ONOMÁZEIN
Revista semestral de lingüística, filología y traducción

PONTIFICIA UNIVERSIDAD
CATÓLICA DE CHILE
FACULTAD DE LETRAS

The role of mathematics in physics: Building knowledge and describing the empirical world

Y. J. Doran
The University of Sydney
Australia

ONOMÁZEIN Número especial SFL (2017): 209-226
DOI: 10.7764/onomazein.sfl.08

Y. J. Doran: Department of Linguistics, The University of Sydney, Australia.
| Correo electrónico: yaegan.doran@sydney.edu.au

Fecha de recepción: 10 de agosto de 2015
Fecha de aceptación: 29 de enero de 2016

Número especial
SFL

Abstract

This paper considers why mathematics is used in physics. It traces the use of mathematics in physics through primary school, junior high school and senior high school in NSW, Australia, considering its role from the point of view of Systemic Functional Linguistics and Legitimation Code Theory. To understand the development of mathematics, two genres that play differing roles in the discipline of physics are introduced: 'derivation' and 'quantification'. Through an analysis using the concepts of semantic density and semantic gravity from Legitimation Code Theory, these genres are shown to aid physics in developing new knowledge and linking its theory to the empirical world. This paper contributes to the growing body of research considering forms of knowledge in academic disciplines and the role of non-linguistic semiotic resources in organizing this knowledge.

Keywords: mathematics; physics; semantic density; semantic gravity; genre; Systemic Functional Linguistics; Legitimation Code Theory.

1. Mathematics in Science

Mathematics is pervasive through many scientific disciplines. It is used in both schooling and research, and it forms part of the high stakes texts students read to learn science and those they write for assessment. But why is it used in science? This question has come to prominence from a recent concern in educational linguistics and social realist sociology with the structure of knowledge in academic disciplines (Christie & Martin, 2007; Christie & Maton, 2011). Various studies of science within the tradition of Systemic Functional Linguistics (SFL) have shown that the natural sciences, along with academic discourse in general, are largely uncommon sense and far removed from our everyday discourse (e.g. Martin & Veel, 1998; Lemke, 1990). The sciences tend to involve distinct sets of factual genres and use language to construe both large sequences of causality and deep taxonomies of composition and classification (Halliday & Martin, 1993; Martin & Rose, 2008). From the viewpoint of Bernstein's code theory, they are characterized by 'hierarchical knowledge structures' that attempt to create very general propositions and theories, integrating knowledge to account for an expanding range of different phenomena (Bernstein, 1999: 162; see Martin & Maton, this issue). According to Legitimation Code Theory (Maton, 2014), the principles underlying these knowledge structures of science emphasize epistemic relations between knowledge and its object of study, and downplay social relations between knowledge and its author or subject. At the same time, one of the most salient features of scientific discourse is its heavy use of non-linguistic semiotic resources, in particular mathematics (Parodi, 2010). Mathematics organizes its meanings in considerably different ways to language and thus offers a complementary system for construing the knowledge of science (O'Halloran, 2005).

The high use of mathematics and the distinctive structuring of scientific knowledge begs the question whether these two attributes are related. Does mathematics contribute to science's ability to develop integrated and abstracted models of the natural world, and does it aid in linking these models to empirical studies of their object of study? If so, how does mathematics do this? This paper will consider these questions by tracing mathematics through schooling as it develops in physics, the natural science in which mathematics is most widely used (Parodi, 2010). It will follow mathematics as it shifts through primary (elementary) school, junior high school and senior high school, in New South Wales, Australia, to understand the changing forms of mathematics and what this means for knowledge in physics.

