$$(6) \quad r = 4\pi\epsilon_0 \frac{n^2\hbar^2}{mZe^2} = \frac{n^2}{Z} a_0$$

Like expressions, symbols can also be iterated indefinitely. Indeed it is here that most of the expansion occurs.

We can thus again consider this type of formalism to be organised around a univariate structure. Like system networks and tree diagrams, mathematics offers multiple avenues for indefinite iteration. In this case, both levels of iteration realise the same field relation: interdependencies between itemised properties. Table 4 summarises these possibilities for mathematical symbolism.

Mathematical symbolism

Univariate structures in grammar	Field-specific meanings realised
expressions within statements	interdependencies between itemised properties
symbols within expressions	

Table 4. Grammatical organisation and field relations in mathematical symbolism.

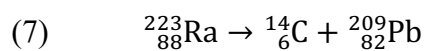
Nuclear equations

As with the other three formalisms, nuclear equations are also organised through multiple components with a univariate structure. Like system networks and tree diagrams, but unlike mathematics, the two univariate components organise different meanings at field. However

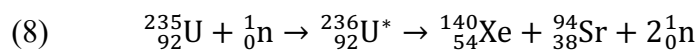
¹³ In the case of multiplication, the \times is typically elided, so that an expression $\frac{1}{2}mv^2$ means $\frac{1}{2} \times m \times v^2$.

whereas each component in system networks and tree diagrams organise slightly different facets of the same broad area of field – the depth, breadth and number of classification taxonomies in system networks, and the depth and breadth of composition taxonomies in tree diagrams – the two avenues for expansion in nuclear equations organise quite different field relations: the momenting of activities and the breadth of composition taxonomies.

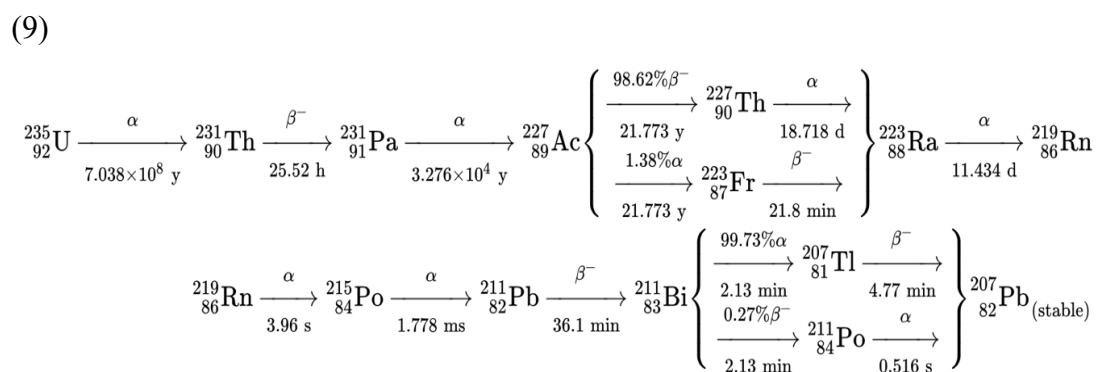
At the highest level, a basic nuclear equation is organised through two expressions on either side of an arrow \rightarrow . In the following equation the two expressions are $^{223}_{88}\text{Ra}$ (signifying Radium-223) and $^{14}_6\text{C} + ^{209}_{82}\text{Pb}$ (Carbon-14 plus Lead-209):



As we've already seen, nuclear equations can have more than two expressions, with each new expression being linked by another \rightarrow , as in the three expressions in:



Just like each of the formalisms we've seen so far, the number of expressions can be repeated indefinitely. A well-known decay chain, known as the actinium chain, for example, can be represented as follows:¹⁴

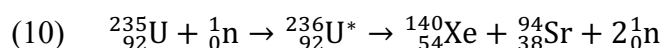


As discussed above, by expanding nuclear equations in this way, the text is able to indefinitely *moment* the activities of nuclear reaction and decay. The discipline is able to specify in as much or as little detail the steps between two states.

¹⁴ This particular nuclear equation is taken from the Wikipedia page for Uranium 235.

<https://en.wikipedia.org/wiki/Uranium-235> Accessed 11/02/19

The second avenue for expansion occurs within each expression and has to do with what particles occur at each stage of the activity. In the following equation, each state is represented by a set of symbols that represent particles. The first involves two symbols ${}^{235}_{92}\text{U}$ and ${}^1_0\text{n}$, the second involves just a single symbol ${}^{236}_{92}\text{U}^*$ and the third involves three symbols ${}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2{}^1_0\text{n}$.