Through schooling there is a distinct evolution in how mathematics is used. The changes across the years correlate with different roles mathematics plays in organizing the knowledge of physics. In order to understand the impact these changes have on the knowledge of physics, the mathematics in use will be viewed from two complementary angles. First, a model of mathematics developed from Systemic Functional Linguistics (Doran, 2016) will be used to map changes in the types of mathematics and its uses in texts. These changes will be illustrated primarily in terms of the distinct mathematical genres deployed (types of text with distinct structures and linguistic and mathematical configurations), as well as mathematics' interaction with language. By using this model, a metalanguage for mathematics becomes available that allows description and understanding of various textual patterns. Second, to explore how these patterns organize the knowledge of physics, they will be interpreted using the Semantics dimension of Legitimation Code Theory (LCT) (Maton, 2014). Semantics is concerned with two main variables: *semantic gravity*, which explores the degree to which meanings are dependent on their context, and *semantic density*, which explores the degree of condensation of meaning in a practice (Maton 2014; see also Martin & Maton, this issue; Maton & Doran, this issue). Each of these will be developed in further detail as they become relevant in the paper. Utilizing these concepts from Semantics enables a nuanced understanding of how the various resources of mathematics allow physics to build integrated and generalized knowledge, while at the same time remaining in contact with its empirical object of study. Bringing the two approaches together provides a method for understanding the kinds of mathematics used in physics, why they are used, and what the payoff is for physics as a discipline.

To organize the paper, a section will be devoted to each of the primary, junior high and senior high sectors of schooling (as these are organized in the state of New South Wales in Australia). At each stage, the mathematics in use will be introduced through the SFL framework of mathematics before being interpreted in terms of Semantics from LCT. This progression will develop an expanding understanding of the utility of mathematics. The final section will pull together the strands raised in each section to characterize physics as a whole when viewed from mathematics.

2. Development of Mathematics in Physics

2.1. Primary School

The late primary school years (ages ~10-12) are the first to introduce mathematics in the service of physics[1]. At this stage, mathematics is not a prominent feature of the discourse; the physics covered depends more heavily on language and images to construe its knowledge. Nonetheless, the mathematics that is used gives a glimpse of how it will organize the knowledge of physics in later stages. In order to contribute to the knowledge of physics, however, it first must be invested with technical meaning from physics. Text 1 shows an example of how this can take place, via mathematics' interaction with language. In this text mathematics

1 In primary school in New South Wales, Australia, physics is not a stand-alone subject. Rather, it forms part of a core science syllabus that also includes other natural sciences such as chemistry, biology (Board of Studies NSW, 2012). From early primary school, mathematics is taught separately as a topic area independent of scientific concerns.

is being used to introduce the relationship between force, mass and acceleration known as Newton's Second Law.

TEXT 1

Primary school *F=ma* text (Farndon, 2003: 19)

FORCE EQUATION

The relationship between force (F), mass (m) and acceleration (a) is summed up in the equation:

$$F = ma$$

This shows the force of an object depends on the combination of its mass and acceleration. This is why the impact of a slow-moving truck and a fast-moving bullet are equally devastating. Both have tremendous force—the truck because of its large mass, the bullet because of its huge acceleration. The equation can also be swapped:

$$a = F/m$$

This shows the acceleration goes up with the force but down with the mass.

The opening sentence of this text introduces three mathematical symbols, *F*, *m* and *a*. Each of these symbols are named using linguistic technicality: *F* is force, *m* is mass and *a* is acceleration. By naming these symbols, the text is investing them with technical meaning from the field. By encoding the symbols with instances of technicality in language, the symbols and linguistic technicality in effect become synonymous. The result is that changes in meaning in one semiotic resource, whether language or mathematics, necessarily changes the meaning of the other. For example, when Text 1 specifies that *the force of an object depends on the combination of its mass and acceleration*, language is indicating an unspecified dependency between *force*, *mass* and *acceleration*. As *F*, *m* and *a* have been made synonymous with these terms, this dependency necessarily transfers to the mathematical symbols. Similarly, any meanings built around these symbols in mathematics automatically implicates language. In $F = ma$ the relationship between the three symbols is given more precisely, specifying the dependency mentioned in language. If we were to translate this equation into language, we would say that *F* is proportional to both *m* and *a*. This means crudely that as either of *m* or *a* increases, *F* does too at the same rate. Similarly, the other equation introduced, $a = F/m$, indicates that *a* is inversely proportional to *m*, meaning that if *a* decreases *m* increases and vice-versa. As the sentence following this equation explains: *This shows the acceleration goes up with the force but down with the mass*. In this text, the meanings of each of *force*, *mass* and *acceleration* are now linked to the meanings of the others. This interaction between language and mathematics is a vital first step for mathematics to contribute to the knowledge of physics. Before it can perform the functions it does in later years, it must be invested with meaning in the field of physics.

Even at this early stage, Text 1 shows that full mathematical equations are used. Equations that are used link symbols through sets of relations such as addition (+), multiplication (×) and division (÷). In order to understand this, it is important to now pause briefly and discuss how equations work (i.e. how they link symbols and what type of structure they rely on the build their meanings). This will provide an initial insight into the utility of mathematics when compared to language, but will also give a point of departure for describing the expansion of mathematics in subsequent stages of physics.

Mathematics is organized through sets of relations that are both precise and indefinitely iterative (Doran, 2016). In SFL terms, they show a univariate structure (Halliday, 2015 [1965]). These relations include the arithmetic relations of addition (+), multiplication (×), division (÷) noted above. In the equation $\boldsymbol{F = ma}$, for example, *m* and *a* are linked through multiplication (though the × is elided by convention). The *ma* relation is then linked to *F* through the equals sign =. In the other equation in Text 1, $\boldsymbol{a = F/m}$, *F* is related to *m* through division (/), with both equated to *a*. Through the combination of the arithmetic relations (+, ×, ÷, −) and the choice of the equals sign = (as opposed to, say, *not-equal* ≠, *approximately equal* ≈, *larger than* > etc.) another set of relations arise, namely that *F* is proportional to *m* and *a*, while a is inversely proportional to *m*. As Text 1 describes, this means that holding all other things equal, as *m* or *a* increases, *F* also increases, but as *m* increases *a* decreases. These two relations of proportionality and inverse-proportionality remain the same across both the equations given. In this sense, $\boldsymbol{F = ma}$ and $\boldsymbol{a = F/m}$ convey the same relations using different equations.

For the field of physics, the mathematics thus sets up precise relations between symbols that hold across the entire field. Indeed, when the relations between these symbols change, a change in the field is also signified. For example, $\boldsymbol{F = ma}$ is applicable for classical 'Newtonian' mechanics—which, to put it crudely, involves the study of motion on a scale of size and speed comparable to that we experience in our everyday life. However, when moving to other fields of physics such as special relativity (concerning situations where speeds are close to the speed of light) and quantum mechanics (concerning the workings of very small things) the relations among these symbols are different. For each subfield of physics, relations specified in mathematics constitute one part of the knowledge of the field.

Something that works powerfully for mathematics as far as its role in physics is concerned, is the possibility for indefinite iteration of symbols within equations. $\boldsymbol{F = ma}$ involves a relatively small number of symbols (three: *F, m* and *a*), related by the equals sign = and multiplication. In primary school, equations do not expand much larger than this, but the nature of mathematics is such that each side of the equation could be expanded indefinitely. In later years physics involves equations such as: $\boldsymbol{K + U = \frac{1}{2}mv^2 + \frac{-GMm}{r}}$. In such equations large sets of symbols (in this case eleven) are related in a single equation. The effect of this is that large sets of technical relations can be distilled into small snapshots.

As we have seen, even in this early stage, there is a give and take between language and mathematics in physics. Mathematics gains meaning by being encoded with technical meaning from the field. At the same time it develops meaning by setting up novel and precise relations among symbols that have been given technical meaning.

To conceptualize this burgeoning of meaning, we can enact the concept of *semantic density* from LCT (Maton, 2014). Semantic density is concerned with the degree of condensation of meaning in an item. If an item has more meaning, it is said to have stronger semantic density (SD+); if it has less meaning it has weaker semantic density (SD–). A key metric for determining whether something has stronger or weaker semantic density is its degree of relationality (Maton & Doran, this issue): how many relations the item has with other items in a field. For example when introducing a term such as *energy*, it can be specified that it has subtypes of *potential energy* and *kinetic energy*. This sets up relations of classification between each of the terms, thus increasing their relationality and strengthening their semantic density.

Viewed from this perspective, mathematics as used in primary school physics primarily works to increase the semantic density of physics—to build technical meaning. First, the individual symbols are invested with meaning from technicality in language, e.g. *F* is given the meanings of *force*. This strengthens the semantic density of the mathematical symbols. Beyond this, the symbols are developed in equations, such as in $F = ma$. These equations specify sets of relations between symbols, further strengthening their semantic density. Since these symbols are associated with linguistic technicality, this semantic density is transferred over to the linguistic realm as well. That is, the relations between *F*, *m* and *a* specified in the mathematics transfers back to the relations between their linguistic correlates, *force*, *mass* and *acceleration*. As these meanings constitute part of the field, this interplay between mathematics and language strengthens the semantic density of the field itself.