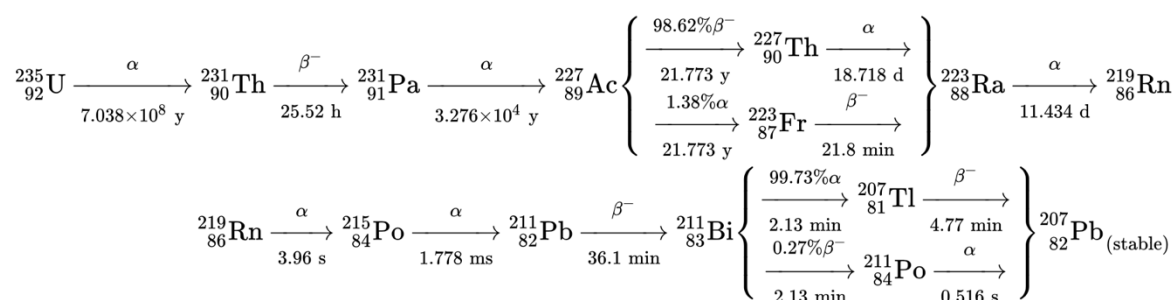
The number of symbols in each expression can again in principle be indefinitely expanded. And each perform the same function in the sense that they can be rearranged in any order without change to their ideational meanings (only textual meanings). In this sense, this provides a second avenue for univariate expansion. In this case, the expansion does not moment an activity, but rather expands the breadth of the compositional taxonomy at this point of the activity. It is important to note that unlike tree diagrams, nuclear equations cannot expand in depth; they cannot iterate further parts of parts of parts. Any depth is specified precisely by the numbers to the side of the symbols, such that ${}^{235}_{92}\text{U}$ indicates the Uranium involves 235 nucleons and 92 protons. The nuclear equation cannot specify what constitutes these nucleons or protons (i.e. quarks), nor can it group different sets of protons or nucleons together in other levels of a compositional taxonomy. The only iterative expansion available here is an expansion in the *breadth* of the compositional taxonomy at a single level.¹⁵

Finally, there is one further small dimension of iteration that occurs in nuclear equations. If we look again at equation (9) reproduced below, the expression following ${}^{227}_{89}\text{Ac}$ (Actinium-227) provides two alternative pathways for the decay. It may decay to ${}^{227}_{90}\text{Th}$ (Thorium-227) or to ${}^{233}_{87}\text{Fr}$ (Francium-233). Similarly, following ${}^{211}_{84}\text{Bi}$ (Bismuth-211), two alternative pathways are given via ${}^{207}_{81}\text{Tl}$ (Thallium-207) or ${}^{211}_{84}\text{Po}$ (Polonium-211). These alternatives are

¹⁵ Similarly, unlike tree diagrams, the order of the symbols does not specify any spatial ordering or location; ${}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2{}^1_0\text{n}$ does not mean that there is spatial layout that orders Xenon-140 and then Strontium-94 then 2 neutrons. The expression simply notes that these particles comprise the system at that point.

shown through the brace { and their vertical arrangement. From the perspective of field, these realise alternative activities construing the particular pathways of the decay. In principle multiple alternatives can be shown, however in practice, this is restricted to the specific possibilities for decay that any particular isotope has, and so any more than two is unusual. Nonetheless, the formalism enables a realisation of multiple parallel activities (similar to system networks enabling multiple classification taxonomies).

(11)



The possibilities for nuclear equations are summarised in Table 5.

Nuclear equations

Univariate structures in the grammar	Field-specific meanings realised
expressions in the equation	momenting of activities
alternative expressions	number of activities
symbols in an expression	breadth of a compositional taxonomy

Table 5. Grammatical organisation and field relations in nuclear equations.

Field and grammar of academic formalisms

This overview has shown that each resource construes a small set of specific relations in field, but that their grammar has evolved to iterate these meanings indefinitely. This enables the functionality of each resource for the particular technical knowledge that their respective

discipline needs, while at the same time enabling the expansion and integration of this technical knowledge into a coherent whole. As far as our concern with semiotic typology is concerned, this overview opens a series of questions that could be used to organise a more elaborated typology in this area (paralleling the questions presented in Martin and Quiroz this volume). From this perspective we may ask of any formalism:

- Does the formalism involve iterative (univariate) structures?
 - If so, how many?
- Do distinct iterative structures realise the same meanings at field? Or do they realise different meanings?
- What meanings do these iterative structures organise?
 - Taxonomy?
 - Only one or many?
 - Composition or classification?
 - Breadth (co-type/co-part) or depth (type-subtype/whole/part)?
 - Activity?
 - Only one or many?
 - Do they moment these activities or not?
 - Property?
 - Only one or many?
 - Do they realise independencies between these properties?