At primary school, then, mathematics works to extend the semantic density of the field of physics. As mentioned above mathematics is only rarely used at this level. Physics at this stage relies more heavily on language and image to construe its knowledge. It is when moving into junior high school that mathematics comes into its own as a crucial component of physics. Not only is it used to a much larger degree, but the mathematics begins to be developed in specifically mathematical genres. This allows mathematics to function considerably differently to the way it does in primary school.

2.2. Junior High School

Physics in junior high school (years 7-10, ages ~12-16) significantly increases its use of mathematics. While still relatively marginal in comparison to the use of language and images, it takes on new forms that offer new possibilities for construing the knowledge of physics. The mathematics in junior high school presents opportunities to reach toward the empirical world through a genre that will be called the *quantification*. This new genre is the key mathematical innovation for this stage and builds upon the basis for knowledge development introduced in primary school to further enhance the possibilities for knowledge building of physics. As in primary school, mathematical equations and symbols are introduced and named, and thereby invested with technical meaning, which increases the semantic density of the field. An example of this, again involving the equation $F = ma$, is shown in Text 2.

TEXT 2

Junior high school *a=F/m* text (Haire et al., 2000: 118)

Newton's Second Law of Motion describes how the mass on an object affects the way that it moves when acted upon by one or more forces. In symbols, Newton's second law can be expressed as:

$$a = \frac{F}{m}$$

where a = acceleration

F = the total force on the object

m = the mass of the object.

If the total force is measured in newtons (N) and the mass is measured in kilograms (kg), the acceleration can be determined in metres per second squared (m/s^2). This formula describes the observation that larger masses accelerate less rapidly than smaller masses acted on by the same total force. It also describes how a particular object accelerates more rapidly when a larger total force is applied. When all of the forces on an object are balanced, the total force is zero. Newton's second law is often expressed as $F = ma$.

Text 2 again encodes technical meaning in individual symbols: *a* is equated with *acceleration*, *F* with *the total force on the object* and *m* with *the mass of the object*. As well as this, the full equation is named as *Newton's second law*. The text continues to build meaning into the symbols and equation in the final paragraph, strengthening their semantic density.

Moving further into the page in which Text 2 is situated, mathematics is again used. This time, however, the text is not concerned with condensing meaning into the symbolism, but with using the mathematics to calculate the acceleration of a space shuttle taking off, as shown in Text 3(a).

TEXT 3(a)

Junior high school *a=F/m* quantification (Haire et al., 2000: 119)

Newton's second law can be used to estimate the acceleration of the space shuttle at blast off:

$$a = \frac{F}{m}$$
$$= \frac{\text{7 000 000 N upwards}}{\text{2 200 000 kg}}$$
$$= 3.2\ \text{m/s}^2$$

In other words the space shuttle is gaining speed at the rate of only 3.2 m/s (or 11.5 km/h) each second.

Text 3(a) shows an example of a recurrent configuration of mathematics and language that occurs throughout physics—exemplifying the bi-modal genre of the *quantification*. Quantifications aim to produce numerical results that measure a specific instance of the object of study (which for physics is the physical world) and have a relatively consistent structure along the lines of that shown below by Text 3(b). The initial language and opening mathematical equation $a = \frac{F}{m}$ together realize a stage called the Situation. This stage orients the text to the situation it is calculating (in this case the acceleration of the space shuttle) as well as specifying the equation to be used. The following line, $= \frac{\text{7 000 000 N upwards}}{\text{2 200 000 kg}}$, realizes the stage Substitution. This involves the replacement of the symbols in the previous line (*m* and *F*) with numbers (7 000 000 and 2 200 000 with the *N* and *kg* giving the units of measurement of force and mass respectively, and *upwards* specifying the direction of the force). In the next line the mathematics provides the Numerical Result, $=3.2\ \text{m/s}^2$, completing the calculation. Finally, a stage I have named the Physical Conclusion reinterprets the Numerical Result linguistically, where it is related back to the acceleration of the space shuttle specified in the opening stage. The entire genre interprets a specific situation numerically and in doing so, relates the abstracted theory shown by the symbolic equation to a specific empirical situation.

The introduction of quantifications is the key innovation that distinguishes the role of mathematics in primary and junior high school physics. To understand the role of quantifications in physics, we can introduce a second concept from the Semantics dimension of LCT: *semantic gravity*. Semantic gravity is concerned with the degree to which meanings are dependent on their context. Stronger semantic gravity (SG+) indicates meanings are less dependent on their context, whereas weaker semantic gravity (SG–) indicates greater context-dependence. Semantic gravity and semantic density are independent variables that allow an understanding of how meanings vary in their relations to other meanings and to their context.

TEXT 3(b)

Genre structure of junior high school *a=F/m* quantification

Newton's second law can be used to estimate the acceleration of the space shuttle at blast off: $a = \frac{F}{m}$	**Situation**
$= \frac{7\ 000\ 000 \text{ N upwards}}{2\ 200\ 000 \text{ kg}}$	**Substitution**
$= 3.2 \text{ m/s}^2$	**Numerical Result**
In other words the space shuttle is gaining speed at the rate of only 3.2 m/s (or 11.5 km/h) each second.	**Physical Conclusion**

Mathematical equations that do not involve numbers are not tied to any particular physical context. $F = ma$, for example, describes an abstract set of relations that hold for a very large set of situations—essentially all physical situations that can occur in our everyday life. The equation does not, however, address any particular situation. It does not, for example, say how much force, acceleration or mass will occur at any particular situation, rather it simply shows their generalized relations. The equation, then, is characterized by relatively weak semantic gravity.

On the other hand, the final equation in numerical form, $(a) = 3.2\frac{\text{m}}{\text{s}^2}$, very precisely describes a specific situation2. The numerical form of the equation does not mention anything about the generalized relationships between force, acceleration and mass, but measures a specific instance of acceleration. It is thereby characterized by relatively strong semantic gravity. The quantification genre thus involves a shift from weaker to stronger semantic gravity; it is a tool for gravitation—for strengthening semantic gravity (Maton, 2014: 129). This allows physics to keep in touch with its object of study.

In junior high school physics, mathematics continues to strengthen semantic density by specifying equations and condensing them with technical meaning from language. At the same time, mathematics' role in quantifications allows physics to strengthen its semantic

2 In the text, the 'a' on the left side of the equation is elided. This is typical of mathematical texts where the left side (the Theme) has been specified in the previous equation (Doran, 2016).

gravity by reaching out to specific empirical situations. Both the use of quantifications and the encoding of technical meaning from language continues into senior high school. There is also a further innovation, discussed below, that highlights the increasing role of mathematics in building knowledge in physics.

2.3. Senior High School

Senior high school (years 11-12, ages ~16-18) physics continues the trend of increasing reliance on mathematics. By this stage, mathematics is a crucial component of the high-stakes assessment in physics. Students must not only read mathematics as it is used in classrooms, textbooks and assessment, and in doing so, gain technical physical meaning, they must also produce mathematics as a means to solving physical problems. In senior high school, the forms of mathematics used in junior high school are consolidated, expanded and built upon. In terms of sheer quantity, there is an enormous increase in the number of equations introduced. Text 4 shows a snapshot of slightly over a quarter of the forty-eight equations specified in the formula sheet of the final state-wide exam of high-school physics.

TEXT 4

Formula sheet for final senior high school physics exam (Board of Studies, Teaching & Educational Standards NSW, 2014: 42)

$$v_{av} = \frac{\Delta r}{\Delta t}$$

$$a_{av} = \frac{\Delta v}{\Delta t} \quad \text{therefore} \quad a_{av} = \frac{v - u}{t}$$

$$\Sigma F = ma$$

$$F = \frac{mv^2}{r}$$

$$E_k = \frac{1}{2}mv^2$$

$$W = Fs$$

$$p = mv$$

$$\text{Impulse} = Ft$$

$$F = \frac{Gm_1 m_2}{d^2}$$

$$E = mc^2$$

$$l_v = l_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m_v = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Each of the equations in this formula sheet must be understood by students to be successful in assessment. Although the equations themselves are given, there is no explication of what they mean, to which situations they apply or how to use them. They are technical equations that students need to understand in relation to the broader field. These equations build a very large complex of relations among technical meanings. The increase in equations arguably accelerates in future years, with a larger part of the technical meaning of physics organized through the mathematics deployed in the field.