Such questions are foundational for the development of a typology of formalisms, and, more broadly, a typology of semiotic resources in general – by providing a principled set of possibilities that go beyond the surface features of resources (such as whether they are images or symbols, language-like or not etc.).

Non-iterative elements

This paper has primarily concerned itself with the iterative structures that form the core of academic formalisms. Each resource also includes a small component of elements that are not iterative but realise crucial field-specific meanings for their discipline. System networks for example include realisation statements that are linked to individual choices, such as:

optative

☒ + Subject; Subject:: *let's*

Here, the choice *optative* leads to the insertion of a Subject and the lexicalisation of this Subject as *let's*. This example is best read as realising a small relation of composition, formalising as it does that optative clauses are composed of (amongst other things) a Subject lexicalised as *let's*. However these realisation rules are not iterative in the sense that one cannot specify further depth in composition within the Subject through multiple realisation rules for a single feature. That is, in the optative example above, one could not put another arrow \searrow below the realisation rule to put forward another element within the Subject or *let's* and then indefinitely repeat this down to the morpheme. To do this would require another set of system networks (Martin 2013).¹⁶

Similarly, as we've seen, tree diagrams include a non-iterative component whereby the final node is replaced by a lexical item. In the diagram below, the *N* is replaced with *ball* and the *T* is replaced with *the*. However one cannot insert another lexical item once *ball* or *the* has been inserted.

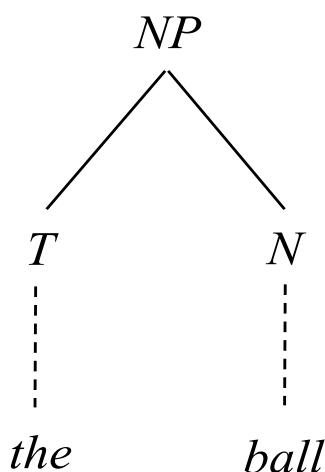


Figure 10. Lexical substitution in tree diagrams

¹⁶ Though see Bateman (2008) for a suggestion that SFL consider using tree fragments as realisation rules, which would enable an iteration of composition along these lines – essentially combining system networks with tree diagrams.

The same applies for mathematics and nuclear equations. Both include small components that construe meaning that cannot be repeated. In the case of mathematics, small modifications of symbols can indicate distinctions in classification. In the following example taken from a high school physics classroom, the subscripts indicate three types of E (energy): $E_{emitted}$ (energy emitted), E_i (initial energy) and E_f (final energy):

$$(12) \quad E_{emitted} = E_i - E_f$$

Again, these subscripts cannot be iterated (a symbol such as $E_{emitted_{i_f_2}}$ cannot occur).

In the case of nuclear equations, the numbers to the left of symbols, such as $^{227}_{90}\text{Th}$ indicate a small compositional taxonomy. We will step this through using 2^1_0n , the final symbol in the equation $^{235}_{92}\text{U} + {}^1_0\text{n} \rightarrow ^{236}_{92}\text{U}^* \rightarrow ^{140}_{54}\text{Xe} + ^{94}_{38}\text{Sr} + 2^1_0\text{n}$. The three numbers and their meanings are:

- The subscript 0, known as the atomic number, gives the number of *protons* in the particle (in this case zero);
- The superscript 1, known as the mass number, gives the total number of *nucleons* in the particle (which group together the *protons* and *neutrons*) (in this case one)
- The number on the left 2, gives the total number of these particles (in this case two).

This establishes a small composition taxonomy whereby the group of particles include a particular number of nucleons, which in turn comprise a particular number of protons. But, this cannot be iterated; the symbol cannot have superscripts on superscripts or subscripts on subscripts that indicate what is in the protons. So, like the other formalisms, nuclear equations complement their iterative components with small non-iterative components that realise distinct field-specific meanings.

From the perspective of developing a semiotic typology, this allows us to ask of any formalism:

- Does the resource couple iterative structures with non-iterative structures?
- If so, what type of structures are used and how many are there?

- What meanings are they being used for? Taxonomy? Activity? Property?

Semiotic typology

The questions raised in the previous sections by no means exhaust the possibilities for variation across semiosis. They deal in particular with academic formalisms and the meanings they organise for technical disciplines. One obvious question not asked above is why some formalisms seem to use an imagic mode (such as system networks and tree diagrams) and others use a more symbolic mode (mathematics, nuclear equations). For reasons of space, we cannot explore this question here; however it does flag work to be done to understand the range of meaning-making systems used in social life. Recent decades of research in Social Semiotics and Systemic Functional Semiotics have dramatically pushed our understanding of semiotic resources to the point where we can now begin to compare and contrast them, and develop a typology. For this to be successful, we need principled means of comparison that allows us to see both the similarities and the differences, and does not assume that certain categories such as metafunction, rank etc. will necessarily occur as they do for language.

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