Complementing the increase in the use of mathematics, the complexity of quantifications also increases. In junior high school, it is typical for single quantifications to occur in isolation, but in senior high school it is common for larger strings of quantifications to occur that aim to produce a single result. Following Martin (2015 [1994]) these strings of quantifications comprise larger *quantification complexes*. Text 5 shows an example of this from a senior high school textbook. Each of the three quantifications is framed with dotted boxes.

TEXT 5

Senior high school quantification complex (De Jong et al. 1990: 249; dotted boxes added)

On the earth:

$$\begin{aligned} \boldsymbol{W}_{\text{earth}} &= \boldsymbol{m}\boldsymbol{g}_{\text{earth}} \\ &= 50 \times 9.8 \\ &= 490 \text{ N downwards} \end{aligned}$$

On Mars:

$$\begin{aligned} \boldsymbol{W}_{\text{Mars}} &= \boldsymbol{m}\boldsymbol{g}_{\text{Mars}} \\ &= 50 \times 3.6 \\ &= 180 \text{ N downwards} \end{aligned}$$

Loss of weight = 490 – 180 = 310 N. But there is no loss of mass!

The quantifications in this text all work toward achieving a single final result. The aim of the text is to determine the loss of weight of a girl moving from Earth to Mars. The final quantification in the bottom box calculates this as 310 N. The loss of weight is the weight on Mars subtracted from the weight on Earth (shown as 490 – 180 in the final quantification). In order to do this calculation, however, the text must first work through the weight on both Earth and Mars (shown by $\boldsymbol{W}_{\text{earth}}$ and $\boldsymbol{W}_{\text{Mars}}$ in the top two boxes). Using values given in the previous co-text (not shown), calculating the weight on Earth and weight on Mars each require their own quantification. In sum, three quantifications are needed to achieve the final result.

The importance of quantification complexes such as the one in Text 5 is that they allow physics to calculate numerical results from relatively distant starting points. In single quantifications numerical values must be available for every symbol other than the one being calculated. For example, in Text 3, discussed in relation to junior high school, the aim was to calculate *a* (acceleration) from the equation $\boldsymbol{a} = \frac{\boldsymbol{F}}{\boldsymbol{m}}$. The numerical values of both *F* and *m* were known from the previous co-text, leaving only *a* to be determined. This allowed *a* to be calculated with a single quantification. In Text 5 from senior high school, on the other hand, calculation of the loss of weight required that both the weight on Earth and the weight on Mars be known. As indicated above, these were not specified in the text and so required calculation through other quantifications. Each of these quantifications used the formula: $\boldsymbol{W}=m\boldsymbol{g}$. The symbols *m* (mass) and *g* (gravity on earth or Mars) were both known from the previous co-text allowing the weight on both Mars and Earth to be calculated. So based on the previously differentiated knowledge of the mass and gravity on both Mars and Earth, a sequence of quantifications could be used to calculate the loss of weight.

With regard to the knowledge of physics, the advent of quantification complexes in senior high school builds upon the role of single quantifications used in junior high school. Single quantifications allowed physics to reach from generalized theory to specific empirical situations. Based on single quantifications, however, empirical situations could be explored only if a relatively specific set of numerical knowledge was available. Quantification complexes, on the other hand, allow a larger range of possible starting points to be used to calculate specific situations. This allows knowledge that is further removed from the empirical object of study to be put to use. As the number of quantifications that can occur in a complex is in principle indefinite, this complexing provides a powerful tool for physics to reach toward its object of study from very distant starting points.

As well as the introduction of quantification complexes and a greater reliance on mathematics, senior high school sees another use of mathematics come to prominence. New types of text appear that are concerned not with measuring specific empirical instances, but rather with developing new mathematical relations. Text 6(a) shows an example of this from a senior high school textbook.

TEXT 6

Senior high school derivation (Warren, 2000: 123)

$$\vec{F} = m\vec{a}$$

$$\text{But } \vec{a} = \frac{\vec{v} - \vec{u}}{\Delta t} \text{, therefore}$$

$$\vec{F} = \frac{m(\vec{v} - \vec{u})}{\Delta t}$$

$$= \frac{m\vec{v} - m\vec{u}}{\Delta t}$$

$$= \frac{\Delta m\vec{v}}{\Delta t}$$

Force is the time rate of change of momentum as stated by Newton!

As in quantifications, texts such as this are recurrent configurations of mathematics and language that realize a distinct genre. We will call this genre a *derivation*. Derivations aim not to find a numerical result, but to produce a new symbolic equation. In doing so, they make explicit relations in the field that may not have previously been specified. Derivations have a relatively consistent structure exemplified below by Text 6(b). The opening two lines provide the initial equations that by this stage of the text are technical and well-known to students. As in quantifications, they provide the initial Situation upon which the rest of the text is based. Following this, the two lines, $\vec{F} = \frac{m(\vec{v}-\vec{u})}{\Delta t}$ and $= \frac{m\vec{v}-m\vec{u}}{\Delta t}$ reorganize the equations given in the Situation in a stage called the Reorganization. In this case, the $\frac{\vec{v}-\vec{u}}{\Delta t}$ of the second line is substituted for the a $\vec{a}$ of the first line, producing $\vec{F} = \frac{m(\vec{v}-\vec{u})}{\Delta t}$. The final line of mathematics gives the Symbolic Result, again in symbolic form. The Reorganization and Symbolic Result stages indicate the difference between quantifications and derivations. Whereas quantifications insert numbers after the Situation, derivations remain in symbolic form. Thus the derivation works to produce a new equation, which is then reinterpreted in language within the Physical Conclusion. The structure of this derivation is shown below in Text 6(b).

TEXT 6(b)

Structure of a senior high school derivation

$\vec{F} = m\vec{a}$ But $\vec{a} = \frac{\vec{v}-\vec{u}}{\Delta t}$, therefore	**Situation**
$\vec{F} = \frac{m(\vec{v}-\vec{u})}{\Delta t}$ $= \frac{m\vec{v}-m\vec{u}}{\Delta t}$	**Reorganization**
$= \frac{\Delta m\vec{v}}{\Delta t}$	**Symbolic Result**
Force is the time rate of change of momentum as stated by Newton!	**Physical Conclusion**

The new equation developed in the derivation makes explicit relations between symbols implied but not yet specified in the field. Derivations are thus deployed to deepen the technical knowledge of physics. They develop and specify new sets of relations that become part of the field. These new relations can in turn be used in quantifications to extend the range of empirical situations accounted for by physics. Derivations are thus used not just to build new mathematical relations, but also to contribute to the development of new linguistic technicality. Text 7, which follows immediately after the excerpt shown in Text 6, provides an example of this.

TEXT 7

Impulse derivation (Warren, 2000: 123; boxes in original)

IMPULSE

As we have just seen, we can write Newton's Second Law as:

$$\vec{F} = \frac{(\Delta m\vec{v})}{\Delta t}$$

Rearranging this equation we can write:

$$\vec{F}\Delta t = \Delta(m\vec{v})$$

Impulse $(I = \vec{F}\Delta t)$ *is equal to the change in momentum of the object upon which the force is applied.*

Text 7 begins with another short derivation. The opening equation $\vec{F} = \frac{(\Delta m\vec{v})}{\Delta t}$ is taken from the Result of the previous equation. This is then reorganized to produce the final Result shown in the box: $\vec{F}\Delta t = \Delta(m\vec{v})$. Crucially for our discussion, the final Physical Conclusion reinterprets this Result in language, and in doing so, introduces a new piece of linguistic technicality, *Impulse*. Impulse is used to name one of the relations developed in the derivation $(\vec{F}\Delta t)$, and is immediately elaborated linguistically in relation to other technical terms (i.e. *change in momentum* and *force*). The derivations used in Texts 6 and 7 have not only developed new relations within language but also engendered new linguistic technicality.

Derivations develop new equations by making relations which are otherwise implied in the field explicit. With the growth of technical symbols and equations, a large combinatorial potential arises. Each symbol carries around a large set of implied relations that can be brought to bear in any particular situation. For example, as discussed throughout, the equation $F = ma$ specifies relations between *F*, *m* and *a*. These relations remain even when one of the symbols is mentioned without the others, such as for *F* in $F = \frac{E_k}{s}$ (E_k is glossed as kinetic energy, *s* is displacement). These two equations, $F = ma$ and $F = \frac{E_k}{s}$, set up relations between *F* and *ma*, and *F* and $\frac{E_k}{s}$ respectively. As both sets of relations hold at the same time, relations between *ma* and $\frac{E_k}{s}$ can also be specified as $ma = \frac{E_k}{s}$, or rearranged, $E_k = mas$. Derivations bring implicit relations between symbols into actuality.

We can again interpret this in terms of the LCT dimension of Semantics. Derivations are tools that make explicit new relations, and lay a platform for the introduction of new linguistic technicality. In this way, they work to build meaning in the field. Whereas in earlier years, mathematics tends to encode technicality developed in language, in senior high school derivations build relations which have not yet been specified. Derivations thus strengthen the semantic density of the field; that is, they are a tool for epistemological condensation (Maton, 2014; Maton & Doran, this issue). This condensation role is particularly powerful. It allows mathematics to make explicit relations not previously known and to push into the new areas, thereby expanding the horizons of knowledge. When used in conjunction with quantifications, this new knowledge can be tested to see how usefully it construes the empirical world.

Based on this understanding of derivations, we can now review them in relation to the increasing use of mathematics, and its relation to language. As we saw in primary school (and continued through junior and senior high school), language initially works to invest mathematics with technical meaning of physics. Drawing on this investment, derivations can then produce new relations which have not previously been made explicit. The relations in mathematics, and the symbols involved in them, can then be named in language. This transfers the meanings developed in mathematics back to language, which can in turn utilize its own ways of meaning- making. By handing meaning back and forth in this way, mathematics and language work in tandem to considerably strengthen the semantic density of the field.

3. Mathematics in the Knowledge Structure of Physics

The survey of mathematics in physics schooling undertaken here reveals its powerful utility. Through its interaction with language, each symbol can garner technical meaning, which can then be related to an indefinite number of symbols in a single snapshot. When used in derivations, physics can employ the large combinatory potential of mathematics to bring together relations in the field that have not yet been specified, and in doing so develop new knowledge in the field. When used in quantifications, mathematics can use these relations to account for specific instances in the real world. This opens the possibility for theory to be tested against empirical data and the physical world to be predicted.

In LCT terms, the mathematics used in physics is a tool for both condensation (strengthening semantic density) and gravitation (strengthening semantic gravity). This allows meanings to be related and proliferated in a large number of combinations, while maintaining the capacity to connect with the empirical world. Without the strong potential for condensation, technical meanings would have a limited possibility for combination, meaning they would be tied to their contexts and become segmentalized. Physics would thus lose the potential to develop generalizable theories that account for a broad range of phenomena. In contrast, without the possibility for gravitation, physics would have no capacity to reconnect with its object of study; there would be no counter-balance to ensure the proliferation of theory maintains relevance in the study of the physical world. The condensation and gravitation shown by mathematics arise from the two genres discussed in this paper. On the one hand, derivations strengthen the semantic density of both individual symbols and the field. The semantic density developed in mathematics can be condensed into language and vice versa, allowing each semiotic resource to utilize its own meaning-making resources. Mathematics thus provides a platform for sciences' hierarchical knowledge structure by incorporating a tool for creating general propositions and theories, and integrating knowledge across a range of phenomena (Bernstein, 1999). On the other hand, quantifications strengthen the semantic gravity of a text and give physics the ability to link abstracted theory to specific instances. This expanded semantic range gives an avenue for physics to strengthen its epistemic relations between its knowledge and its object of study (Maton, 2014). Through mathematics, theory can be tested by data, and data can be predicted by theory.

So we can now return to the original question: why is mathematics used in science? Mathematics is used because it provides tools for both theoretical development and for bringing theory to bear on data. It is thus an instrument for expanding the frontiers of knowledge and for keeping that knowledge in touch with the empirical world.

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